

THE RUNNING SPECTRAL INDEX AS A PROBE OF PHYSICS AT HIGH SCALES

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- ★ INFLATION : Simple (SUSY) Model
- ★ WMAP Indications for a Running Scalar Spectral Index
- ★ Attempts to produce $d n/d \ln k$
- ★ General Remarks

Based on

G. Ballesteros, J.A. Casas, J.R.E. [hep-ph/0601134] JCAP 0603(2006)001
+ R. Ruiz de Austri [work in progress]

INFLATION

Can explain flatness, homogeneity and isotropy plus huge entropy of our Universe, through a period of superluminal expansion driven by a new scalar field ϕ with nearly constant potential energy $a \rightarrow e^{Ht} a_0$ $H \equiv \dot{a}/a$

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V}{\partial \phi} \quad H^2 = \frac{1}{3M_{Pl}^2} \left(V + \frac{1}{2} \dot{\phi}^2 \right)$$

$$3H\dot{\phi} \approx -\frac{\partial V}{\partial \phi} \quad H^2 = \frac{1}{3M_{Pl}^2} V$$

Slow-Roll

$$\epsilon = \frac{1}{2} M_{Pl}^2 \left(\frac{V'}{V} \right)^2$$

Determined by

$$\eta = M_{Pl}^2 \frac{V''}{V} \ll 1$$

* e-folds of expansion

$$N_e = \int_{t_i}^{t_e} H dt \quad \sim 50-60$$

SIMPLE EXAMPLE

Binetruy-Dvali '96

Use flat directions, ubiquitous in SUSY models

$U(1)$ gauged $\phi(0) \quad H_+ (+1) \quad H_- (-1)$

$$W = \lambda \tilde{\phi} \hat{H}_+ \hat{H}_- - \mu^2 \tilde{\phi}$$

with

$$\Rightarrow V_F = |\lambda H_+ H_- - \mu^2|^2 + \lambda^2 |\phi|^2 (|H_+|^2 + |H_-|^2)$$

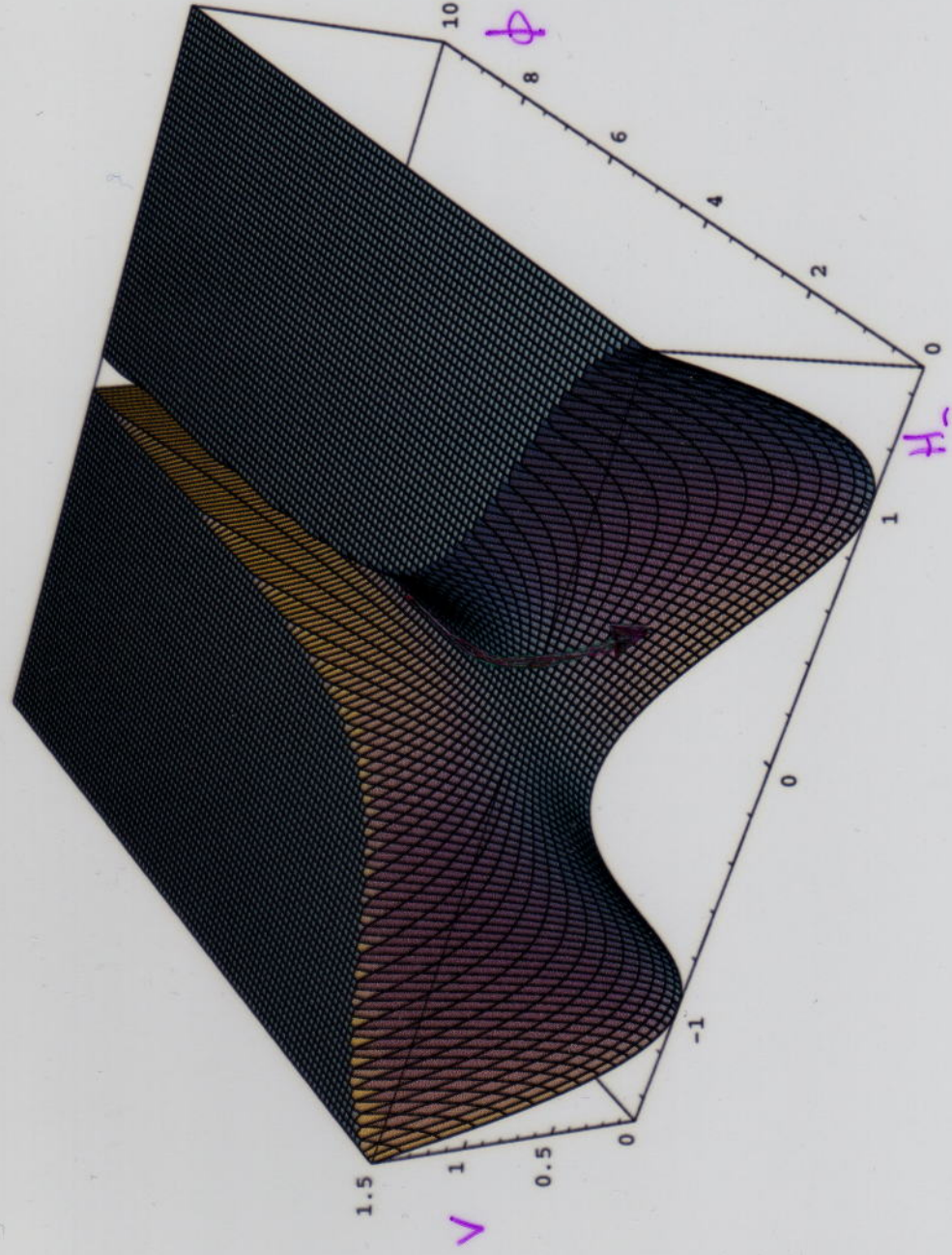
$$V_D = \frac{1}{2} g^2 [|H_+|^2 - |H_-|^2 + \frac{k}{g^2}]^2$$

↑ Fayet-Iliopoulos term ($\sim M^2$)

$\therefore$ Simple and well motivated from particle physics POV.

$\Rightarrow$ D-term hybrid inflation model

INFLATON VALLEY



Along valley ($H_+ = H_- = 0, \phi \neq 0$)

$$V \approx \mu^4 + \frac{1}{2} g^2 \xi_D^2$$

Tree-level flatness
lifted by rad. corr.

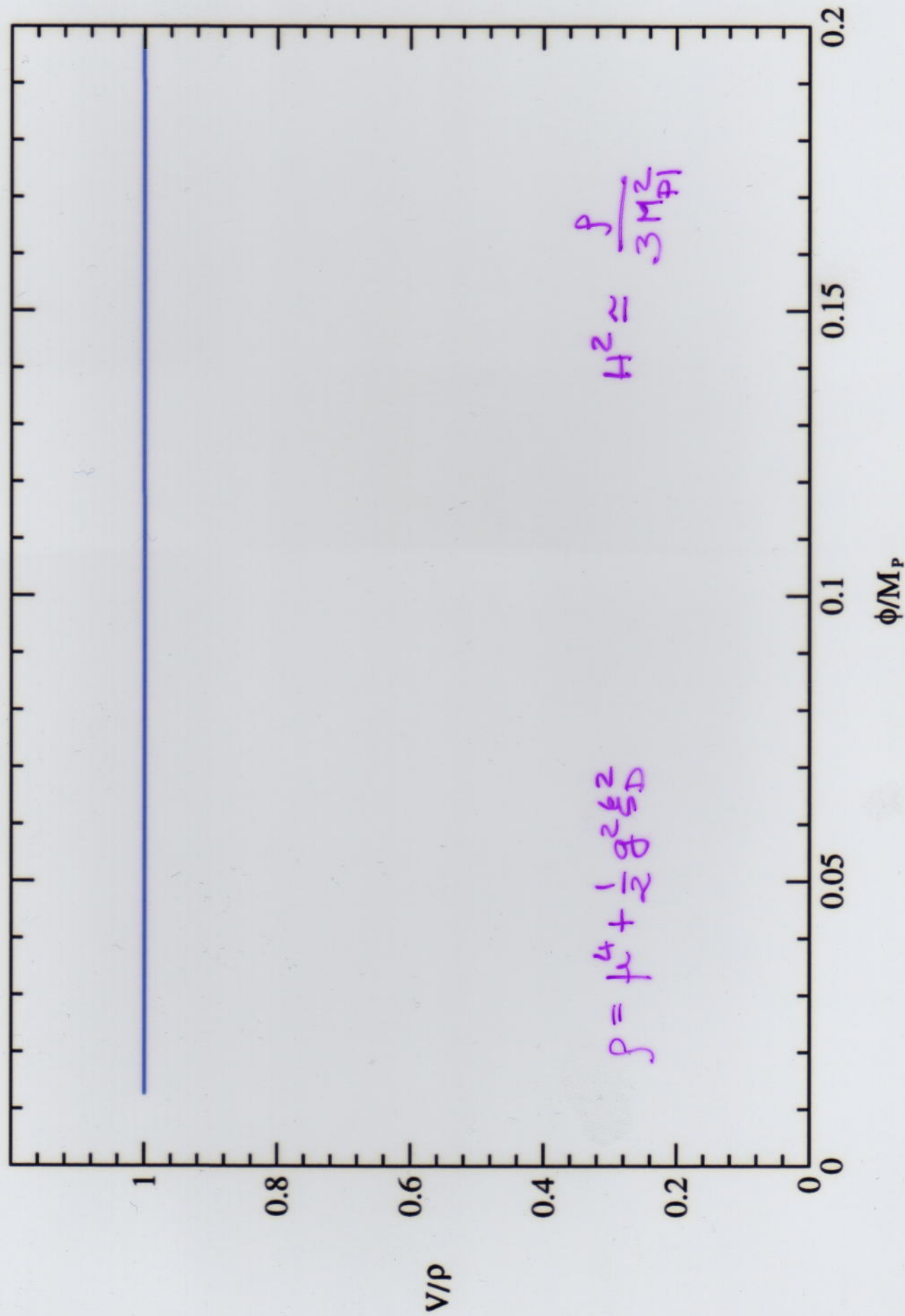


$$V = f + \beta \log(\phi/a)$$

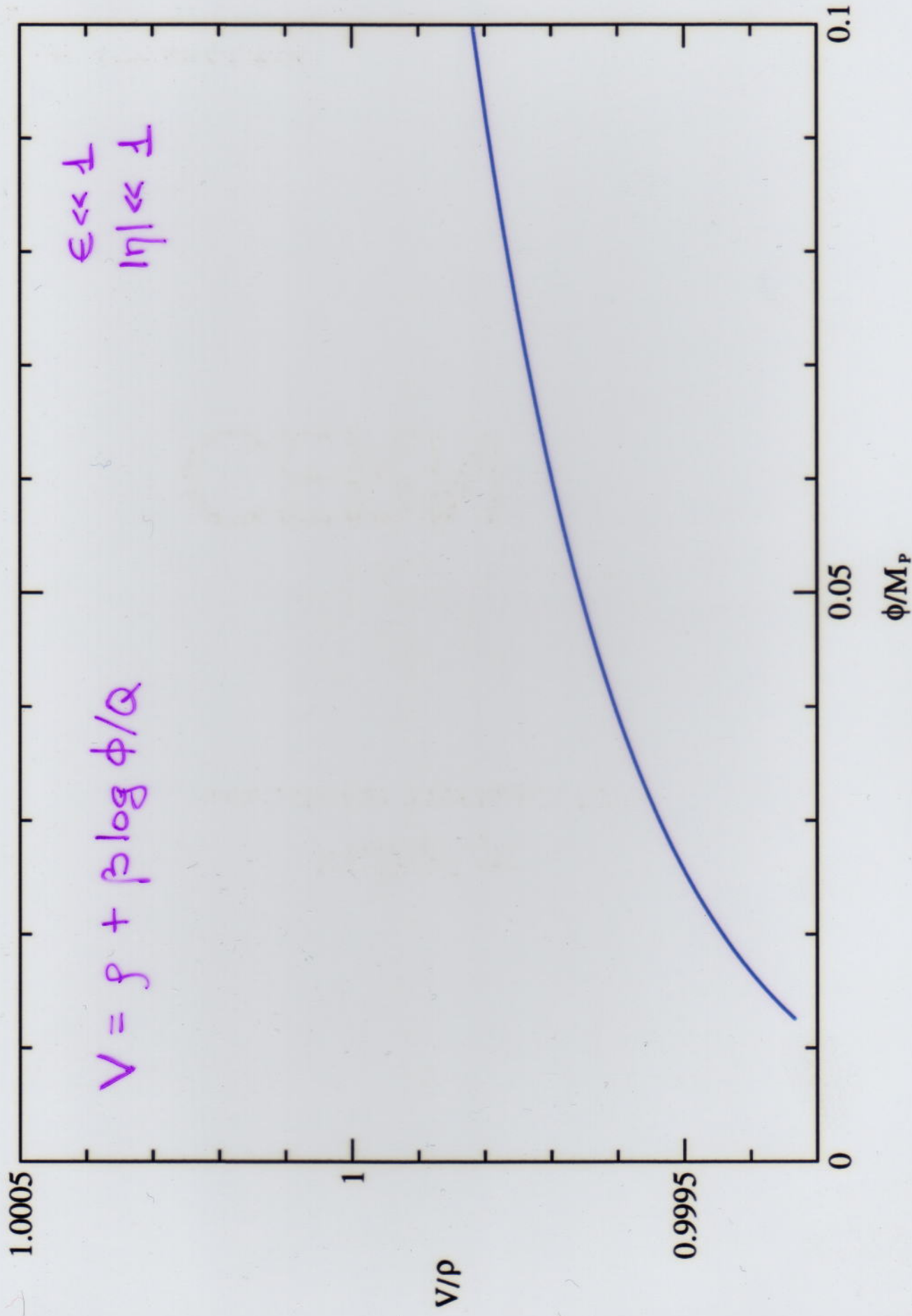


SMALL SLOPE

POTENTIAL ALONG ϕ



POTENTIAL ALONG ϕ



STRUCTURE FORMATION

Through the stretching of quantum fluctuations of ϕ

$$\hat{\phi}(x,t) = \phi(t) + \varphi(x,t)$$

frozen at superhorizon scales with amplitudes det by H and a flat spectrum

$$\langle \varphi^2 \rangle = \frac{H^2}{(2\pi)^2} \int \frac{dk}{k}$$

$k \rightarrow$ comoving momentum (scale $\sim 1/k$)

Diff: $\varphi \Rightarrow$ Diff $t_{\text{end}} \Rightarrow \delta t = \varphi / \dot{\phi}$

$\Rightarrow$ Inhomogeneities $\delta \rho / \rho \sim H \delta t \sim H^2 / \dot{\phi}$

$$\Rightarrow \frac{\delta \rho}{\rho} = c \cdot \frac{V^{3/2}}{M_{\text{Pl}}^3 V'} \Big|_{\text{end}} \quad \sim \text{nearby } k\text{-indep}$$

Parametrize departure from scale independence with

SCALAR SPECTRAL INDEX n

$$\left(\frac{\delta P}{P}\right)^2 \propto k^{n-1}$$

$n = 1 \Rightarrow$ SCALE INDEP SPECTRUM.

In terms of slow-roll parameters

$$n \approx 1 + 2\eta - 6\epsilon$$

Inflation typically predicts $n \approx 1$

WMAP INDICATIONS FOR RUNNING n

Λ CDM, constant n :

$$n(0.002) = 0.951 \pm 0.016$$

Λ CDM, running n :

$$n(0.002) = 1.050^{+0.059}_{-0.058}$$

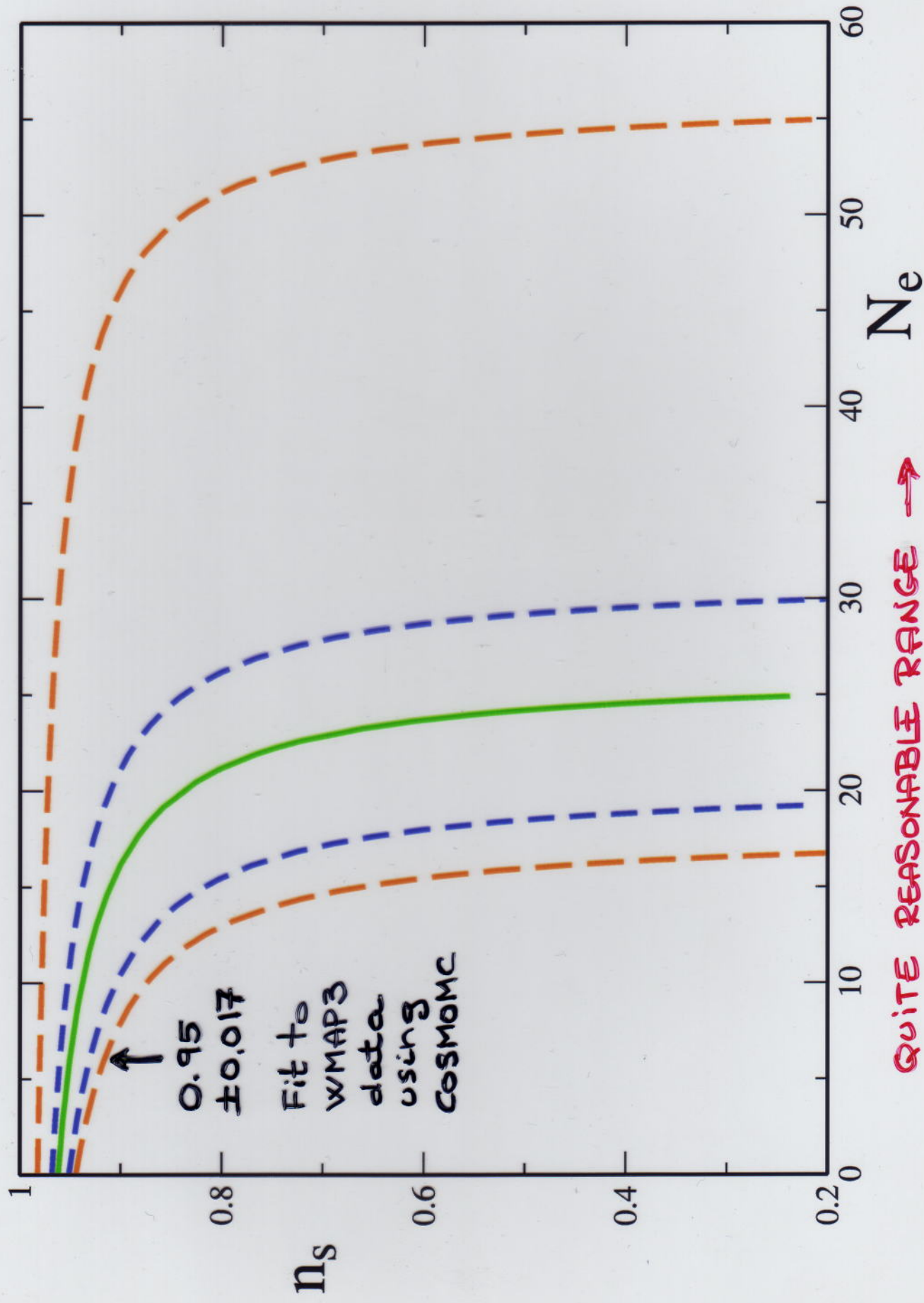
$$\frac{dn}{d \ln k} = -0.055^{+0.030}_{-0.031}$$

Still with us
after 3 year's
worth of data

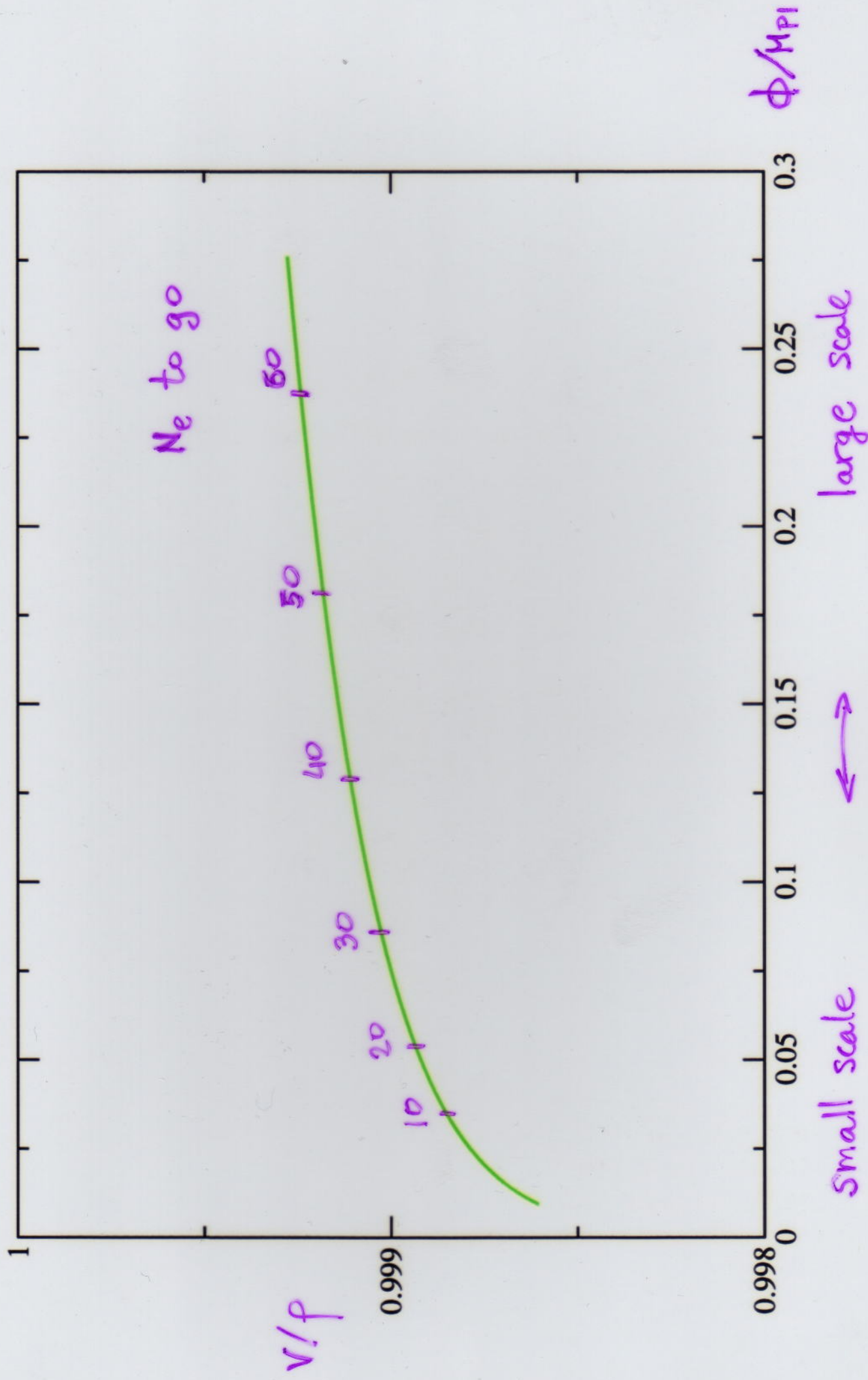
Large $\frac{dn}{d \ln k} \Rightarrow$ gives $n < 1$ for larger k .



$$n = 1 - \frac{1}{N_e - \ln(k/k_*)} \Rightarrow n < 1, \quad \frac{dn}{d \ln k} = -(n-1)^2$$



ROLLING ϕ SCANS IN ENERGY AND $1/k$



CAN INFLATION ACCOMMODATE $\frac{dn}{d\ln k}$ NATURALLY?

- Start with simple D-term hybrid model
- Try to modify* the potential at high energy to reproduce $dn/d\ln k$ at large scales
- * in a well motivated way!

- In terms of slow-roll parameters

$$\frac{dn}{d\ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\xi$$

with $\xi \equiv M_{Pl}^4 \frac{V' V'''}{V^2}$

SIZEABLE $\frac{dn}{d\ln k}$ requires sizeable $\xi \rightarrow$ effect in V'''

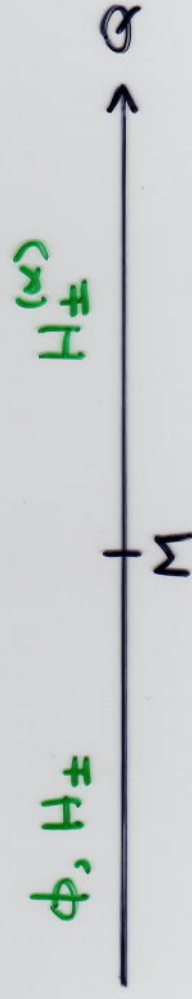
FIRST ATTEMPT

Couplings in $V(\phi)$ getting strong in the UV

→ changes shape of inflaton potential

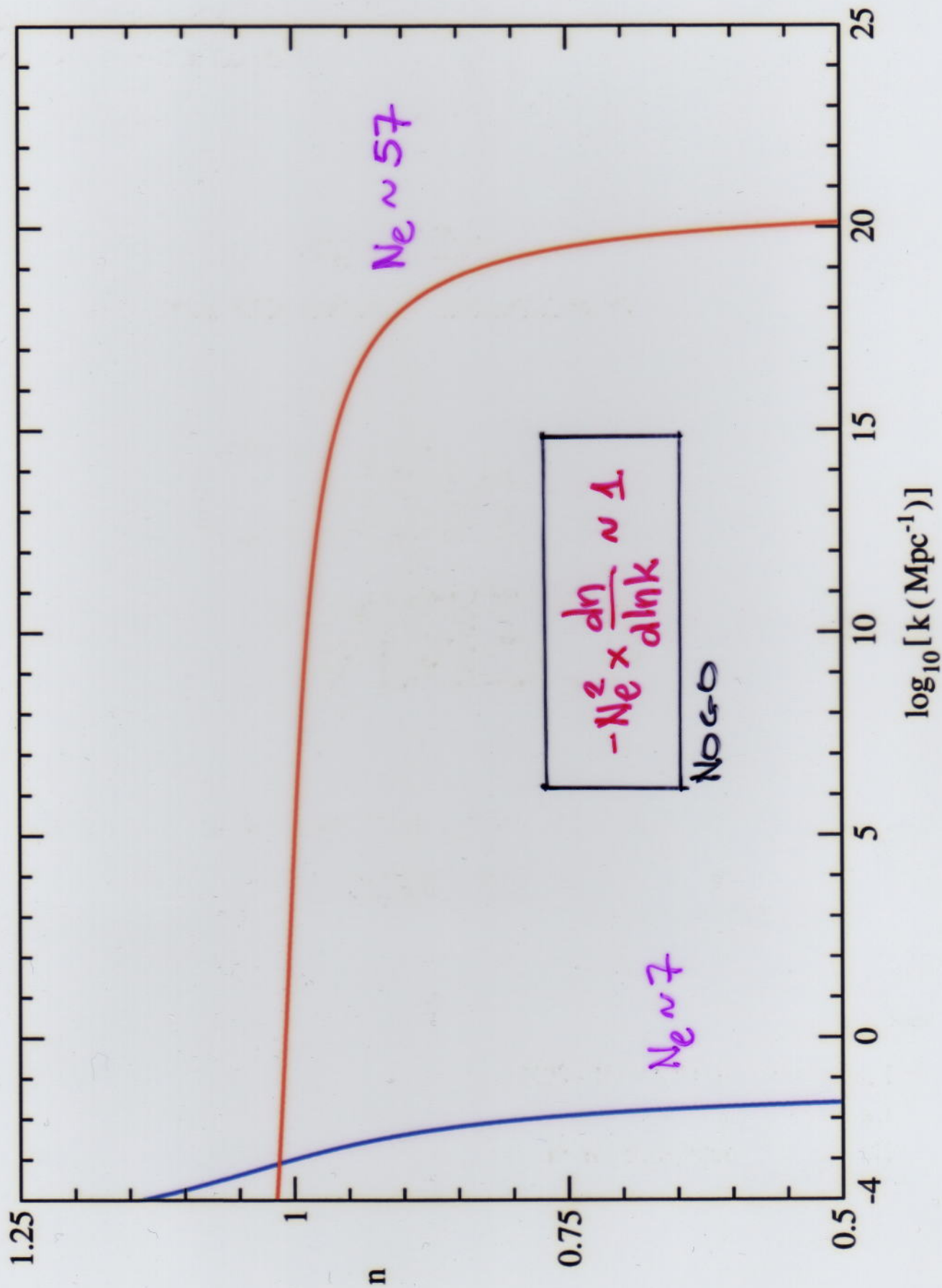
SECOND ATTEMPT

New Physics threshold crossed by ϕ



$$V = \rho + \beta \log \frac{\phi}{Q} + \frac{1}{2} \tilde{\beta} \log \frac{M^2 + \chi^2 \phi^2}{Q^2}$$

CAN GET LARGE $dn/dlnk$ BUT $N_e \downarrow$



THIRD (SUCCESSFUL!) ATTEMPT

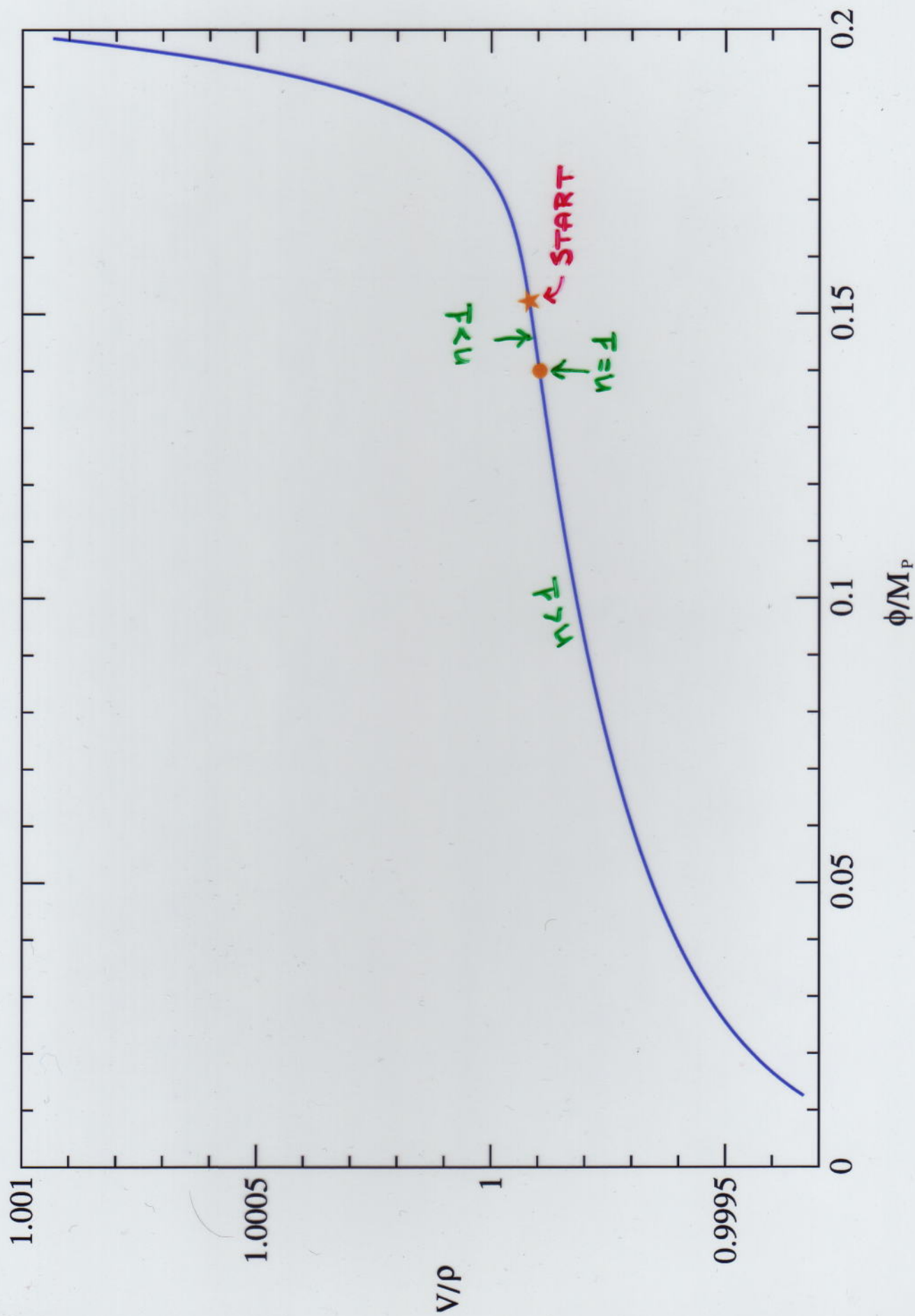
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Effect of heavy physics through NROs

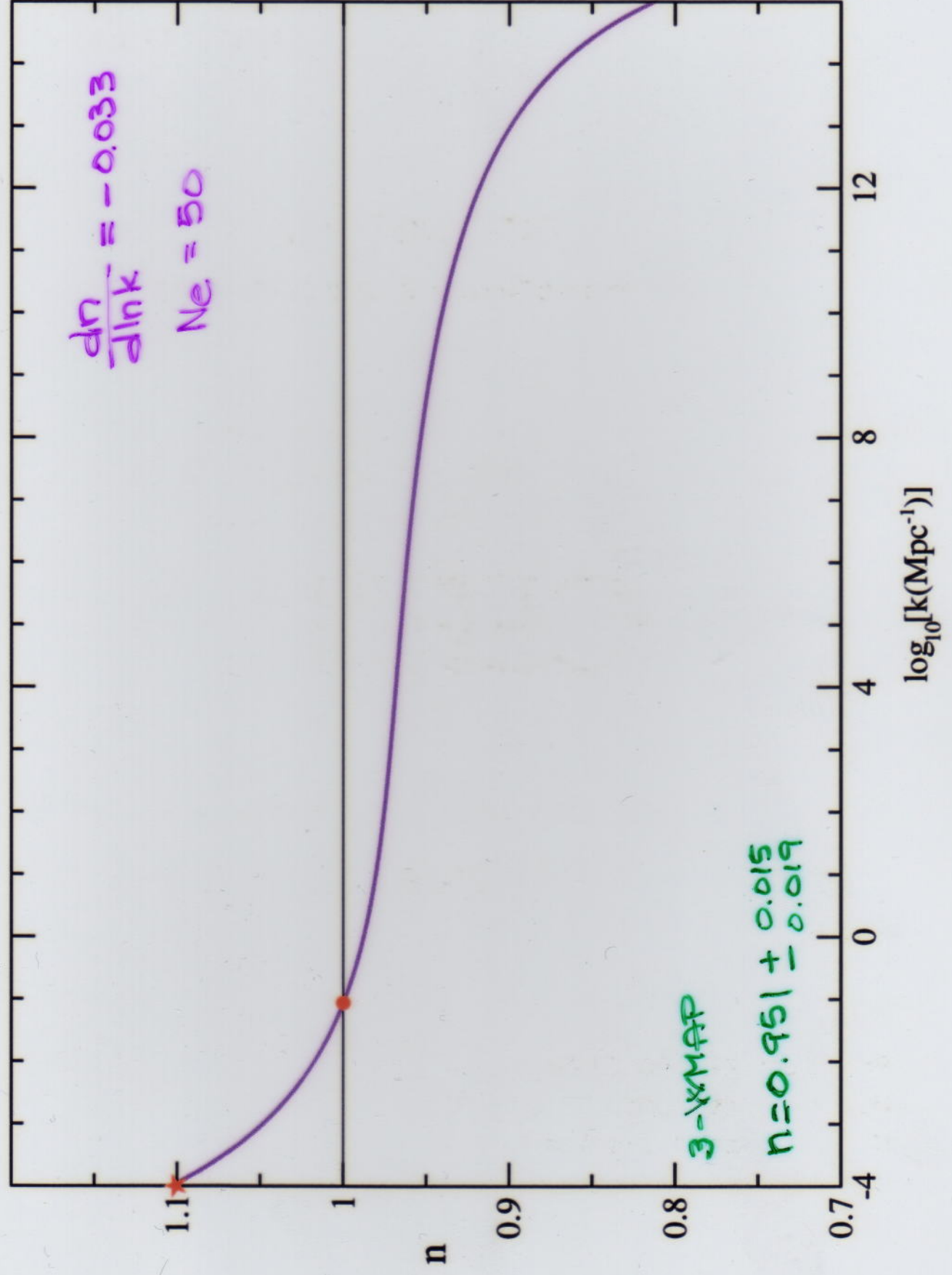
$$V = g + \beta \log \phi / Q + \phi^4 \frac{\phi^{2N}}{M^{2N}}$$

NOTE : choose $\phi_{in} \ll M$ for effective theory
to be valid

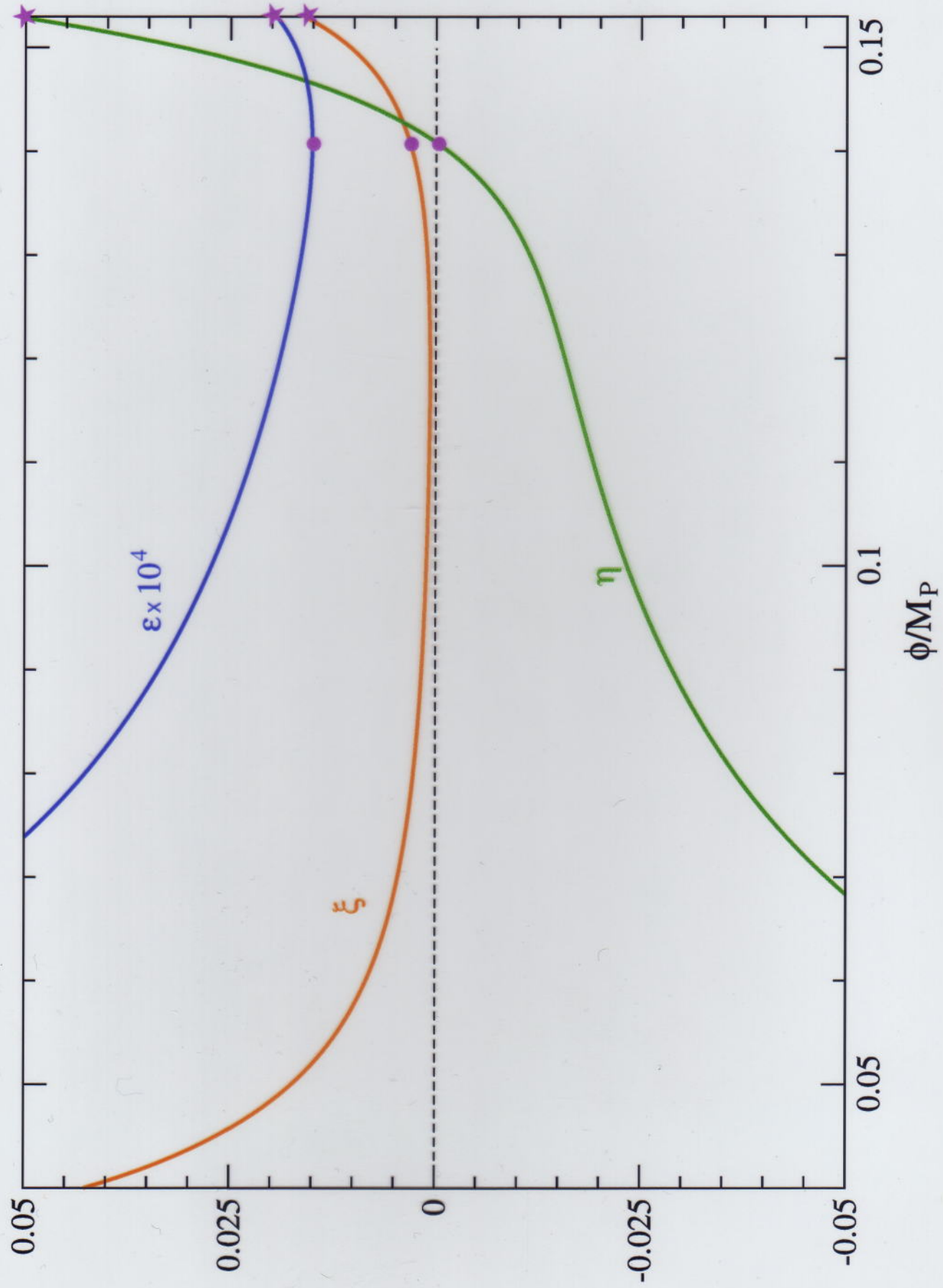
INFLATON POTENTIAL WITH NRO



RUNNING SPECTRAL INDEX ($N=9$)



SLOW-ROLL PARAMETERS



SOME COMMENTS

- Effect of NRO can be important because the potential is so flat
- Effect on 3rd derivative enhanced wrt to 1st and 2nd : allows to keep ϵ, η small but ξ larger
 $\Rightarrow$ helps in $\frac{dn}{dlnk} \uparrow$ keeping N_e large

SOME COMMENTS (CONT'D)

- Lifting of susy flat directions by high-order NROs is naturally expected (MSSM, strings)
- V can arise from reasonable superpotentials like

$$W = \lambda \phi H_+ H_- + \frac{1}{2} m \phi^2 + \frac{1}{(P+2)} \phi' \phi^2 \frac{\phi^P}{M^P}$$



$$V = V_D + \phi^4 \frac{\phi^2}{m^2} \frac{\phi^{4P}}{M^{4P}}$$

⇒ Gives $N=9$ for $P=4$

REMARKS / CONCLUSIONS

★ WMAP INDICATION OF RUNNING n IS
VERY INTERESTING FOR PHYSICS AT
HIGH ENERGY SCALES

★ NOT TOO DIFFICULT TO ACCOMMODATE
IN SIMPLE AND WELL MOTIVATED MODELS

★ STILL,... IT IS PUZZLING :

WHY SHOULD THE RUNNING OCCUR NEAR $n \approx 60$?

→ NEW COINCIDENCE PROBLEM ?

(Mersini-Houghton)