ICE CSIC IEEC

Cosmological Casimir effect and beyond

EMILIO ELIZALDE

ICE/CSIC & IEEC, UAB, Barcelona

web: google → emilio elizalde

DSU2006, UAM, Madrid, June 20, 2006

Outline of the talk

- Zeta Function $\zeta_A(s)$ and Determinant $\det_\zeta A$

Outline of the talk

- Zeta Function $\zeta_A(s)$ and Determinant $\det_\zeta A$
- The Wodzicki Residue

Outline of the talk

- Zeta Function $\zeta_A(s)$ and Determinant $\det_\zeta A$
- The Wodzicki Residue
- Multiplicative (or Noncommutative) Anomaly

Outline of the talk

- Zeta Function $\zeta_A(s)$ and Determinant $\det_\zeta A$
- The Wodzicki Residue
- Multiplicative (or Noncommutative) Anomaly
- Vacuum Fluctuations and the Cosmol Const

Outline of the talk

- Zeta Function $\zeta_A(s)$ and Determinant $\det_\zeta A$
- The Wodzicki Residue
- Multiplicative (or Noncommutative) Anomaly
- Vacuum Fluctuations and the Cosmol Const
- New CC Theories & Cosmo-topol Casimir Effect

Outline of the talk

- Zeta Function $\zeta_A(s)$ and Determinant $\det_\zeta A$
- The Wodzicki Residue
- Multiplicative (or Noncommutative) Anomaly
- Vacuum Fluctuations and the Cosmol Const
- New CC Theories & Cosmo-topol Casimir Effect
- A Simple Model with large & small dimensions

Outline of the talk

- Zeta Function $\zeta_A(s)$ and Determinant $\det_\zeta A$
- The Wodzicki Residue
- Multiplicative (or Noncommutative) Anomaly
- Vacuum Fluctuations and the Cosmol Const
- New CC Theories & Cosmo-topol Casimir Effect
- A Simple Model with large & small dimensions
- Realistic Models: – dS & AdS Braneworlds
  – Supergraviton Theories

Outline of the talk

- Zeta Function $\zeta_A(s)$ and Determinant $\det_\zeta A$
- The Wodzicki Residue
- Multiplicative (or Noncommutative) Anomaly
- Vacuum Fluctuations and the Cosmol Const
- New CC Theories & Cosmo-topol Casimir Effect
- A Simple Model with large & small dimensions
- Realistic Models: – dS & AdS Braneworlds
  – Supergraviton Theories
- Conclusions

Existence of ζ_A for A a ΨDO

- A a positive-definite elliptic ΨDO of positive order $m \in \mathbb{R}$
- A acts on the space of smooth sections of
- E, n-dim vector bundle over
- M closed n-dim manifold

Existence of ζ_A for A a ΨDO

– A a positive-definite elliptic ΨDO of positive order $m \in \mathbb{R}$

– A acts on the space of smooth sections of

– E, n-dim vector bundle over

– M closed n-dim manifold

(a) The zeta function is defined as $\zeta_A(s) = \text{tr}\, A^{-s} = \sum_j \lambda_j^{-s}$, $\quad \text{Re}\, s > \frac{n}{m} \equiv s_0$

$\{\lambda_j\}$ ordered spect of A, $s_0 = \dim M / \text{ord}\, A$ abscissa of converg of $\zeta_A(s)$

Existence of ζ_A for A a ΨDO

– A a positive-definite elliptic ΨDO of positive order $m \in \mathbb{R}$

– A acts on the space of smooth sections of

– E, n-dim vector bundle over

– M closed n-dim manifold

(a) The zeta function is defined as $\zeta_A(s) = \text{tr}\, A^{-s} = \sum_j \lambda_j^{-s}, \quad \text{Re}\, s > \frac{n}{m} \equiv s_0$

$\{\lambda_j\}$ ordered spect of A, $s_0 = \dim M / \text{ord}\, A$ abscissa of converg of $\zeta_A(s)$

(b) $\zeta_A(s)$ has a meromorphic continuation to the whole complex plane $\mathbb{C}$ (regular at $s = 0$), provided the principal symbol of A, $a_m(x,\xi)$, admits a spectral cut: $L_\theta = \{\lambda \in \mathbb{C} \,|\, \text{Arg}\, \lambda = \theta, \theta_1 < \theta < \theta_2\}$, Spec $A \cap L_\theta = \emptyset$

(Agmon-Nirenberg condition)

Existence of ζ_A for A a ΨDO

– A a positive-definite elliptic ΨDO of positive order $m \in \mathbb{R}$

– A acts on the space of smooth sections of

– E, n-dim vector bundle over

– M closed n-dim manifold

(a) The zeta function is defined as $\zeta_A(s) = \text{tr } A^{-s} = \sum_j \lambda_j^{-s}, \quad \text{Re } s > \frac{n}{m} \equiv s_0$

$\{\lambda_j\}$ ordered spect of A, $s_0 = \dim M/\text{ord } A$ abscissa of converg of $\zeta_A(s)$

(b) $\zeta_A(s)$ has a meromorphic continuation to the whole complex plane $\mathbb{C}$ (regular at $s = 0$), provided the principal symbol of A, $a_m(x, \xi)$, admits a spectral cut: $L_\theta = \{\lambda \in \mathbb{C} \,|\, \text{Arg } \lambda = \theta, \theta_1 < \theta < \theta_2\}$, Spec $A \cap L_\theta = \emptyset$

(Agmon-Nirenberg condition)

(c) The definition of $\zeta_A(s)$ depends on the position of the cut L_θ

Existence of ζ_A for A a ΨDO

- A a positive-definite elliptic ΨDO of positive order $m \in \mathbb{R}$
- A acts on the space of smooth sections of
- E, n-dim vector bundle over
- M closed n-dim manifold

(a) The zeta function is defined as $\zeta_A(s) = \text{tr } A^{-s} = \sum_j \lambda_j^{-s}, \quad \text{Re}\, s > \frac{n}{m} \equiv s_0$

$\{\lambda_j\}$ ordered spect of A, $s_0 = \dim M / \text{ord}\, A$ abscissa of converg of $\zeta_A(s)$

(b) $\zeta_A(s)$ has a meromorphic continuation to the whole complex plane $\mathbb{C}$ (regular at $s = 0$), provided the principal symbol of A, $a_m(x, \xi)$, admits a spectral cut: $L_\theta = \{\lambda \in \mathbb{C} \,|\, \text{Arg}\, \lambda = \theta, \theta_1 < \theta < \theta_2\}$, Spec $A \cap L_\theta = \emptyset$

(Agmon-Nirenberg condition)

(c) The definition of $\zeta_A(s)$ depends on the position of the cut L_θ

(d) The only possible singularities of $\zeta_A(s)$ are simple poles at

$$s_k = (n-k)/m, \qquad k = 0, 1, 2, \ldots, n-1, n+1, \ldots$$

Definition of Determinant

H ΨDO operator $\{\varphi_i, \lambda_i\}$ spectral decomposition

Definition of Determinant

H ΨDO operator $\{\varphi_i, \lambda_i\}$ spectral decomposition

$\prod_{i \in I} \lambda_i$?! $\ln \prod_{i \in I} \lambda_i = \sum_{i \in I} \ln \lambda_i$

Definition of Determinant

H ΨDO operator $\{\varphi_i, \lambda_i\}$ spectral decomposition

$\prod_{i\in I} \lambda_i$?! $\ln \prod_{i\in I} \lambda_i = \sum_{i\in I} \ln \lambda_i$

Riemann zeta func: $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$, $Re\, s > 1$ (& analytic cont)

$\Longrightarrow$ Definition: zeta function of H $\zeta_H(s) = \sum_{i\in I} \lambda_i^{-s} = \text{tr}\, H^{-s}$

As Mellin transform: $\zeta_H(s) = \frac{1}{\Gamma(s)} \int_0^\infty dt\, t^{s-1}\, \text{tr}\, e^{-tH}$, $Re\, s > D/2$

Derivative: $\zeta_H'(0) = -\sum_{i\in I} \ln \lambda_i$

Definition of Determinant

H ΨDO operator $\{\varphi_i, \lambda_i\}$ spectral decomposition

$\prod_{i\in I} \lambda_i$?! $\ln \prod_{i\in I} \lambda_i = \sum_{i\in I} \ln \lambda_i$

Riemann zeta func: $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}, \ Re\, s > 1$ (& analytic cont)

$\Longrightarrow$ Definition: zeta function of H $\zeta_H(s) = \sum_{i\in I} \lambda_i^{-s} = \text{tr}\, H^{-s}$

As Mellin transform: $\zeta_H(s) = \frac{1}{\Gamma(s)} \int_0^{\infty} dt\, t^{s-1}\, \text{tr}\, e^{-tH}, \quad Re\, s > D/2$

Derivative: $\zeta'_H(0) = -\sum_{i\in I} \ln \lambda_i$

$\Longrightarrow$ Determinant: Ray & Singer, '67

$$\det_\zeta H = \exp\left[-\zeta'_H(0)\right]$$

Definition of Determinant

H ΨDO operator $\{\varphi_i, \lambda_i\}$ spectral decomposition

$\prod_{i\in I} \lambda_i$?! $\ln \prod_{i\in I} \lambda_i = \sum_{i\in I} \ln \lambda_i$

Riemann zeta func: $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}, \; Re\, s > 1$ (& analytic cont)

$\Longrightarrow$ Definition: zeta function of H $\zeta_H(s) = \sum_{i\in I} \lambda_i^{-s} = \text{tr}\, H^{-s}$

As Mellin transform: $\zeta_H(s) = \frac{1}{\Gamma(s)} \int_0^\infty dt\, t^{s-1}\, \text{tr}\, e^{-tH}, \quad Re\, s > D/2$

Derivative: $\zeta'_H(0) = -\sum_{i\in I} \ln \lambda_i$

$\Longrightarrow$ Determinant: Ray & Singer, '67

$$\det_\zeta H = \exp\left[-\zeta'_H(0)\right]$$

Weierstrass definition: subtract leading behavior of λ_i in i, as $i \to \infty$, until the series $\sum_{i\in I} \ln \lambda_i$ converges

$\Longrightarrow$ non-local counterterms !!

Properties

- The definition of the determinant $\det_\zeta A$ only depends on the homotopy class of the cut

Properties

- The definition of the determinant $\det_\zeta A$ only depends on the homotopy class of the cut
- A zeta function (and corresponding determinant) with the same meromorphic structure in the complex s-plane and extending the ordinary definition to operators of complex order $m \in \mathbb{C} \backslash \mathbb{Z}$ (they do not admit spectral cuts), has been obtained [Kontsevich and Vishik]

Properties

- The definition of the determinant $\det_\zeta A$ only depends on the homotopy class of the cut
- A zeta function (and corresponding determinant) with the same meromorphic structure in the complex s-plane and extending the ordinary definition to operators of complex order $m \in \mathbb{C}\backslash\mathbb{Z}$ (they do not admit spectral cuts), has been obtained [Kontsevich and Vishik]
- Asymptotic expansion for the heat kernel

Properties

- The definition of the determinant $\det_\zeta A$ only depends on the homotopy class of the cut

- A zeta function (and corresponding determinant) with the same meromorphic structure in the complex s-plane and extending the ordinary definition to operators of complex order $m \in \mathbb{C} \backslash \mathbb{Z}$ (they do not admit spectral cuts), has been obtained [Kontsevich and Vishik]

- Asymptotic expansion for the heat kernel

$$\text{tr}\, e^{-tA} = \sum'_{\lambda \in \text{Spec}\, A} e^{-t\lambda}$$

$$\sim \alpha_n(A) + \sum_{n \neq j \geq 0} \alpha_j(A) t^{-s_j} + \sum_{k \geq 1} \beta_k(A) t^k \ln t, \quad t \downarrow 0$$

$$\alpha_n(A) = \zeta_A(0), \quad \alpha_j(A) = \Gamma(s_j)\, \text{Res}_{s=s_j} \zeta_A(s), \; s_j \notin -\mathbb{N}$$

$$\alpha_j(A) = \frac{(-1)^k}{k!} \left[\text{PP}\, \zeta_A(-k) + \psi(k+1)\, \text{Res}_{s=-k} \zeta_A(s)\right], \quad s_j = -k, \; k \in \mathbb{N}$$

$$\beta_k(A) = \frac{(-1)^{k+1}}{k!}\, \text{Res}_{s=-k} \zeta_A(s), \quad k \in \mathbb{N} \backslash \{0\}$$

$$\text{PP}\, \phi = \lim_{s \to p} \left[\phi(s) - \frac{\text{Res}_{s=p}\, \phi(s)}{s-p}\right]$$

The Wodzicki Residue

- The Wodzicki (or noncommutative) residue is the only extension of the Dixmier trace to ΨDOs which are not in $\mathcal{L}^{(1,\infty)}$

The Wodzicki Residue

- The Wodzicki (or noncommutative) residue is the only extension of the Dixmier trace to ΨDOs which are not in $\mathcal{L}^{(1,\infty)}$
- Only trace one can define in the algebra of ΨDOs (up to multipl const)

The Wodzicki Residue

- The Wodzicki (or noncommutative) residue is the only extension of the Dixmier trace to ΨDOs which are not in $\mathcal{L}^{(1,\infty)}$
- Only trace one can define in the algebra of ΨDOs (up to multipl const)
- Definition: $\text{res}\, A = 2\, \text{Res}_{s=0}\, \text{tr}\, (A\Delta^{-s})$, Δ Laplacian

The Wodzicki Residue

- The Wodzicki (or noncommutative) residue is the only extension of the Dixmier trace to ΨDOs which are not in $\mathcal{L}^{(1,\infty)}$
- Only trace one can define in the algebra of ΨDOs (up to multipl const)
- Definition: $\text{res}\, A = 2\, \text{Res}_{s=0}\, \text{tr}\, (A\Delta^{-s})$, Δ Laplacian
- Satisfies the trace condition: $\text{res}\,(AB) = \text{res}\,(BA)$

The Wodzicki Residue

- The Wodzicki (or noncommutative) residue is the only extension of the Dixmier trace to ΨDOs which are not in $\mathcal{L}^{(1,\infty)}$
- Only trace one can define in the algebra of ΨDOs (up to multipl const)
- Definition: $\text{res}\, A = 2\, \text{Res}_{s=0}\, \text{tr}\, (A\Delta^{-s}),$ Δ Laplacian
- Satisfies the trace condition: $\text{res}\,(AB) = \text{res}\,(BA)$
- Important ! it can be expressed as an integral (local form)

$$\text{res}\, A = \int_{S^*M} \text{tr}\, a_n(x,\xi)\, d\xi$$

with $S^*M \subset T^*M$ the co-sphere bundle on M (some authors put a coefficient in front of the integral: Adler-Manin residue)

The Wodzicki Residue

- The Wodzicki (or noncommutative) residue is the only extension of the Dixmier trace to ΨDOs which are not in $\mathcal{L}^{(1,\infty)}$
- Only trace one can define in the algebra of ΨDOs (up to multipl const)
- Definition: $\text{res}\, A = 2\, \text{Res}_{s=0}\, \text{tr}\,(A\Delta^{-s})$, Δ Laplacian
- Satisfies the trace condition: $\text{res}\,(AB) = \text{res}\,(BA)$
- Important ! it can be expressed as an integral (local form)

  $$\text{res}\, A = \int_{S^*M} \text{tr}\, a_n(x,\xi)\, d\xi$$

  with $S^*M \subset T^*M$ the co-sphere bundle on M (some authors put a coefficient in front of the integral: Adler-Manin residue)
- If $\dim M = n = -\,\text{ord}\, A$ (M compact Riemann, A elliptic, $n \in \mathbb{N}$) it coincides with the Dixmier trace, and $\text{Res}_{s=1}\zeta_A(s) = \frac{1}{n}\,\text{res}\, A^{-1}$

The Wodzicki Residue

- The Wodzicki (or noncommutative) residue is the only extension of the Dixmier trace to ΨDOs which are not in $\mathcal{L}^{(1,\infty)}$
- Only trace one can define in the algebra of ΨDOs (up to multipl const)
- Definition: $\text{res}\, A = 2\, \text{Res}_{s=0}\, \text{tr}\,(A\Delta^{-s})$, Δ Laplacian
- Satisfies the trace condition: $\text{res}\,(AB) = \text{res}\,(BA)$
- Important ! it can be expressed as an integral (local form)

$$\text{res}\, A = \int_{S^*M} \text{tr}\, a_n(x,\xi)\, d\xi$$

  with $S^*M \subset T^*M$ the co-sphere bundle on M (some authors put a coefficient in front of the integral: Adler-Manin residue)

- If $\dim M = n = -\,\text{ord}\, A$ (M compact Riemann, A elliptic, $n \in \mathbb{N}$) it coincides with the Dixmier trace, and $\text{Res}_{s=1}\zeta_A(s) = \frac{1}{n}\,\text{res}\, A^{-1}$
- The Wodzicki res makes sense for ΨDOs of arbitrary order. Even if symbols $a_j(x,\xi)$, $j < m$, are not coordinate invariant, the integral is, and defines a trace

Multiplicative Anomaly (or Defect)

- Given A, B, and AB ψDOs, even if ζ_A, ζ_B, and ζ_{AB} exist, it turns out that, in general,

$$\det_\zeta(AB) \neq \det_\zeta A \, \det_\zeta B$$

Multiplicative Anomaly (or Defect)

- Given A, B, and AB ψDOs, even if ζ_A, ζ_B, and ζ_{AB} exist, it turns out that, in general,

$$\det{}_\zeta(AB) \neq \det{}_\zeta A \, \det{}_\zeta B$$

- The multiplicative (or noncommutative) anomaly (or defect) is defined as

$$\delta(A,B) = \ln \left[\frac{\det_\zeta(AB)}{\det_\zeta A \ \det_\zeta B} \right] = -\zeta'_{AB}(0) + \zeta'_A(0) + \zeta'_B(0)$$

Multiplicative Anomaly (or Defect)

- Given A, B, and AB ψDOs, even if ζ_A, ζ_B, and ζ_{AB} exist, it turns out that, in general,

$$\det{}_\zeta(AB) \neq \det{}_\zeta A \, \det{}_\zeta B$$

- The multiplicative (or noncommutative) anomaly (or defect) is defined as

$$\delta(A,B) = \ln\left[\frac{\det_\zeta(AB)}{\det_\zeta A \;\det_\zeta B}\right] = -\zeta'_{AB}(0) + \zeta'_A(0) + \zeta'_B(0)$$

- Wodzicki formula

$$\delta(A,B) = \frac{\text{res}\left\{[\ln \sigma(A,B)]^2\right\}}{2\,\text{ord}\,A\;\text{ord}\,B\,(\text{ord}\,A + \text{ord}\,B)}$$

where $\quad \sigma(A,B) = A^{\text{ord}\,B} B^{-\text{ord}\,A}$

Consequences of the mult. anom.

$$\mathcal{L}(x) = -\frac{1}{2}\partial_\mu\Phi(x)\partial_\mu\Phi(x) - \frac{1}{2}m_1^2\Phi(x)^2 + V_1(\Phi(x))$$
$$-\frac{1}{2}\partial_\mu\Xi(x)\partial_\mu\Xi(x) - \frac{1}{2}m_2^2\Xi(x)^2 + V_2(\Xi(x)) + V_3(\Psi(x),\Xi(x))$$

Consequences of the mult. anom.

$$\begin{aligned}\mathcal{L}(x) &= -\frac{1}{2}\partial_\mu\Phi(x)\partial_\mu\Phi(x) - \frac{1}{2}m_1^2\Phi(x)^2 + V_1(\Phi(x)) \\ &\quad -\frac{1}{2}\partial_\mu\Xi(x)\partial_\mu\Xi(x) - \frac{1}{2}m_2^2\Xi(x)^2 + V_2(\Xi(x)) + V_3(\Psi(x),\Xi(x))\end{aligned}$$

Partition function

$$\begin{aligned}e^{-Z} &= \int D\Phi\, D\Xi\, \exp\left\{-\int d^{d+1}x\, \left[\frac{1}{2}\Phi(-\partial^2 + m_1^2)\Phi + \frac{1}{2}\Xi(-\partial^2 + m_2^2)\Xi\right.\right. \\ &\qquad \left.\left. -V_1(\Phi) - V_2(\Xi) - V_3(\Phi,\Xi)\right]\right\}\end{aligned}$$

where the x-dependence is no more explicitly depicted (for simplification)

Consequences of the mult. anom.

$$\begin{aligned}\mathcal{L}(x) &= -\frac{1}{2}\partial_\mu\Phi(x)\partial_\mu\Phi(x) - \frac{1}{2}m_1^2\Phi(x)^2 + V_1(\Phi(x)) \\ &\quad -\frac{1}{2}\partial_\mu\Xi(x)\partial_\mu\Xi(x) - \frac{1}{2}m_2^2\Xi(x)^2 + V_2(\Xi(x)) + V_3(\Psi(x),\Xi(x))\end{aligned}$$

Partition function

$$\begin{aligned}e^{-Z} &= \int D\Phi\, D\Xi\, \exp\left\{-\int d^{d+1}x\, \left[\frac{1}{2}\Phi(-\partial^2+m_1^2)\Phi + \frac{1}{2}\Xi(-\partial^2+m_2^2)\Xi \right.\right. \\ &\qquad \left.\left. -V_1(\Phi) - V_2(\Xi) - V_3(\Phi,\Xi)\right]\right\}\end{aligned}$$

where the x-dependence is no more explicitly depicted (for simplification)

Background quantization: $\Phi = \Phi_0 + \varphi, \qquad \Xi = \Xi_0 + \chi$

Consequences of the mult. anom.

$$\mathcal{L}(x) = -\frac{1}{2}\partial_\mu\Phi(x)\partial_\mu\Phi(x) - \frac{1}{2}m_1^2\Phi(x)^2 + V_1(\Phi(x))$$
$$-\frac{1}{2}\partial_\mu\Xi(x)\partial_\mu\Xi(x) - \frac{1}{2}m_2^2\Xi(x)^2 + V_2(\Xi(x)) + V_3(\Psi(x),\Xi(x))$$

Partition function

$$e^{-Z} = \int D\Phi\, D\Xi\, \exp\left\{-\int d^{d+1}x \left[\frac{1}{2}\Phi(-\partial^2+m_1^2)\Phi + \frac{1}{2}\Xi(-\partial^2+m_2^2)\Xi \right.\right.$$
$$\left.\left. -V_1(\Phi) - V_2(\Xi) - V_3(\Phi,\Xi)\right]\right\}$$

where the x-dependence is no more explicitly depicted (for simplification)

Background quantization: $\Phi = \Phi_0 + \varphi,\qquad \Xi = \Xi_0 + \chi$

Gaussian approximation (one loop)

$$e^{-Z_1} = \int D\varphi\, D\chi\, \exp\left\{-\int d^{d+1}x \left[(\varphi\ \ \chi)\begin{pmatrix} A & C \\ C & B \end{pmatrix}\begin{pmatrix} \varphi \\ \chi \end{pmatrix}\right]\right\}$$
$$= \left|\det\begin{pmatrix} A & C \\ C & B \end{pmatrix}\right|^{-1/2}$$

with

$$
\begin{aligned}
A &\equiv -\partial^2 + m_1^2 - V_1''(\Phi_0) - \partial_1^2 V_3(\Phi_0, \Xi_0), \\
B &\equiv -\partial^2 + m_2^2 - V_2''(\Xi_0) - \partial_2^2 V_3(\Phi_0, \Xi_0), \\
C &\equiv -\partial_1 \partial_2 V_3(\Phi_0, \Xi_0).
\end{aligned}
$$

with
$$\begin{aligned} A &\equiv -\partial^2 + m_1^2 - V_1''(\Phi_0) - \partial_1^2 V_3(\Phi_0, \Xi_0), \\ B &\equiv -\partial^2 + m_2^2 - V_2''(\Xi_0) - \partial_2^2 V_3(\Phi_0, \Xi_0), \\ C &\equiv -\partial_1 \partial_2 V_3(\Phi_0, \Xi_0). \end{aligned}$$

After some calculations it turns out that the determinant can be expressed as:

$$\det\left(AB - C^2\right) = \det\left[(-\partial^2 + \alpha)(-\partial^2 + \beta)\right],$$

with

$$A \equiv -\partial^2 + m_1^2 - V_1''(\Phi_0) - \partial_1^2 V_3(\Phi_0, \Xi_0),$$
$$B \equiv -\partial^2 + m_2^2 - V_2''(\Xi_0) - \partial_2^2 V_3(\Phi_0, \Xi_0),$$
$$C \equiv -\partial_1 \partial_2 V_3(\Phi_0, \Xi_0).$$

After some calculations it turns out that the determinant can be expressed as:

$$\det\left(AB - C^2\right) = \det\left[(-\partial^2 + \alpha)(-\partial^2 + \beta)\right],$$

with

$$\alpha \equiv a - \sqrt{b^2 + c^2}, \qquad \beta \equiv a + \sqrt{b^2 + c^2},$$

with
$$
\begin{aligned}
A &\equiv -\partial^2 + m_1^2 - V_1''(\Phi_0) - \partial_1^2 V_3(\Phi_0, \Xi_0), \\
B &\equiv -\partial^2 + m_2^2 - V_2''(\Xi_0) - \partial_2^2 V_3(\Phi_0, \Xi_0), \\
C &\equiv -\partial_1 \partial_2 V_3(\Phi_0, \Xi_0).
\end{aligned}
$$

After some calculations it turns out that the determinant can be expressed as:

$$
\det\left(AB - C^2\right) = \det\left[(-\partial^2 + \alpha)(-\partial^2 + \beta)\right],
$$

with

$$
\alpha \equiv a - \sqrt{b^2 + c^2}, \qquad \beta \equiv a + \sqrt{b^2 + c^2},
$$

and

$$
a \equiv \frac{c_1 + c_2}{2}, \qquad b \equiv \frac{c_1 - c_2}{2},
$$

$$
c_1 \equiv m_1^2 - V_1''(\Phi_0) - \partial_1^2 V_3(\Phi_0, \Xi_0), \qquad c_2 \equiv m_2^2 - V_2''(\Xi_0) - \partial_2^2 V_3(\Phi_0, \Xi_0),
$$

$$
c \equiv -\partial_1 \partial_2 V_3(\Phi_0, \Xi_0).
$$

This situation has been considered by us before, in much detail. It gives rise to the multiplicative (or non-commutative) anomaly (or defect), namely the fact that, generically,

$$\det\left[(-\partial^2+\alpha)(-\partial^2+\beta)\right] \;\neq\; \det(-\partial^2+\alpha)\;\det(-\partial^2+\beta)$$

This situation has been considered by us before, in much detail. It gives rise to the multiplicative (or non-commutative) anomaly (or defect), namely the fact that, generically,

$$\det\left[(-\partial^2+\alpha)(-\partial^2+\beta)\right] \neq \det(-\partial^2+\alpha)\ \det(-\partial^2+\beta)$$

The anomaly δ (logarithm of the quotient) is expressed as a perturbative series of the difference $\alpha-\beta=-2\sqrt{b^2+c^2}$

$$\delta = \sum_n a_n(\alpha-\beta)^n = \sum_n (-2)^n\, a_n\, (b^2+c^2)^{n/2}$$

This situation has been considered by us before, in much detail. It gives rise to the multiplicative (or non-commutative) anomaly (or defect), namely the fact that, generically,

$$\det\left[(-\partial^2+\alpha)(-\partial^2+\beta)\right] \neq \det(-\partial^2+\alpha)\ \det(-\partial^2+\beta)$$

The anomaly δ (logarithm of the quotient) is expressed as a perturbative series of the difference $\alpha-\beta=-2\sqrt{b^2+c^2}$

$$\delta = \sum_n a_n(\alpha-\beta)^n = \sum_n (-2)^n\, a_n\, (b^2+c^2)^{n/2}$$

$\Longrightarrow$ Note that b depends on the mass difference, the difference in the second derivatives (at the background) of the individual potentials and on the difference between the two partial, second-order derivatives of the double potential, while c is given by the cross derivative of this potential.

This situation has been considered by us before, in much detail. It gives rise to the multiplicative (or non-commutative) anomaly (or defect), namely the fact that, generically,

$$\det\left[(-\partial^2+\alpha)(-\partial^2+\beta)\right] \neq \det(-\partial^2+\alpha)\ \det(-\partial^2+\beta)$$

The anomaly δ (logarithm of the quotient) is expressed as a perturbative series of the difference $\alpha-\beta=-2\sqrt{b^2+c^2}$

$$\delta = \sum_n a_n(\alpha-\beta)^n = \sum_n (-2)^n\, a_n\,(b^2+c^2)^{n/2}$$

$\Longrightarrow$ Note that b depends on the mass difference, the difference in the second derivatives (at the background) of the individual potentials and on the difference between the two partial, second-order derivatives of the double potential, while c is given by the cross derivative of this potential.

$\Longrightarrow$ In some cases the anomaly proves to be genuine: it persists after renormalization is performed.

Zero point energy

QFT vacuum to vacuum transition: $\langle 0|H|0\rangle$

Zero point energy

QFT vacuum to vacuum transition: $\langle 0 | H | 0 \rangle$

Spectrum, normal ordering:

$$H = \left(n + \frac{1}{2} \right) \lambda_n \, a_n \, a_n^\dagger$$

Zero point energy

QFT vacuum to vacuum transition: $\langle 0|H|0\rangle$

Spectrum, normal ordering:

$$H = \left(n + \frac{1}{2}\right) \lambda_n \, a_n \, a_n^\dagger$$

$$\langle 0|H|0\rangle = \frac{(\hbar c)}{2} \sum_n \lambda_n = \frac{1}{2} \,\mathrm{tr}\, H$$

Zero point energy

QFT vacuum to vacuum transition: $\langle 0|H|0\rangle$

Spectrum, normal ordering:

$$H = \left(n + \frac{1}{2}\right) \lambda_n \, a_n \, a_n^\dagger$$

$$\langle 0|H|0\rangle = \frac{(\hbar c)}{2} \sum_n \lambda_n = \frac{1}{2} \, \mathrm{tr}\, H$$

gives ∞ physical meaning?

Zero point energy

QFT vacuum to vacuum transition: $\langle 0|H|0\rangle$

Spectrum, normal ordering:

$$H = \left(n + \frac{1}{2}\right) \lambda_n \, a_n \, a_n^\dagger$$

$$\langle 0|H|0\rangle = \frac{(\hbar c)}{2} \sum_n \lambda_n = \frac{1}{2} \, \mathrm{tr} \, H$$

gives ∞ physical meaning?

Regularization $+$ Renormalization (cut-off, dim, ζ)

Zero point energy

QFT vacuum to vacuum transition: $\langle 0|H|0\rangle$

Spectrum, normal ordering:

$$H = \left(n + \frac{1}{2}\right) \lambda_n \, a_n \, a_n^\dagger$$

$$\langle 0|H|0\rangle \;=\; \frac{(\hbar c)}{2} \sum_n \lambda_n \;=\; \frac{1}{2} \, \mathrm{tr}\, H$$

gives ∞ physical meaning?

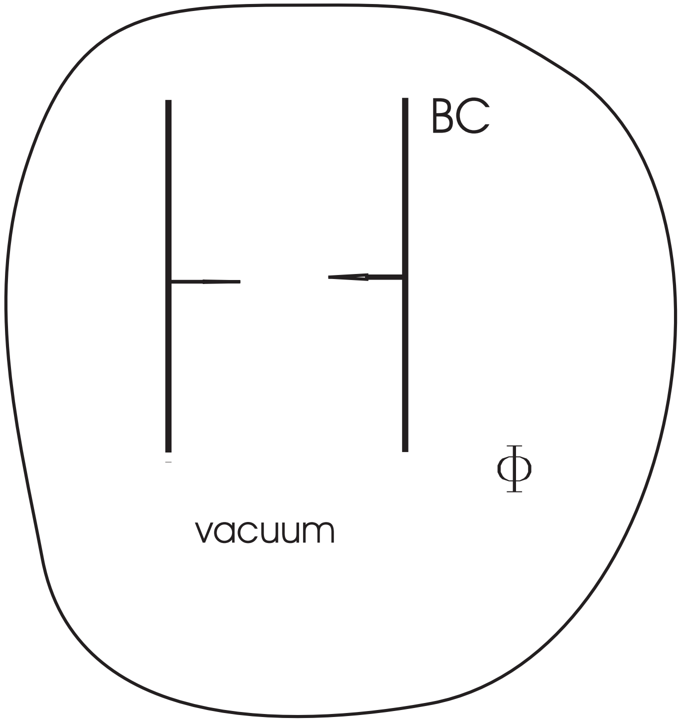
Regularization $+$ Renormalization (cut-off, dim, ζ)

Even then: Has the final value any meaning??

The Casimir Effect

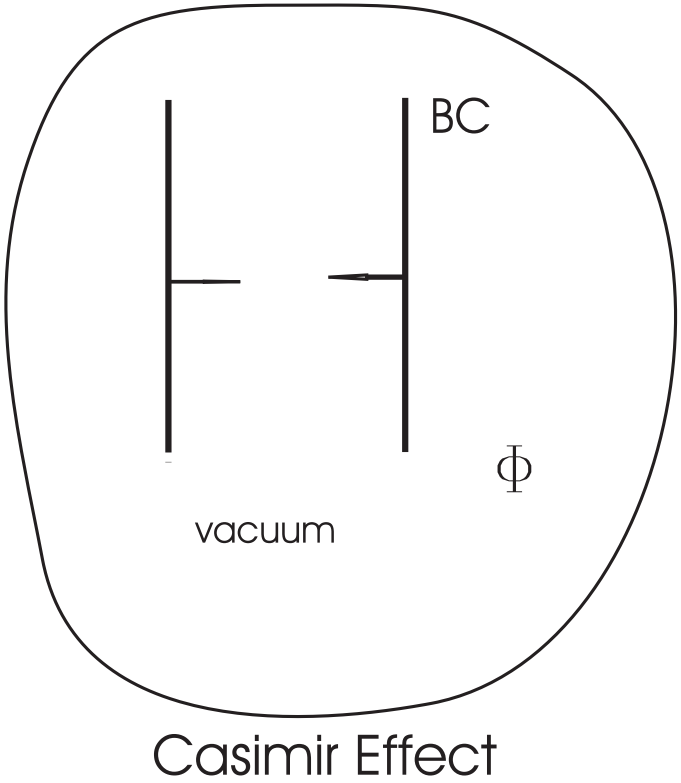
The Casimir Effect

BC e.g. periodic



Casimir Effect

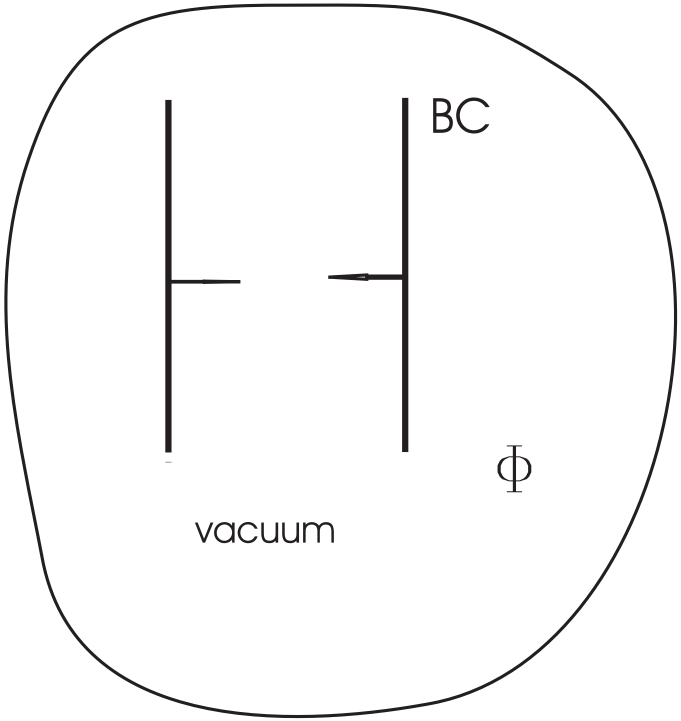
The Casimir Effect



BC e.g. periodic

$\Longrightarrow$ all kind of fields

The Casimir Effect



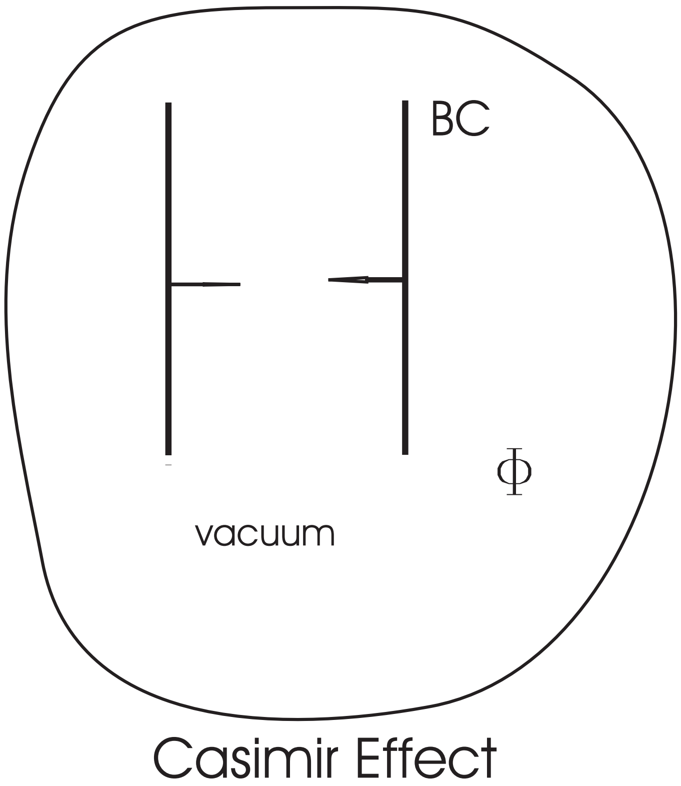
Casimir Effect

BC e.g. periodic

$\Longrightarrow$ all kind of fields

$\Longrightarrow$ curvature or topology

The Casimir Effect



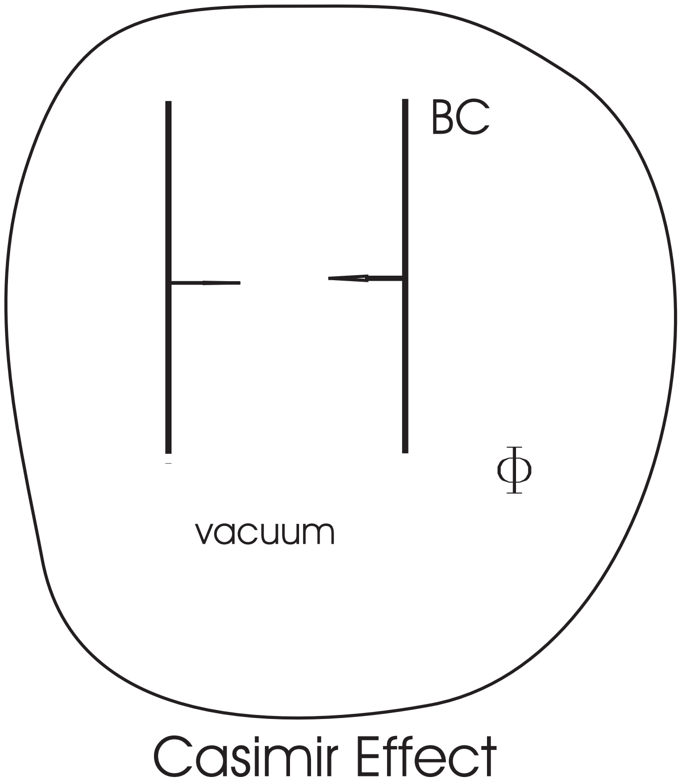
BC e.g. periodic

$\Longrightarrow$ all kind of fields

$\Longrightarrow$ curvature or topology

Universal process:

The Casimir Effect



BC e.g. periodic

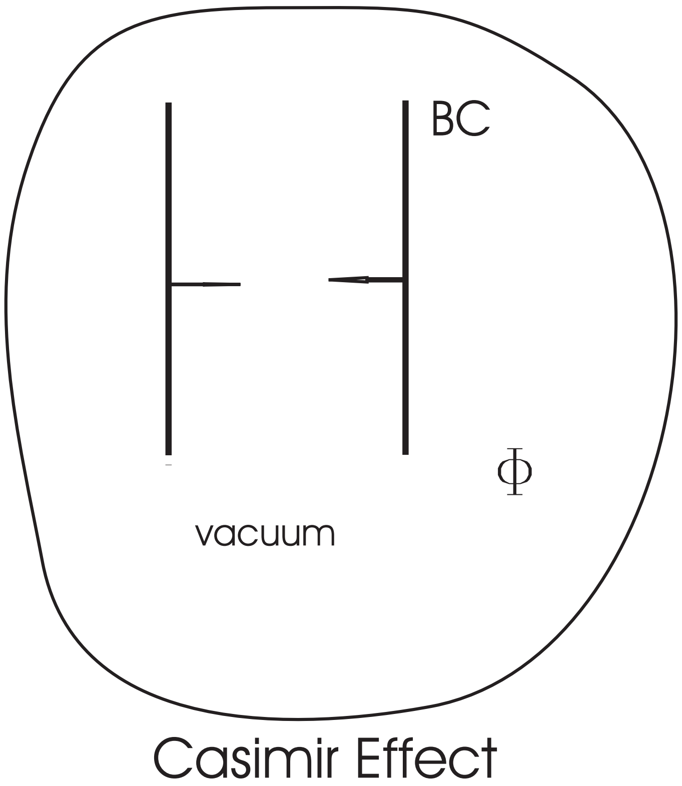
$\Longrightarrow$ all kind of fields

$\Longrightarrow$ curvature or topology

Universal process:

- Sonoluminiscence (Schwinger)
- Cond. matter (wetting ^{3}He alc.)
- Optical cavities
- Direct experim. confirmation

The Casimir Effect



BC e.g. periodic

$\Longrightarrow$ all kind of fields

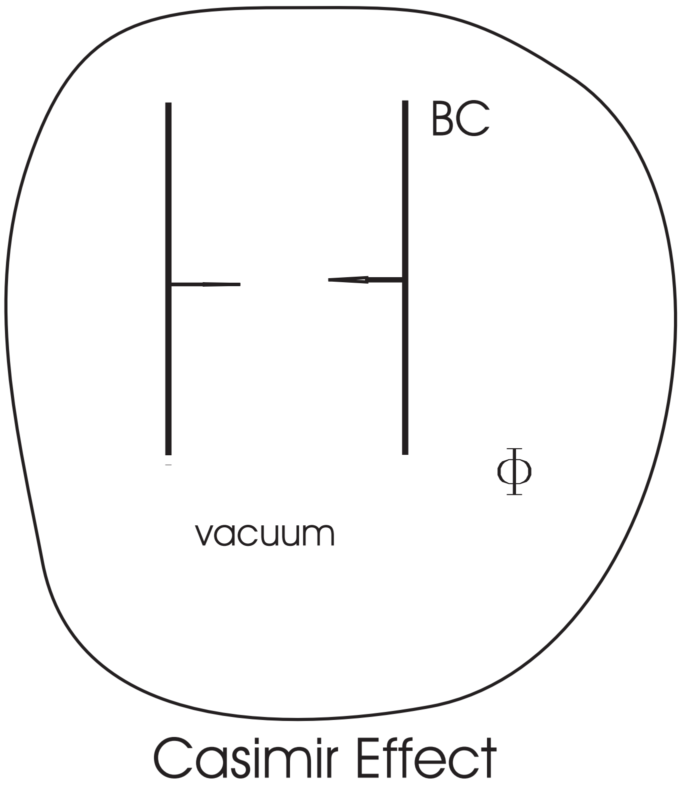
$\Longrightarrow$ curvature or topology

Universal process:

- Sonoluminiscence (Schwinger)
- Cond. matter (wetting ^{3}He alc.)
- Optical cavities
- Direct experim. confirmation

Van der Waals, Lipschitz theory

The Casimir Effect



BC e.g. periodic

$\Longrightarrow$ all kind of fields

$\Longrightarrow$ curvature or topology

Universal process:

- Sonoluminiscence (Schwinger)
- Cond. matter (wetting ^{3}He alc.)
- Optical cavities
- Direct experim. confirmation

Van der Waals, Lipschitz theory

- Dynamical CE
- Lateral CE
- Extract energy from vacuum
- CE and the cosmological constant

Vacuum energy density and the CC

- The main issue:

  energy ALWAYS gravitates, therefore the energy density of the vacuum, more precisely, the vacuum expectation value of the stress-energy tensor $\langle T_{\mu\nu} \rangle \equiv -\mathcal{E} g_{\mu\nu}$

Vacuum energy density and the CC

- The main issue:

  energy ALWAYS gravitates, therefore the energy density of the vacuum, more precisely, the vacuum expectation value of the stress-energy tensor $\langle T_{\mu\nu} \rangle \equiv -\mathcal{E} g_{\mu\nu}$

- Appears on the rhs of Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G (\tilde{T}_{\mu\nu} - \mathcal{E} g_{\mu\nu})$$

It affects cosmology: $\tilde{T}_{\mu\nu}$ excitations above the vacuum

Vacuum energy density and the CC

- The main issue:

  energy ALWAYS gravitates, therefore the energy density of the vacuum, more precisely, the vacuum expectation value of the stress-energy tensor $\langle T_{\mu\nu}\rangle \equiv -\mathcal{E} g_{\mu\nu}$

- Appears on the rhs of Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G(\tilde{T}_{\mu\nu} - \mathcal{E} g_{\mu\nu})$$

  It affects cosmology: $\tilde{T}_{\mu\nu}$ excitations above the vacuum

- Equivalent to a cosmological constant $\lambda = 8\pi G\mathcal{E}$

Vacuum energy density and the CC

- The main issue:

  energy ALWAYS gravitates, therefore the energy density of the vacuum, more precisely, the vacuum expectation value of the stress-energy tensor $\langle T_{\mu\nu} \rangle \equiv -\mathcal{E} g_{\mu\nu}$

- Appears on the rhs of Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G(\tilde{T}_{\mu\nu} - \mathcal{E} g_{\mu\nu})$$

  It affects cosmology: $\tilde{T}_{\mu\nu}$ excitations above the vacuum

- Equivalent to a cosmological constant $\lambda = 8\pi G \mathcal{E}$

- Recent observations: M. Tegmark *et al.* [SDSS Collab.] PRD 2004

$$\lambda \ = \ (2.14 \pm 0.13 \times 10^{-3} \text{ eV})^4 \quad \sim \ 4.32 \times 10^{-9} \text{ erg/cm}^3$$

Vacuum energy density and the CC

- The main issue:

  energy ALWAYS gravitates, therefore the energy density of the vacuum, more precisely, the vacuum expectation value of the stress-energy tensor $\langle T_{\mu\nu}\rangle \equiv -\mathcal{E} g_{\mu\nu}$

- Appears on the rhs of Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G (\tilde{T}_{\mu\nu} - \mathcal{E} g_{\mu\nu})$$

  It affects cosmology: $\tilde{T}_{\mu\nu}$ excitations above the vacuum

- Equivalent to a cosmological constant $\lambda = 8\pi G \mathcal{E}$
- Recent observations: M. Tegmark *et al.* [SDSS Collab.] PRD 2004

$$\lambda = (2.14 \pm 0.13 \times 10^{-3}\ \text{eV})^4 \sim 4.32 \times 10^{-9}\ \text{erg/cm}^3$$

- Idea: zero point fluctuations do contribute to the cosmological constant

Cosmo-Topological Casimir Effect (I)

- Zeta techniques used in calcul the contribution of the vacuum energy of quantum fields pervading the universe to the cosmolog const [cc]

Cosmo-Topological Casimir Effect (I)

- Zeta techniques used in calcul the contribution of the vacuum energy of quantum fields pervading the universe to the cosmolog const [cc]
- Direct calculations of the absolute contributions of the known fields (all couple to gravity) lead to a value which is off by roughly 120 orders of magnitude $[\rho_\Lambda^{obs}/(\rho_{Pl})^4 \sim 10^{-120}]$
  $\Longrightarrow$ kind of a modern (and very thick!) aether

Cosmo-Topological Casimir Effect (I)

- Zeta techniques used in calcul the contribution of the vacuum energy of quantum fields pervading the universe to the cosmolog const [cc]

- Direct calculations of the absolute contributions of the known fields (all couple to gravity) lead to a value which is off by roughly 120 orders of magnitude $[\rho_\Lambda^{obs}/(\rho_{Pl})^4 \sim 10^{-120}]$

  $\Longrightarrow$ kind of a modern (and very thick!) aether

- Observational tests see nothing (or very little) of it:

  $\Longrightarrow$ (new) cosmological constant problem.

Cosmo-Topological Casimir Effect (I)

- Zeta techniques used in calcul the contribution of the vacuum energy of quantum fields pervading the universe to the cosmolog const [cc]
- Direct calculations of the absolute contributions of the known fields (all couple to gravity) lead to a value which is off by roughly 120 orders of magnitude $[\rho_\Lambda^{obs}/(\rho_{Pl})^4 \sim 10^{-120}]$

  $\Longrightarrow$ kind of a modern (and very thick!) aether
- Observational tests see nothing (or very little) of it:

  $\Longrightarrow$ (new) cosmological constant problem.
- Very difficult to solve and we do not address this question directly [Baum, Hawking, Coleman, Polchinsky, Weinberg,...]

Cosmo-Topological Casimir Effect (I)

- Zeta techniques used in calcul the contribution of the vacuum energy of quantum fields pervading the universe to the cosmolog const [cc]
- Direct calculations of the absolute contributions of the known fields (all couple to gravity) lead to a value which is off by roughly 120 orders of magnitude $[\rho_\Lambda^{obs}/(\rho_{Pl})^4 \sim 10^{-120}]$
  $\Longrightarrow$ kind of a modern (and very thick!) aether
- Observational tests see nothing (or very little) of it:
  $\Longrightarrow$ (new) cosmological constant problem.
- Very difficult to solve and we do not address this question directly [Baum, Hawking, Coleman, Polchinsky, Weinberg,...]
- What we do consider —with relative success in some different approaches— is the additional contribution to the cc coming from the non-trivial topology of space or from specific boundary conditions imposed on braneworld models:
  $\Longrightarrow$ kind of cosmological Casimir effect

Cosmo-Topological Casimir Effect (II)

- Assuming one will be able to prove (in the future) that the ground value of the cc is zero (as many had suspected until recently), we will be left with this incremental value coming from the topology or BCs

  * L. Parker & A. Raval, VCDM, vacuum energy density
  * C.P. Burgess, hep-th/0606020 & 0510123: Susy Large Extra Dims (SLED), two $10^{-2}\mu$m dims, bulk vs brane Susy breaking scales
  * T. Padmanabhan, gr-qc/0606061: Holographic Perspective, CC is an intg const, no response of gravity to changes in bulk vac energy dens

Cosmo-Topological Casimir Effect (II)

- Assuming one will be able to prove (in the future) that the ground value of the cc is zero (as many had suspected until recently), we will be left with this incremental value coming from the topology or BCs

  * L. Parker & A. Raval, VCDM, vacuum energy density
  * C.P. Burgess, hep-th/0606020 & 0510123: Susy Large Extra Dims (SLED), two $10^{-2}\mu$m dims, bulk vs brane Susy breaking scales
  * T. Padmanabhan, gr-qc/0606061: Holographic Perspective, CC is an intg const, no response of gravity to changes in bulk vac energy dens

- We show (with different examples) that this value acquires the correct order of magnitude —corresponding to the one coming from the observed acceleration in the expansion of our universe— in some reasonable models involving:

Cosmo-Topological Casimir Effect (II)

- Assuming one will be able to prove (in the future) that the ground value of the cc is zero (as many had suspected until recently), we will be left with this incremental value coming from the topology or BCs

  * L. Parker & A. Raval, VCDM, vacuum energy density
  * C.P. Burgess, hep-th/0606020 & 0510123: Susy Large Extra Dims (SLED), two $10^{-2}\mu$m dims, bulk vs brane Susy breaking scales
  * T. Padmanabhan, gr-qc/0606061: Holographic Perspective, CC is an intg const, no response of gravity to changes in bulk vac energy dens

- We show (with different examples) that this value acquires the correct order of magnitude —corresponding to the one coming from the observed acceleration in the expansion of our universe— in some reasonable models involving:
  - (a) small and large compactified scales

Cosmo-Topological Casimir Effect (II)

- Assuming one will be able to prove (in the future) that the ground value of the cc is zero (as many had suspected until recently), we will be left with this incremental value coming from the topology or BCs

  * L. Parker & A. Raval, VCDM, vacuum energy density
  * C.P. Burgess, hep-th/0606020 & 0510123: Susy Large Extra Dims (SLED), two $10^{-2}\mu$m dims, bulk vs brane Susy breaking scales
  * T. Padmanabhan, gr-qc/0606061: Holographic Perspective, CC is an intg const, no response of gravity to changes in bulk vac energy dens

- We show (with different examples) that this value acquires the correct order of magnitude —corresponding to the one coming from the observed acceleration in the expansion of our universe— in some reasonable models involving:
  - (a) small and large compactified scales
  - (b) dS & AdS worldbranes

Cosmo-Topological Casimir Effect (II)

- Assuming one will be able to prove (in the future) that the ground value of the cc is zero (as many had suspected until recently), we will be left with this incremental value coming from the topology or BCs

  * L. Parker & A. Raval, VCDM, vacuum energy density
  * C.P. Burgess, hep-th/0606020 & 0510123: Susy Large Extra Dims (SLED), two $10^{-2}\mu$m dims, bulk vs brane Susy breaking scales
  * T. Padmanabhan, gr-qc/0606061: Holographic Perspective, CC is an intg const, no response of gravity to changes in bulk vac energy dens

- We show (with different examples) that this value acquires the correct order of magnitude —corresponding to the one coming from the observed acceleration in the expansion of our universe— in some reasonable models involving:
  - (a) small and large compactified scales
  - (b) dS & AdS worldbranes
  - (c) supergraviton theories (discret dims, deconstr)

A. Simple model: large & small dim's

- Space-time:

$$\mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{T}^q, \qquad \mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{S}^q, \qquad \cdots$$

A. Simple model: large & small dim's

- Space-time:

$$\mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{T}^q, \qquad \mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{S}^q, \qquad \dots$$

- Scalar field, ϕ, pervading the universe

A. Simple model: large & small dim's

- Space-time:

$$\mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{T}^q, \qquad \mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{S}^q, \qquad \ldots$$

- Scalar field, ϕ, pervading the universe
- ρ_ϕ contribution to ρ_V from this field

$$\rho_\phi = \frac{1}{2} \sum_i \lambda_i = \frac{1}{2} \sum_{\mathbf{k}} \frac{1}{\mu} \left(k^2 + M^2\right)^{1/2}$$

A. Simple model: large & small dim's

- Space-time:

$$\mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{T}^q, \qquad \mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{S}^q, \qquad \ldots$$

- Scalar field, ϕ, pervading the universe
- ρ_ϕ contribution to ρ_V from this field

$$\rho_\phi = \frac{1}{2} \sum_i \lambda_i = \frac{1}{2} \sum_{\mathbf{k}} \frac{1}{\mu} \left(k^2 + M^2\right)^{1/2}$$

- $\sum_i$ and $\sum_{\mathbf{k}}$ are generalized sums, μ mass-dim parameter, $\hbar = c = 1$

A. Simple model: large & small dim's

- Space-time:

$$\mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{T}^q, \qquad \mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{S}^q, \qquad \cdots$$

- Scalar field, ϕ, pervading the universe
- ρ_ϕ contribution to ρ_V from this field

$$\rho_\phi = \frac{1}{2} \sum_i \lambda_i = \frac{1}{2} \sum_{\mathbf{k}} \frac{1}{\mu} \left(k^2 + M^2\right)^{1/2}$$

- $\sum_i$ and $\sum_{\mathbf{k}}$ are generalized sums, μ mass-dim parameter, $\hbar = c = 1$
- M mass of the field arbitrarily small (a tiny mass for the field can never be excluded); see

L. Parker & A. Raval, PRL86 749 (2001); PRD62 083503 (2000)

- For d–open, (p,q)–toroidal universe:

$$\rho_\phi = \frac{\pi^{-d/2}}{2^d\Gamma(d/2)\prod_{j=1}^p a_j \prod_{h=1}^q b_h} \int_0^\infty dk\, k^{d-1} \sum_{\mathbf{n}_p=-\infty}^{\infty} \sum_{\mathbf{m}_q=-\infty}^{\infty} \left[\sum_{j=1}^p \left(\frac{2\pi n_j}{a_j}\right)^2 + \sum_{h=1}^q \left(\frac{2\pi m_h}{b_h}\right)^2 + \mathbf{k}_d^2 + M^2\right]^{1/2}$$

$$\simeq \frac{1}{a^p b^q} \sum_{\mathbf{n}_p,\mathbf{m}_q=-\infty}^{\infty} \left(\frac{1}{a^2}\sum_{j=1}^p n_j^2 + \frac{1}{b^2}\sum_{h=1}^q m_h^2 + M^2\right)^{\frac{d+1}{2}+1}$$

- For d–open, (p,q)–toroidal universe:

$$\rho_\phi = \frac{\pi^{-d/2}}{2^d\Gamma(d/2)\prod_{j=1}^p a_j \prod_{h=1}^q b_h}\int_0^\infty dk\, k^{d-1}$$

$$\sum_{\mathbf{n}_p=-\infty}^{\infty}\sum_{\mathbf{m}_q=-\infty}^{\infty}\left[\sum_{j=1}^p\left(\frac{2\pi n_j}{a_j}\right)^2+\sum_{h=1}^q\left(\frac{2\pi m_h}{b_h}\right)^2+\mathbf{k}_d^2+M^2\right]^{1/2}$$

$$\simeq \frac{1}{a^p b^q}\sum_{\mathbf{n}_p,\mathbf{m}_q=-\infty}^{\infty}\left(\frac{1}{a^2}\sum_{j=1}^p n_j^2+\frac{1}{b^2}\sum_{h=1}^q m_h^2+M^2\right)^{\frac{d+1}{2}+1}$$

- For d–open, (p–toroidal, q–spherical) universe:

$$\rho_\phi = \frac{\pi^{-d/2}}{2^d\Gamma(d/2)\prod_{j=1}^p a_j\, b^q}\int_0^\infty dk\, k^{d-1}\sum_{\mathbf{n}_p=-\infty}^{\infty}\sum_{l=1}^{\infty}P_{q-1}(l)$$

$$\left[\sum_{j=1}^p\left(\frac{2\pi n_j}{a_j}\right)^2+\frac{Q_2(l)}{b^2}+\mathbf{k}_d^2+M^2\right]^{1/2}$$

$[P_{q-1}(l)$ poly in l deg $q-1]$

$$\simeq \frac{1}{a^p b^q}\sum_{\mathbf{n}_p=-\infty}^{\infty}\sum_{l=1}^{\infty}P_{q-1}(l)\left(\frac{4\pi^2}{a^2}\sum_{j=1}^p n_j^2+\frac{l(l+q)}{b^2}+\frac{M^2}{4\pi^2}\right)^{\frac{d+1}{2}+1}$$

Regularize: zeta function

$$\zeta(s) = \frac{1}{2}\sum_i \lambda_i^{-s} = \frac{1}{2}\sum_{\mathbf{k}} \left(\frac{k^2 + M^2}{\mu^2}\right)^{-s/2} \Longrightarrow \rho_\phi = \zeta(-1)$$

- Regularize: zeta function

$$\zeta(s) = \frac{1}{2}\sum_i \lambda_i^{-s} = \frac{1}{2}\sum_{\mathbf{k}} \left(\frac{k^2+M^2}{\mu^2}\right)^{-s/2} \Longrightarrow \rho_\phi = \zeta(-1)$$

- No further subtraction or renormalization needed here
  [E.E., J. Math. Phys. **35**, 3308, 6100 (1994)]

- Regularize: zeta function

$$\zeta(s) = \frac{1}{2}\sum_i \lambda_i^{-s} = \frac{1}{2}\sum_{\mathbf{k}} \left(\frac{k^2+M^2}{\mu^2}\right)^{-s/2} \Longrightarrow \rho_\phi = \zeta(-1)$$

- No further subtraction or renormalization needed here [E.E., J. Math. Phys. **35**, 3308, 6100 (1994)]

- Here spectrum of the Hamiltonian op. known explicitly. Corresponds to generalized Chowla-Selberg expression [E.E., Commun. Math. Phys. **198**, 83 (1998) E.E., J. Phys. **A30**, 2735 (1997)]

- Regularize: zeta function

$$\zeta(s) = \frac{1}{2}\sum_i \lambda_i^{-s} = \frac{1}{2}\sum_{\mathbf{k}} \left(\frac{k^2+M^2}{\mu^2}\right)^{-s/2} \Longrightarrow \rho_\phi = \zeta(-1)$$

- No further subtraction or renormalization needed here [E.E., J. Math. Phys. **35**, 3308, 6100 (1994)]

- Here spectrum of the Hamiltonian op. known explicitly. Corresponds to generalized Chowla-Selberg expression [E.E., Commun. Math. Phys. **198**, 83 (1998) E.E., J. Phys. **A30**, 2735 (1997)]

- For the zeta function (Re $s > p/2$):

$$\zeta_{A,\vec{c},q}(s) = \sum_{\vec{n}\in\mathbb{Z}^p}{}' \left[\frac{1}{2}(\vec{n}+\vec{c})^T A(\vec{n}+\vec{c}) + q\right]^{-s}$$

$$\equiv \sum_{\vec{n}\in\mathbb{Z}^p}{}' \left[Q(\vec{n}+\vec{c}) + q\right]^{-s}$$

- Prime means that point $\vec{n} = \vec{0}$ is excluded from sum (not important)

- Prime means that point $\vec{n} = \vec{0}$ is excluded from sum (not important)
- Gives (analyt cont of) multidim zeta function in terms of an exponentially convergent multiseries, valid in the whole complex plane

- Prime means that point $\vec{n} = \vec{0}$ is excluded from sum (not important)
- Gives (analyt cont of) multidim zeta function in terms of an exponentially convergent multiseries, valid in the whole complex plane
- Exhibits singularities (simple poles) of meromorphic continuation —with corresponding residua— explicitly

- Prime means that point $\vec{n} = \vec{0}$ is excluded from sum (not important)
- Gives (analyt cont of) multidim zeta function in terms of an exponentially convergent multiseries, valid in the whole complex plane
- Exhibits singularities (simple poles) of meromorphic continuation —with corresponding residua— explicitly
- Only condition on matrix A: corresponds to (non negative) quadratic form, Q. Vector $\vec{c}$ arbitrary, while q (for the moment) positive constant

$$\zeta_{A,\vec{c},q}(s) = \frac{(2\pi)^{p/2} q^{p/2-s}}{\sqrt{\det A}} \frac{\Gamma(s-p/2)}{\Gamma(s)} + \frac{2^{s/2+p/4+2} \pi^s q^{-s/2+p/4}}{\sqrt{\det A}\,\Gamma(s)}$$

$$\sum_{\vec{m}\in\mathbb{Z}^p_{1/2}}{}' \cos(2\pi\vec{m}\cdot\vec{c}) \left(\vec{m}^T A^{-1} \vec{m}\right)^{s/2-p/4} K_{p/2-s}\left(2\pi\sqrt{2q\,\vec{m}^T A^{-1} \vec{m}}\right)$$

- Prime means that point $\vec{n} = \vec{0}$ is excluded from sum (not important)
- Gives (analyt cont of) multidim zeta function in terms of an exponentially convergent multiseries, valid in the whole complex plane
- Exhibits singularities (simple poles) of meromorphic continuation —with corresponding residua— explicitly
- Only condition on matrix A: corresponds to (non negative) quadratic form, Q. Vector $\vec{c}$ arbitrary, while q (for the moment) positive constant

$$\zeta_{A,\vec{c},q}(s) = \frac{(2\pi)^{p/2} q^{p/2-s}}{\sqrt{\det A}} \frac{\Gamma(s-p/2)}{\Gamma(s)} + \frac{2^{s/2+p/4+2} \pi^s q^{-s/2+p/4}}{\sqrt{\det A}\,\Gamma(s)}$$

$$\sum_{\vec{m}\in\mathbb{Z}^p_{1/2}}{}' \cos(2\pi\vec{m}\cdot\vec{c})\, (\vec{m}^T A^{-1}\vec{m})^{s/2-p/4}\, K_{p/2-s}\left(2\pi\sqrt{2q\,\vec{m}^T A^{-1}\vec{m}}\right)$$

- K_ν modified Bessel function of the second kind and the subindex 1/2 in $\mathbb{Z}^p_{1/2}$ means that only half of the vectors $\vec{m} \in \mathbb{Z}^p$ are summed over. That is, if we take an $\vec{m} \in \mathbb{Z}^p$ we must then exclude $-\vec{m}$ (as simple criterion one can, for instance, select those vectors in $\mathbb{Z}^p \backslash \{\vec{0}\}$ whose first non-zero component is positive).

Calcul. of vacuum energy density

Analytic continuation of the zeta function:

$$\zeta(s) = \frac{2\pi^{s/2+1}}{a^{p-(s+1)/2} b^{q-(s-1)/2} \Gamma(s/2)} \sum_{\mathbf{m}_q=-\infty}^{\infty} \sum_{h=0}^{p} \binom{p}{h} 2^h \sum_{\mathbf{n}_h=1}^{\infty} \left(\frac{\sum_{j=1}^{h} n_j^2}{\sum_{k=1}^{q} m_k^2 + M^2} \right)^{\frac{s-1}{4}}$$

$$\times K_{(s-1)/2} \left[\frac{2\pi a}{b} \left(\sum_{j=1}^{h} n_j^2 \right)^{1/2} \left(\sum_{k=1}^{q} m_k^2 + M^2 \right)^{1/2} \right]$$

Calcul. of vacuum energy density

Analytic continuation of the zeta function:

$$\zeta(s) = \frac{2\pi^{s/2+1}}{a^{p-(s+1)/2} b^{q-(s-1)/2} \Gamma(s/2)} \sum_{\mathbf{m}_q=-\infty}^{\infty} \sum_{h=0}^{p} \binom{p}{h} 2^h \sum_{\mathbf{n}_h=1}^{\infty} \left(\frac{\sum_{j=1}^{h} n_j^2}{\sum_{k=1}^{q} m_k^2 + M^2} \right)^{\frac{s-1}{4}}$$

$$\times K_{(s-1)/2} \left[\frac{2\pi a}{b} \left(\sum_{j=1}^{h} n_j^2 \right)^{1/2} \left(\sum_{k=1}^{q} m_k^2 + M^2 \right)^{1/2} \right]$$

Yields the vacuum energy density:

$$\rho_\phi = -\frac{1}{a^p b^{q+1}} \sum_{h=0}^{p} \binom{p}{h} 2^h \sum_{\mathbf{n}_h=1}^{\infty} \sum_{\mathbf{m}_q=-\infty}^{\infty} \sqrt{\frac{\sum_{k=1}^{q} m_k^2 + M^2}{\sum_{j=1}^{h} n_j^2}}$$

$$\times K_1 \left[\frac{2\pi a}{b} \left(\sum_{j=1}^{h} n_j^2 \right)^{1/2} \left(\sum_{k=1}^{q} m_k^2 + M^2 \right)^{1/2} \right]$$

Now, from $$K_\nu(z) \sim \frac{1}{2}\Gamma(\nu)(z/2)^{-\nu}, \qquad z \to 0$$

Now, from $K_\nu(z) \sim \frac{1}{2}\Gamma(\nu)(z/2)^{-\nu}, \qquad z \to 0$

when M is very small

$$\rho_\phi = -\frac{1}{a^p b^{q+1}} \left\{ M\, K_1\left(\frac{2\pi a}{b} M\right) + \sum_{h=0}^{p} \binom{p}{h} 2^h \times \sum_{\mathbf{n}_h=1}^{\infty} \frac{M}{\sqrt{\sum_{j=1}^{h} n_j^2}} K_1\left(\frac{2\pi a}{b} M \sqrt{\sum_{j=1}^{h} n_j^2}\right) + \mathcal{O}\left[q\sqrt{1+M^2} K_1\left(\frac{2\pi a}{b}\sqrt{1+M^2}\right)\right]\right\}$$

Now, from $K_\nu(z) \sim \frac{1}{2}\Gamma(\nu)(z/2)^{-\nu}, \qquad z \to 0$

when M is very small

$$\rho_\phi = -\frac{1}{a^p b^{q+1}} \left\{ M\, K_1\left(\frac{2\pi a}{b} M\right) + \sum_{h=0}^{p} \binom{p}{h} 2^h \times \sum_{\mathbf{n}_h=1}^{\infty} \frac{M}{\sqrt{\sum_{j=1}^{h} n_j^2}} K_1\left(\frac{2\pi a}{b} M \sqrt{\sum_{j=1}^{h} n_j^2}\right) + \mathcal{O}\left[q\sqrt{1+M^2} K_1\left(\frac{2\pi a}{b}\sqrt{1+M^2}\right)\right] \right\}$$

Inserting the $\hbar$ and c factors

$$\rho_\phi = -\frac{\hbar c}{2\pi a^{p+1} b^q} \left[1 + \sum_{h=0}^{p} \binom{p}{h} 2^h \alpha\right] + \mathcal{O}\left[q K_1\left(\frac{2\pi a}{b}\right)\right]$$

Now, from $K_\nu(z) \sim \frac{1}{2}\Gamma(\nu)(z/2)^{-\nu}, \qquad z \to 0$

when M is very small

$$\rho_\phi = -\frac{1}{a^p b^{q+1}} \Bigg\{ M\, K_1\left(\frac{2\pi a}{b} M\right) + \sum_{h=0}^{p} \binom{p}{h} 2^h \times \sum_{\mathbf{n}_h=1}^{\infty} \frac{M}{\sqrt{\sum_{j=1}^h n_j^2}} K_1\left(\frac{2\pi a}{b} M \sqrt{\sum_{j=1}^h n_j^2}\right) + \mathcal{O}\left[q\sqrt{1+M^2} K_1\left(\frac{2\pi a}{b}\sqrt{1+M^2}\right)\right]\Bigg\}$$

Inserting the $\hbar$ and c factors

$$\rho_\phi = -\frac{\hbar c}{2\pi a^{p+1} b^q}\left[1 + \sum_{h=0}^{p}\binom{p}{h} 2^h \alpha\right] + \mathcal{O}\left[q K_1\left(\frac{2\pi a}{b}\right)\right]$$

α some finite constant (explicit geometrical sum in the limit $M \to 0$)

Now, from $K_\nu(z) \sim \frac{1}{2}\Gamma(\nu)(z/2)^{-\nu}, \qquad z \to 0$

when M is very small

$$\rho_\phi = -\frac{1}{a^p b^{q+1}} \left\{ M\, K_1\left(\frac{2\pi a}{b} M\right) + \sum_{h=0}^{p} \binom{p}{h} 2^h \right.$$
$$\times \sum_{\mathbf{n}_h=1}^{\infty} \frac{M}{\sqrt{\sum_{j=1}^{h} n_j^2}} K_1\left(\frac{2\pi a}{b} M \sqrt{\sum_{j=1}^{h} n_j^2}\right)$$
$$\left. + \mathcal{O}\left[q\sqrt{1+M^2} K_1\left(\frac{2\pi a}{b}\sqrt{1+M^2}\right)\right]\right\}$$

Inserting the $\hbar$ and c factors

$$\rho_\phi = -\frac{\hbar c}{2\pi a^{p+1} b^q}\left[1 + \sum_{h=0}^{p}\binom{p}{h} 2^h \alpha\right] + \mathcal{O}\left[q K_1\left(\frac{2\pi a}{b}\right)\right]$$

α some finite constant (explicit geometrical sum in the limit $M \to 0$)

$\Longrightarrow$ Sign may change with BC (e.g., Dirichlet): a problem

Matching the obs. results for the CC

$$b \sim l_{P(lanck)} \qquad a \sim R_{U(niverse)} \qquad a/b \sim 10^{60}$$

Matching the obs. results for the CC

$$b \sim l_{P(lanck)} \qquad a \sim R_{U(niverse)} \qquad a/b \sim 10^{60}$$

ρ_ϕ	$p=0$	$p=1$	$p=2$	$p=3$
$b=l_P$	10^{-13}	10^{-6}	1	10^5
$b=10l_P$	10^{-14}	$[10^{-8}]$	10^{-3}	10
$b=10^2l_P$	10^{-15}	(10^{-10})	10^{-6}	10^{-3}
$b=10^3l_P$	10^{-16}	10^{-12}	$[10^{-9}]$	(10^{-7})
$b=10^4l_P$	10^{-17}	10^{-14}	10^{-12}	10^{-11}
$b=10^5l_P$	10^{-18}	10^{-16}	10^{-15}	10^{-15}

Table 2: Vacuum energy density in units of erg/cm^3, for p large compactified dimensions a, and $q = p+1$ small compactified dimensions b, $p = 0, \ldots, 3$, for different values of b, proportional to the Planck length l_P

Results of this simple model

* Precise (in absolute value) coincidence with the observational value for the cosmological constant with

$$\rho_\phi \longrightarrow [\quad]$$

Results of this simple model

* Precise (in absolute value) coincidence with the observational value for the cosmological constant with

$$\rho_\phi \longrightarrow [\quad]$$

* Approximate coincidence $\longrightarrow (\quad)$

Results of this simple model

* Precise (in absolute value) coincidence with the observational value for the cosmological constant with

$$\rho_\phi \longrightarrow [\quad]$$

* Approximate coincidence $\longrightarrow (\quad)$

* $b \sim$ 10 to 1000 times the Planck length l_P

Results of this simple model

* Precise (in absolute value) coincidence with the observational value for the cosmological constant with

$$\rho_\phi \longrightarrow [\quad]$$

* Approximate coincidence $\longrightarrow (\quad)$

* $b \sim$ 10 to 1000 times the Planck length l_P

* $q = p + 1$: (1,2) and (2,3) compactified dimensions, respectively

Results of this simple model

* Precise (in absolute value) coincidence with the observational value for the cosmological constant with

$$\rho_\phi \longrightarrow [\quad]$$

* Approximate coincidence $\longrightarrow (\quad)$

* $b \sim$ 10 to 1000 times the Planck length l_P

* $q = p + 1$: (1,2) and (2,3) compactified dimensions, respectively

* Everything dictated by two basic lengths:

  Planck value and radius of observable Universe

Results of this simple model

* Precise (in absolute value) coincidence with the observational value for the cosmological constant with

$$\rho_\phi \longrightarrow [\quad]$$

* Approximate coincidence $\longrightarrow (\quad)$

* $b \sim$ 10 to 1000 times the Planck length l_P

* $q = p + 1$: (1,2) and (2,3) compactified dimensions, respectively

* Everything dictated by two basic lengths:

  Planck value and radius of observable Universe

* Both situations correspond to a marginally closed universe

Results of this simple model

* Precise (in absolute value) coincidence with the observational value for the cosmological constant with

$$\rho_\phi \longrightarrow [\quad]$$

* Approximate coincidence $\longrightarrow (\quad)$

* $b \sim$ 10 to 1000 times the Planck length l_P

* $q = p + 1$: (1,2) and (2,3) compactified dimensions, respectively

* Everything dictated by two basic lengths:

  Planck value and radius of observable Universe

* Both situations correspond to a marginally closed universe

$\Longrightarrow$ To examine $\rightarrow$ couplings in GR

$\rightarrow$ alternative theories

B. Realistic models: dS & AdS BW

Elizalde, Nojiri, Odintsov, PRD70 (2004) 043539

Elizalde, Nojiri, Odintsov, Ogushi, PRD67 (2003) 063515

B. Realistic models: dS & AdS BW

Elizalde, Nojiri, Odintsov, PRD70 (2004) 043539

Elizalde, Nojiri, Odintsov, Ogushi, PRD67 (2003) 063515

- Bulk Casimir effect may play a role in radion stabilization of BW
- Bulk Casimir effect (eff. pot.) for a conformal or massive scalar field

B. Realistic models: dS & AdS BW

Elizalde, Nojiri, Odintsov, PRD70 (2004) 043539

Elizalde, Nojiri, Odintsov, Ogushi, PRD67 (2003) 063515

- Bulk Casimir effect may play a role in radion stabilization of BW
- Bulk Casimir effect (eff. pot.) for a conformal or massive scalar field
- Bulk: 5-dim AdS or dS space with 2/1 4-dim dS brane (our universe)

B. Realistic models: dS & AdS BW

Elizalde, Nojiri, Odintsov, PRD70 (2004) 043539

Elizalde, Nojiri, Odintsov, Ogushi, PRD67 (2003) 063515

- Bulk Casimir effect may play a role in radion stabilization of BW
- Bulk Casimir effect (eff. pot.) for a conformal or massive scalar field
- Bulk: 5-dim AdS or dS space with 2/1 4-dim dS brane (our universe)
- Consistent with observational data even for large extra dimension

B. Realistic models: dS & AdS BW

Elizalde, Nojiri, Odintsov, PRD70 (2004) 043539

Elizalde, Nojiri, Odintsov, Ogushi, PRD67 (2003) 063515

- Bulk Casimir effect may play a role in radion stabilization of BW
- Bulk Casimir effect (eff. pot.) for a conformal or massive scalar field
- Bulk: 5-dim AdS or dS space with 2/1 4-dim dS brane (our universe)
- Consistent with observational data even for large extra dimension

Two dS_4 branes in dS_5 background; becomes a one-brane as $L \to \infty$

B. Realistic models: dS & AdS BW

Elizalde, Nojiri, Odintsov, PRD70 (2004) 043539

Elizalde, Nojiri, Odintsov, Ogushi, PRD67 (2003) 063515

- Bulk Casimir effect may play a role in radion stabilization of BW
- Bulk Casimir effect (eff. pot.) for a conformal or massive scalar field
- Bulk: 5-dim AdS or dS space with 2/1 4-dim dS brane (our universe)
- Consistent with observational data even for large extra dimension

Two dS_4 branes in dS_5 background; becomes a one-brane as $L \to \infty$

$\Longrightarrow$ Casimir energy density and effective potential for a de Sitter (dS) brane in a five-dimensional anti-de Sitter (AdS) background

B. Realistic models: dS & AdS BW

Elizalde, Nojiri, Odintsov, PRD70 (2004) 043539

Elizalde, Nojiri, Odintsov, Ogushi, PRD67 (2003) 063515

- Bulk Casimir effect may play a role in radion stabilization of BW
- Bulk Casimir effect (eff. pot.) for a conformal or massive scalar field
- Bulk: 5-dim AdS or dS space with 2/1 4-dim dS brane (our universe)
- Consistent with observational data even for large extra dimension

Two dS_4 branes in dS_5 background; becomes a one-brane as $L \to \infty$

$\Longrightarrow$ Casimir energy density and effective potential for a de Sitter (dS) brane in a five-dimensional anti-de Sitter (AdS) background

$\Longrightarrow$ Action for conformally inv massless scalar with scalar-gravit coupling

$$\mathcal{S} = \frac{1}{2} \int d^5x \sqrt{g} \left[-g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \xi_5 R^{(5)} \phi^2 \right]$$

$\xi_5 = -3/16$ $\qquad$ $R^{(5)}$ 5-dim scalar curvature

$\Longrightarrow$ Euclidean metric of the 5-dim AdS bulk

$$
\begin{aligned}
ds^2 &= g_{\mu\nu} dx^\mu dx^\nu = \frac{\alpha^2}{\sinh^2 z} \left(dz^2 + d\Omega_4^2 \right) \\
d\Omega_4^2 &= d\xi^2 + \sin^2 \xi d\Omega_3^2
\end{aligned}
$$

α is the AdS radius, related to cc of AdS bulk

$d\Omega_3$ metric on the 3-sphere

$\Longrightarrow$ Euclidean metric of the 5-dim AdS bulk

$$
\begin{aligned}
ds^2 &= g_{\mu\nu} dx^\mu dx^\nu = \frac{\alpha^2}{\sinh^2 z} \left(dz^2 + d\Omega_4^2\right) \\
d\Omega_4^2 &= d\xi^2 + \sin^2 \xi d\Omega_3^2
\end{aligned}
$$

α is the AdS radius, related to cc of AdS bulk

$d\Omega_3$ metric on the 3-sphere

CASIMIR ENERGY

$\Longrightarrow$ Euclidean metric of the 5-dim AdS bulk

$$
\begin{aligned}
ds^2 &= g_{\mu\nu} dx^\mu dx^\nu = \frac{\alpha^2}{\sinh^2 z} \left(dz^2 + d\Omega_4^2\right) \\
d\Omega_4^2 &= d\xi^2 + \sin^2 \xi d\Omega_3^2
\end{aligned}
$$

α is the AdS radius, related to cc of AdS bulk

$d\Omega_3$ metric on the 3-sphere

CASIMIR ENERGY

(a) One-brane Casimir energy $= 0$

$\Longrightarrow$ Euclidean metric of the 5-dim AdS bulk

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \frac{\alpha^2}{\sinh^2 z}\left(dz^2 + d\Omega_4^2\right)$$
$$d\Omega_4^2 = d\xi^2 + \sin^2\xi d\Omega_3^2$$

α is the AdS radius, related to cc of AdS bulk

$d\Omega_3$ metric on the 3-sphere

CASIMIR ENERGY

(a) One-brane Casimir energy $= 0$

(b) Bulk Casimir energy (L brane separation, $\mathcal{R}$ brane radius)

$$\zeta(s|L_5) = \frac{\mu^{-2s}}{6}\sum_{n,l=1}^{\infty}(l+1)(l+2)(2l+3)\left[\left(\frac{\pi n}{L}\right)^2 + \mathcal{R}^{-2}\left(l^2+3l+\frac{9}{4}\right)\right]^{-s}$$

$\Longrightarrow$ Euclidean metric of the 5-dim AdS bulk

$$
\begin{aligned}
ds^2 &= g_{\mu\nu}dx^\mu dx^\nu = \frac{\alpha^2}{\sinh^2 z}\left(dz^2 + d\Omega_4^2\right) \\
d\Omega_4^2 &= d\xi^2 + \sin^2\xi d\Omega_3^2
\end{aligned}
$$

α is the AdS radius, related to cc of AdS bulk

$d\Omega_3$ metric on the 3-sphere

CASIMIR ENERGY

(a) One-brane Casimir energy $= 0$

(b) Bulk Casimir energy (L brane separation, $\mathcal{R}$ brane radius)

$$
\zeta(s|L_5) = \frac{\mu^{-2s}}{6}\sum_{n,l=1}^{\infty}(l+1)(l+2)(2l+3)\left[\left(\frac{\pi n}{L}\right)^2 + \mathcal{R}^{-2}\left(l^2+3l+\frac{9}{4}\right)\right]^{-s}
$$

The Casimir energy density (pressure) follows (non-trivially!)

$\Longrightarrow$ Euclidean metric of the 5-dim AdS bulk

$$\begin{aligned} ds^2 &= g_{\mu\nu}dx^\mu dx^\nu = \frac{\alpha^2}{\sinh^2 z}\left(dz^2 + d\Omega_4^2\right) \\ d\Omega_4^2 &= d\xi^2 + \sin^2\xi \, d\Omega_3^2 \end{aligned}$$

α is the AdS radius, related to cc of AdS bulk

$d\Omega_3$ metric on the 3-sphere

CASIMIR ENERGY

(a) One-brane Casimir energy $= 0$

(b) Bulk Casimir energy (L brane separation, $\mathcal{R}$ brane radius)

$$\zeta(s|L_5) = \frac{\mu^{-2s}}{6}\sum_{n,l=1}^{\infty}(l+1)(l+2)(2l+3)\left[\left(\frac{\pi n}{L}\right)^2 + \mathcal{R}^{-2}\left(l^2+3l+\frac{9}{4}\right)\right]^{-s}$$

The Casimir energy density (pressure) follows (non-trivially!)

$$\mathcal{E}_{\text{Cas}} = \frac{\hbar c}{2L\mathcal{R}^4}\zeta\left(-\frac{1}{2}|L_5\right) = -\frac{\hbar c \pi^3}{36L^6}\left[\frac{\pi^2}{315} - \frac{1}{240}\left(\frac{L}{\mathcal{R}}\right)^2 + \mathcal{O}\left(\frac{L}{\mathcal{R}}\right)^4\right]$$

C. Supergraviton Theories

- ⟹ Cognola, Elizalde, Zerbini, PLB624 (2005) 70
- ⟹ Cognola, Elizalde, Nojiri, Odintsov, Zerbini, MPLA19 (2004) 1435
- ⟹ Boulanger, Damour, Gualtieri, Henneaux, NPB597 (2001) 127
- ⟹ Sugamoto, Grav. Cosmol. 9 (2003) 91
- ⟹ Arkani-Hamed, Cohen, Georgi, PRL86 (2001) 4757
- ⟹ Arkani-Hamed, Georgi, Schwartz, Ann. Phys. (NY) 305 (2003) 96
- ⟹ Hill, Pokorski, Wang; Damour, Kogan, Papazoglou; Deffayet, Mourad

- Effective potential for a multi-graviton model with supersymmetry (discretized dim's, deconstruction)

C. Supergraviton Theories

$\Longrightarrow$ Cognola, Elizalde, Zerbini, PLB624 (2005) 70

$\Longrightarrow$ Cognola, Elizalde, Nojiri, Odintsov, Zerbini, MPLA19 (2004) 1435

$\Longrightarrow$ Boulanger, Damour, Gualtieri, Henneaux, NPB597 (2001) 127

$\Longrightarrow$ Sugamoto, Grav. Cosmol. 9 (2003) 91

$\Longrightarrow$ Arkani-Hamed, Cohen, Georgi, PRL86 (2001) 4757

$\Longrightarrow$ Arkani-Hamed, Georgi, Schwartz, Ann. Phys. (NY) 305 (2003) 96

$\Longrightarrow$ Hill, Pokorski, Wang; Damour, Kogan, Papazoglou; Deffayet, Mourad

- Effective potential for a multi-graviton model with supersymmetry (discretized dim's, deconstruction)

- The bulk is a flat manifold with torus topology $\mathbb{R} \times \mathbb{T}^3$
$\Longrightarrow$ shown the induced cosmological constant could be positive due to topological contributions

C. Supergraviton Theories

$\Longrightarrow$ Cognola, Elizalde, Zerbini, PLB624 (2005) 70

$\Longrightarrow$ Cognola, Elizalde, Nojiri, Odintsov, Zerbini, MPLA19 (2004) 1435

$\Longrightarrow$ Boulanger, Damour, Gualtieri, Henneaux, NPB597 (2001) 127

$\Longrightarrow$ Sugamoto, Grav. Cosmol. 9 (2003) 91

$\Longrightarrow$ Arkani-Hamed, Cohen, Georgi, PRL86 (2001) 4757

$\Longrightarrow$ Arkani-Hamed, Georgi, Schwartz, Ann. Phys. (NY) 305 (2003) 96

$\Longrightarrow$ Hill, Pokorski, Wang; Damour, Kogan, Papazoglou; Deffayet, Mourad

- Effective potential for a multi-graviton model with supersymmetry (discretized dim's, deconstruction)
- The bulk is a flat manifold with torus topology $\mathbb{R} \times \mathbb{T}^3$ $\Longrightarrow$ shown the induced cosmological constant could be positive due to topological contributions
- Previously considered in $\mathbb{R}^4$

$\Longrightarrow$ Allow for non-nearest-neighbor couplings

$\Longrightarrow$ Allow for non-nearest-neighbor couplings

$\Longrightarrow$ Multi-graviton model defined by taking $N-$copies of fields, with graviton $h_{n\mu\nu}$ and Stückelberg fields $A_{n\mu}$, φ_n

$\Longrightarrow$ Allow for non-nearest-neighbor couplings

$\Longrightarrow$ Multi-graviton model defined by taking $N-$copies of fields, with graviton $h_{n\mu\nu}$ and Stückelberg fields $A_{n\mu}$, φ_n

$\Longrightarrow$ Lagrangian of theory generalization of Kan-Shiraishi

$$\mathcal{L} = \sum_{n=0}^{N-1} \Big[-\frac{1}{2}\partial_\lambda h_{n\mu\nu}\partial^\lambda h_n^{\mu\nu} + \partial_\lambda h_{n\mu}^\lambda \partial_\nu h_n^{\mu\nu} - \partial_\mu h_n^{\mu\nu}\partial_\nu h_n + \frac{1}{2}\partial_\lambda h_n \partial^\lambda h_n$$
$$-\frac{1}{2}\left(m^2 \Delta h_{n\mu\nu}\Delta h_n^{\mu\nu} - \left(\Delta h_n\right)^2 \right) - 2\left(m\Delta^\dagger A_n^\mu + \partial^\mu \varphi_n \right)\left(\partial^\nu h_{n\mu\nu} - \partial_\mu h_n\right)$$
$$-\frac{1}{2}\left(\partial_\mu A_{n\nu} - \partial_\nu A_{n\mu}\right)\left(\partial^\mu A_n^\nu - \partial^\nu A_n^\mu\right)\Big]$$

Δ, $\Delta^\dagger$ difference operators

$\Longrightarrow$ Allow for non-nearest-neighbor couplings

$\Longrightarrow$ Multi-graviton model defined by taking $N-$copies of fields, with graviton $h_{n\mu\nu}$ and Stückelberg fields $A_{n\mu}$, φ_n

$\Longrightarrow$ Lagrangian of theory generalization of Kan-Shiraishi

$$\mathcal{L} = \sum_{n=0}^{N-1} \Big[-\frac{1}{2}\partial_\lambda h_{n\mu\nu}\partial^\lambda h_n^{\mu\nu} + \partial_\lambda h_{n\mu}^{\lambda}\partial_\nu h_n^{\mu\nu} - \partial_\mu h_n^{\mu\nu}\partial_\nu h_n + \frac{1}{2}\partial_\lambda h_n \partial^\lambda h_n$$
$$-\frac{1}{2}\left(m^2 \Delta h_{n\mu\nu}\Delta h_n^{\mu\nu} - (\Delta h_n)^2\right) - 2\left(m\Delta^\dagger A_n^\mu + \partial^\mu \varphi_n\right)\left(\partial^\nu h_{n\mu\nu} - \partial_\mu h_n\right)$$
$$-\frac{1}{2}\left(\partial_\mu A_{n\nu} - \partial_\nu A_{n\mu}\right)\left(\partial^\mu A_n^\nu - \partial^\nu A_n^\mu\right)\Big]$$

Δ, $\Delta^\dagger$ difference operators operate on the indices n as

$$\Delta\phi_n \equiv \sum_{k=0}^{N-1} a_k \phi_{n+k},\ \Delta^\dagger\phi_n \equiv \sum_{k=0}^{N-1} a_k\phi_{n-k},\ \sum_{k=0}^{N-1} a_k = 0,$$

a_k are N constants and ϕ_n can be identified with periodic fields on a lattice with N sites if periodic boundary conditions, $\phi_{n+N} = \phi_n$, are imposed

$\Longrightarrow$ Allow for non-nearest-neighbor couplings

$\Longrightarrow$ Multi-graviton model defined by taking $N-$copies of fields, with graviton $h_{n\mu\nu}$ and Stückelberg fields $A_{n\mu}$, φ_n

$\Longrightarrow$ Lagrangian of theory generalization of Kan-Shiraishi

$$\mathcal{L} = \sum_{n=0}^{N-1} \Big[-\frac{1}{2}\partial_\lambda h_{n\mu\nu}\partial^\lambda h_n^{\mu\nu} + \partial_\lambda h_{n\mu}^\lambda \partial_\nu h_n^{\mu\nu} - \partial_\mu h_n^{\mu\nu}\partial_\nu h_n + \frac{1}{2}\partial_\lambda h_n \partial^\lambda h_n$$
$$-\frac{1}{2}\left(m^2 \Delta h_{n\mu\nu}\Delta h_n^{\mu\nu} - (\Delta h_n)^2\right) - 2\left(m\Delta^\dagger A_n^\mu + \partial^\mu \varphi_n\right)\left(\partial^\nu h_{n\mu\nu} - \partial_\mu h_n\right)$$
$$-\frac{1}{2}\left(\partial_\mu A_{n\nu} - \partial_\nu A_{n\mu}\right)\left(\partial^\mu A_n^\nu - \partial^\nu A_n^\mu\right)\Big]$$

Δ, $\Delta^\dagger$ difference operators operate on the indices n as

$\Delta\phi_n \equiv \sum_{k=0}^{N-1} a_k \phi_{n+k}$, $\Delta^\dagger \phi_n \equiv \sum_{k=0}^{N-1} a_k \phi_{n-k}$, $\sum_{k=0}^{N-1} a_k = 0$,

a_k are N constants and ϕ_n can be identified with periodic fields on a lattice with N sites if periodic boundary conditions, $\phi_{n+N} = \phi_n$, are imposed

(Δ becomes usual differentiation operator in properly defined continuum limit)

- In the multi-graviton model the induced cosmological constant can be positive only if # of massive gravitons is sufficiently large

- In the multi-graviton model the induced cosmological constant can be positive only if # of massive gravitons is sufficiently large
- In the supersymmetric case the cosmological constant can be positive if one imposes anti-periodic BC in the fermionic sector

- In the multi-graviton model the induced cosmological constant can be positive only if # of massive gravitons is sufficiently large
- In the supersymmetric case the cosmological constant can be positive if one imposes anti-periodic BC in the fermionic sector
- Topological effects discussed may also be relevant in the study of electroweak symmetry breaking in models with a similar type of non-nearest-neighbour couplings, for the deconstruction issue

- In the multi-graviton model the induced cosmological constant can be positive only if # of massive gravitons is sufficiently large
- In the supersymmetric case the cosmological constant can be positive if one imposes anti-periodic BC in the fermionic sector
- Topological effects discussed may also be relevant in the study of electroweak symmetry breaking in models with a similar type of non-nearest-neighbour couplings, for the deconstruction issue
- Case of the torus topology: top. contributions to the eff. potential have always a fixed sign, depending on the BC one imposes

- In the multi-graviton model the induced cosmological constant can be positive only if # of massive gravitons is sufficiently large
- In the supersymmetric case the cosmological constant can be positive if one imposes anti-periodic BC in the fermionic sector
- Topological effects discussed may also be relevant in the study of electroweak symmetry breaking in models with a similar type of non-nearest-neighbour couplings, for the deconstruction issue
- Case of the torus topology: top. contributions to the eff. potential have always a fixed sign, depending on the BC one imposes
  - They are negative for periodic fields

- In the multi-graviton model the induced cosmological constant can be positive only if # of massive gravitons is sufficiently large
- In the supersymmetric case the cosmological constant can be positive if one imposes anti-periodic BC in the fermionic sector
- Topological effects discussed may also be relevant in the study of electroweak symmetry breaking in models with a similar type of non-nearest-neighbour couplings, for the deconstruction issue
- Case of the torus topology: top. contributions to the eff. potential have always a fixed sign, depending on the BC one imposes
  - They are negative for periodic fields
  - They are positive for anti-periodic fields.

- In the multi-graviton model the induced cosmological constant can be positive only if # of massive gravitons is sufficiently large
- In the supersymmetric case the cosmological constant can be positive if one imposes anti-periodic BC in the fermionic sector
- Topological effects discussed may also be relevant in the study of electroweak symmetry breaking in models with a similar type of non-nearest-neighbour couplings, for the deconstruction issue
- Case of the torus topology: top. contributions to the eff. potential have always a fixed sign, depending on the BC one imposes
  - They are negative for periodic fields
  - They are positive for anti-periodic fields.
- Topology provides a natural mechanism which permits to have a positive cc in the multi-supergravity model with anti-periodic fermions

- In the multi-graviton model the induced cosmological constant can be positive only if # of massive gravitons is sufficiently large
- In the supersymmetric case the cosmological constant can be positive if one imposes anti-periodic BC in the fermionic sector
- Topological effects discussed may also be relevant in the study of electroweak symmetry breaking in models with a similar type of non-nearest-neighbour couplings, for the deconstruction issue
- Case of the torus topology: top. contributions to the eff. potential have always a fixed sign, depending on the BC one imposes
    - They are negative for periodic fields
    - They are positive for anti-periodic fields.
- Topology provides a natural mechanism which permits to have a positive cc in the multi-supergravity model with anti-periodic fermions
- The value of the cc is regulated by the corresponding size of the torus (one can most naturally use the minimum number, N = 3, of copies of bosons and fermions), and are not far from observational values

Conclusions

- At terrestrial scales, vacuum energy fluctuations are becoming important (MEMs, NEMs, nanotubes)

Conclusions

- At terrestrial scales, vacuum energy fluctuations are becoming important (MEMs, NEMs, nanotubes)
- Topology + Vacuum energy may prove to be a fundamental driving force at cosmological scale

Conclusions

- At terrestrial scales, vacuum energy fluctuations are becoming important (MEMs, NEMs, nanotubes)
- Topology + Vacuum energy may prove to be a fundamental driving force at cosmological scale
- Zeta functions provide a fine, precise, and powerful tool to perform the calculations

Conclusions

- At terrestrial scales, vacuum energy fluctuations are becoming important (MEMs, NEMs, nanotubes)
- Topology + Vacuum energy may prove to be a fundamental driving force at cosmological scale
- Zeta functions provide a fine, precise, and powerful tool to perform the calculations

Thank You !