THE RUNNING SPECTRAL INDEX
AS A PROBE OF PHYSICS AT HIGH SCALES

DSU'06

** INFLATION : Simple (susy) Model
** WMAP Indications for a Running Scalar Spectral Index
** Attempts to produce dn/dlnk
** General Remarks

Based on

+ R. Ruiz de Austri [work in progress]
**INFLATION**

Can explain flatness, homogeneity and isotropy plus huge entropy of our Universe, through a period of superluminal expansion driven by a new scalar field $\phi$ with nearly constant potential energy

$$a \rightarrow e^{Ht} a_0 \quad H = \frac{\dot{a}}{a}$$

\[
\dot{\phi} + 3H\dot{\phi} = -\frac{\partial V}{\partial \phi} \\
H^2 = \frac{1}{3M_{Pl}^2} (V + \frac{1}{2} \phi^2)
\]

**SLOW-ROLL**

\[
3H\dot{\phi} = -\frac{\partial V}{\partial \phi} \\
H^2 = \frac{1}{3M_{Pl}^2} V
\]

determined by

\[
e = \frac{1}{2} M_{Pl}^2 \left(\frac{V'}{V}\right)^2 \\
\eta = M_{Pl}^2 \frac{V''}{V} \ll 1
\]

\* e-folds of expansion

$$Ne = \int_{t_i}^{t_e} H dt \approx 50-60$$
**SIMPLE EXAMPLE**

use flat directions, ubiquitous in SUSY models

$U(1)$ gauged $\phi(0)$ $H_4(+1)$ $H_(-1)$

with

$W = 2\phi \hat{H}_4 \hat{H}_- - \mu^2 \phi$

$V_F = |2H_4H_- - \mu^2|^2 + \chi^2 |\phi|^2 (|H_4|^2 + |H_-|^2)$

$V_D = \frac{1}{2} g^2 \left[ |H_4|^2 - |H_-|^2 + \xi_D \right]^2$

Fayet-Iliopoulos term ($\mu M^2$)

∴ Simple and well motivated from particle physics POV.

∴ D-term hybrid inflation model
Potential along $\phi$

$$\frac{p}{\sqrt{\phi}} \geq \frac{\phi}{\frac{3}{2} M_{pl}^2}$$

$$p = \frac{4}{3} + \frac{1}{2} \frac{g_2}{g_D}$$
POTENTIAL ALONG $\phi$

$$V = g + \beta \log \frac{\phi}{Q}$$
STRUCTURE FORMATION

Through the stretching of quantum fluctuations of $\phi$

$\hat{\phi}(x,t) = \phi(t) + \Phi(x,t)$

Frozen at super-horizon scales with amplitudes set by $H$
and a flat spectrum

$\langle \Phi^2 \rangle = \frac{H^2}{(2\pi)^2} \int \frac{dk}{k}$

$k$ = comoving momentum (scale $\approx 1/k$)

Diff. $\Phi$ $\Rightarrow$ Diff. tend $\Rightarrow$ $\delta t = \Phi/\dot{\Phi}$

$\Rightarrow$ Inhomogeneities $\delta \rho/\rho \sim H\delta t \sim H^2/\dot{\Phi}$

$\Rightarrow$ $\frac{\delta \rho}{\rho} = C \cdot \frac{\sqrt{3/2}}{M_{Pl}^3 V^{1/2}} \left|_{\text{lead}} \right. n \text{ nearly } k \text{-indep}$
Parametrize departure from scale independence with scalar spectral index $n$

$$(\frac{\delta \rho}{\rho})^2 \propto k^{n-1}$$

$n = 1 \Rightarrow$ Scale indep. spectrum.

In terms of slow-roll parameters

$n = 1 + 2\eta - 6\epsilon$

Inflation typically predicts $n \approx 1$
WMAP INDICATIONS FOR RUNNING $n$

$\Lambda$CDM, constant $n$:

$$n(0.002) = 0.951 \pm 0.016$$

$\Lambda$CDM, running $n$:

$$n(0.002) = 1.050 \pm 0.059$$

$$\frac{dn}{dl\ln k} = -0.055 \pm 0.030$$

$$\pm 0.031$$

Still with us after 3 year's worth of data.

Large $\frac{dn}{dl\ln k}$ \(\Rightarrow\) gives $n<1$ for larger $k$.

Expect $\frac{n}{l\ln k}$
\[ n = 1 - \frac{1}{N_e - \ln(k/k_*)} \Rightarrow n < 1, \quad \frac{dn}{d\ln k} = -(n-1)^2 \]

**Figure**

- **Axes:**
  - y-axis: \( n_s \)
  - x-axis: \( N_e \)

- **Curves and Labels:**
  - Green curve labeled: Fit to WMAP3 data using CosmoMC
  - Dashed orange curve labeled: 0.95 ± 0.017

- **Text:**
  - "Quite reasonable range"
ROLLING \( \phi \) SCANS IN ENERGY AND \( 1/k \)

\[ \frac{v}{\varphi} \]

\[ 0.998 \quad 0.999 \]

\[ 0 \quad 0.05 \quad 0.1 \quad 0.15 \quad 0.2 \quad 0.25 \quad 0.3 \]

\[ \text{small scale} \quad \leftrightarrow \quad \text{large scale} \]

\[ \text{Ne to go} \]

\[ \phi/\varphi_\text{Pl} \]
CAN INFLATION ACCOMODATE $\frac{d\eta}{d\ln k}$ NATURALLY?

→ Start with simple D-term hybrid model

→ Try to modify* the potential at high energy to reproduce $d\eta/d\ln k$ at large scales
   * in a well motivated way!

• In terms of slow-roll parameters

$$\frac{d\eta}{d\ln k} = 16 \epsilon \eta - 24 \epsilon^2 - 2 \xi$$

with $\xi = M_{pl}^4 \frac{V' V'''}{V^2}$

SIZEABLE $\frac{d\eta}{d\ln k}$ requires sizeable $\xi \rightarrow$ effect in $V'''$
FIRST ATTEMPT

Couplings in $V(\phi)$ getting strong in the UV

SECOND ATTEMPT

Changes shape of inflaton potential

New Physics threshold crossed by $\phi$

$\frac{\phi}{\phi_0} = \frac{\phi}{\phi_0} + \frac{1}{2\beta} \log \frac{\phi}{\phi_0} + \frac{M^2 + X_0^2}{2\phi_0^2}$
CAN GET LARGE $dn/d\ln k$ BUT $N_e \leq N_e^\ast$
THIRD (SUCCESSFUL!) ATTEMPT

Effect of heavy physics through NRDS

\[ V = g + \beta \log \phi/Q + \phi^4 \frac{\phi^{2n}}{M^{2n}} \]

Note: choose $\phi_{\text{in}} \ll M$ for effective theory to be valid
Running Spectral Index

\[ \frac{\Delta n}{\Delta n_{\text{eq}}} = -0.033 \]

\[ N_{\text{e}} = 50 \]

\[ n = 0.951 \pm 0.019 \]

\[ \log_{10} [k (\text{Mpc}^{-1})] \]
SLOW-ROLL PARAMETERS

\[ \xi, \eta, \epsilon \times 10^4 \]

\[ \frac{\phi}{M_p} \]
some comments

effect of NRO can be important because the potential is so flat
the 3rd derivative enhanced wrt to effect on 3rd derivative allows to keep E, N small

but & larger helps in keeping Ne large
SOME COMMENTS (CONT'D)

- Lifting of susy flat directions by high-order NROS is naturally expected (MSSM, Strings)

- $V$ can arise from reasonable superpotentials like

\[ W = \lambda \phi H_u H_d + \frac{1}{2} m \phi^2 + \frac{1}{(P+2)} \phi^P \phi^2 \frac{\phi^P}{M_P} \]

\[ \Rightarrow \]

\[ V = V_D + \phi^4 \frac{\phi^2}{m^2} \frac{\phi^{4P}}{M_{4P}} \]

- Gives $N=9$ for $P=4$
REMARKS / CONCLUSIONS

★ WMAP INDICATION OF RUNNING $\Delta$ IS VERY INTERESTING FOR PHYSICS AT HIGH ENERGY SCALES

★ NOT TOO DIFFICULT TO ACCOMODATE IN SIMPLE AND WELL MOTIVATED MODELS

★ STILL,... IT IS PUZZLING: WHY SHOULD THE RUNNING OCCUR NEAR $N_{\sim 60}$?

→ NEW COINCIDENCE PROBLEM?

(Mersini-Houghton)