Halos and voids
in a multifractal model of
the dark matter distribution

José Gaite

Instituto de Matemáticas y Física Fundamental,
CSIC, Madrid (Spain)

PLAN OF THE TALK

1. Cold dark matter structure in the adhesion model
2. Random fractals
3. Scaling of voids
4. Analysis of the SDSS catalogue
5. Multifractals: halos and voids
6. Conclusions.

CDM structure in the adhesion model

- Newtonian equations of motion in co-moving coordinates (peculiar velocity $\boldsymbol{u} = \boldsymbol{v} - H\boldsymbol{r}$):

$$\frac{d\boldsymbol{u}}{dt} + H\boldsymbol{u} = \boldsymbol{g}_T - \boldsymbol{g}_b \equiv \boldsymbol{g}\,.$$

- No initial vorticity $\longrightarrow$ Zeldovich approx.: prolong the linear solution $\boldsymbol{x}(t, \boldsymbol{x}_0) = \boldsymbol{x}_0 + b(t)\,\boldsymbol{g}(\boldsymbol{x}_0)$.
  $\tau := b(t) \Rightarrow$ free motion with velocity $\boldsymbol{g}(\boldsymbol{x}_0)$

- Singularities when lines cross $\longrightarrow$ caustics.

CDM structure in the adhesion model

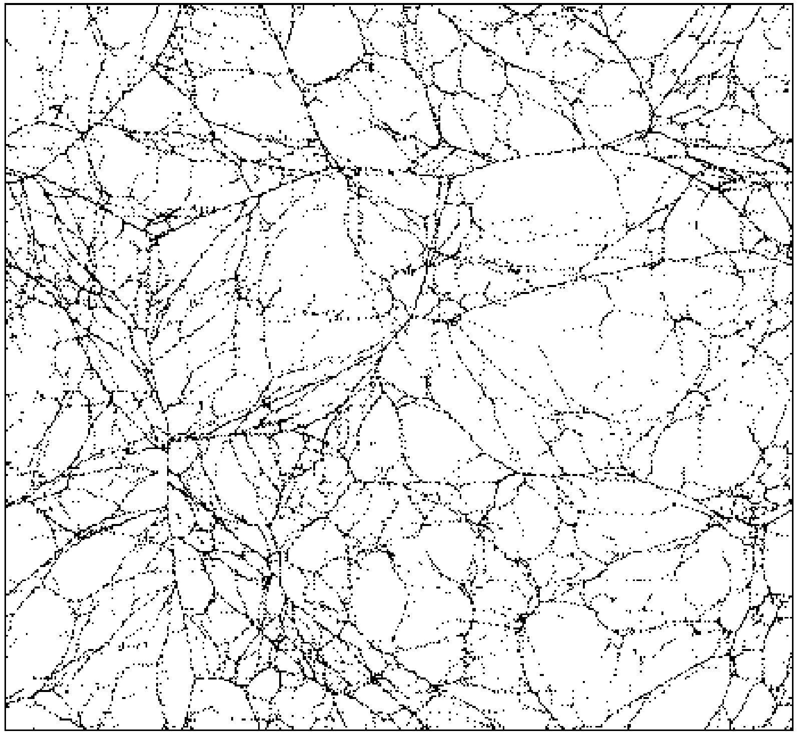
- In caustics particles adhere to each other $\longrightarrow$ viscosity.
- Burgers equation (compressible turbulence):

$$\frac{d\widetilde{\boldsymbol{u}}}{d\tau} = \frac{\partial\widetilde{\boldsymbol{u}}}{\partial\tau} + \widetilde{\boldsymbol{u}} \cdot \nabla\widetilde{\boldsymbol{u}} = \nu\nabla^2\widetilde{\boldsymbol{u}}, \quad \nu \longrightarrow 0,$$
$$\nabla \times \widetilde{\boldsymbol{u}} = 0.$$

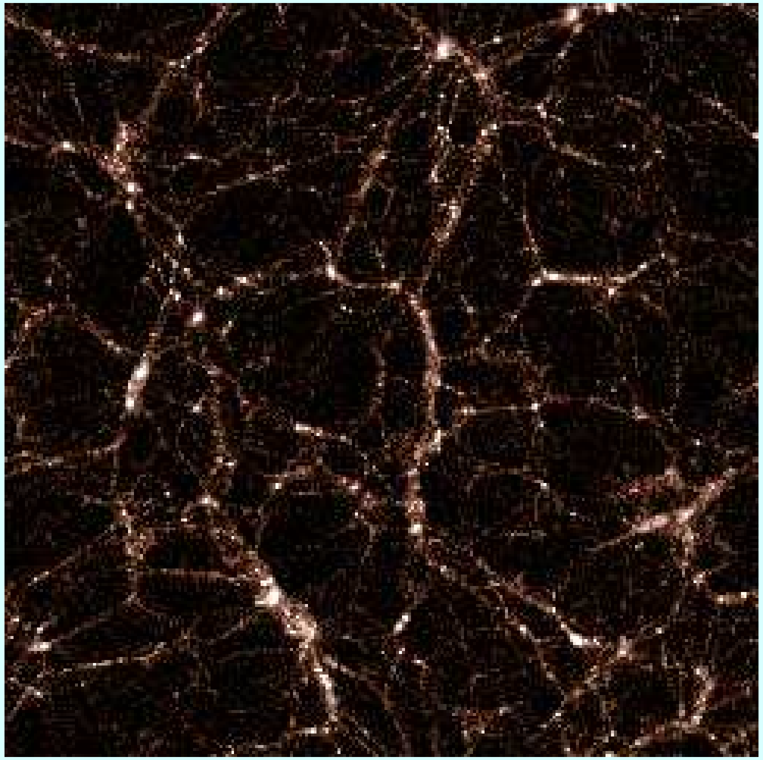
  Stabilization of caustics (pancakes, filaments, clusters).

- Random initial conditions $\longrightarrow$ self-similar distribution of caustics: "cosmic web" $\longrightarrow$ multifractal features (Vergassola et al '94, Aurell et al '97).

Simulations



Burgers eq. with random initial conditions (Vergassola, Dubrulle, Frisch & Noullez)

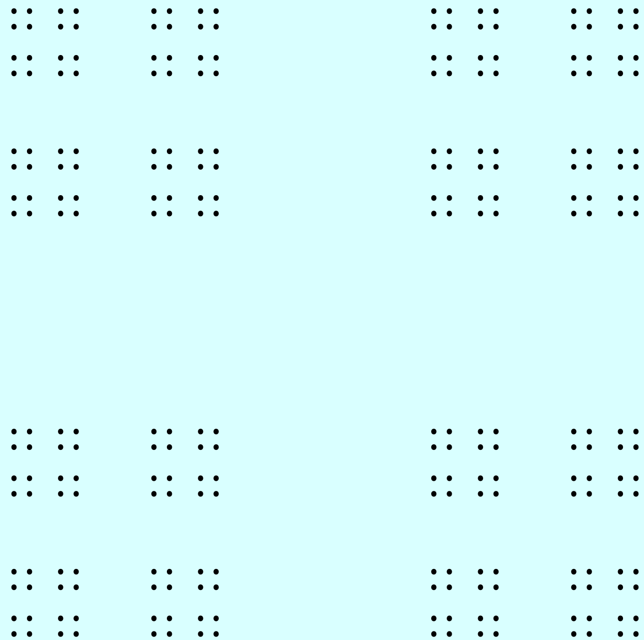


N-body cosmological simulation of Λ-CDM evolution (GIF2)

Fractals

Self-similar fractals

- Cluster hierarchy
- Power-law mass $\propto L^D$, with $D = 2\log 2/\log 3 = 1.26$



Cantor set (2d)

Random fractals

Number function

$$N(r) = B\, r^D \Rightarrow$$

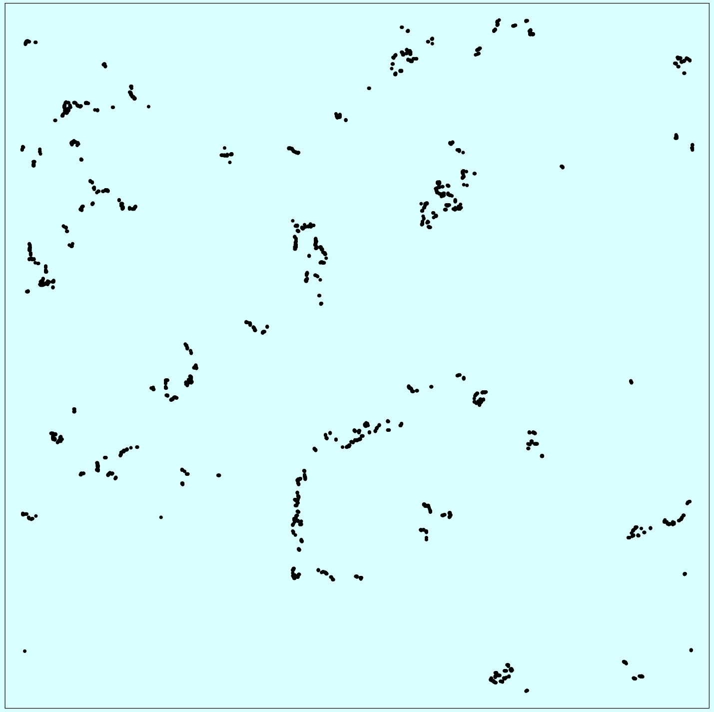
$$\Gamma(r) = \frac{1}{4\pi r^2}\frac{dN(r)}{dr} = \frac{BD}{4\pi} r^{D-3}$$

Transition to homogeneity at r_0:

$$\Gamma = \langle \varrho \rangle (1+\xi).$$

Power-law 2-point correlator:

$$\xi = \left(\frac{r_0}{r}\right)^\gamma >> 1,\ \gamma = D - 3.$$



Random fractal with

$$D = 0.8.$$

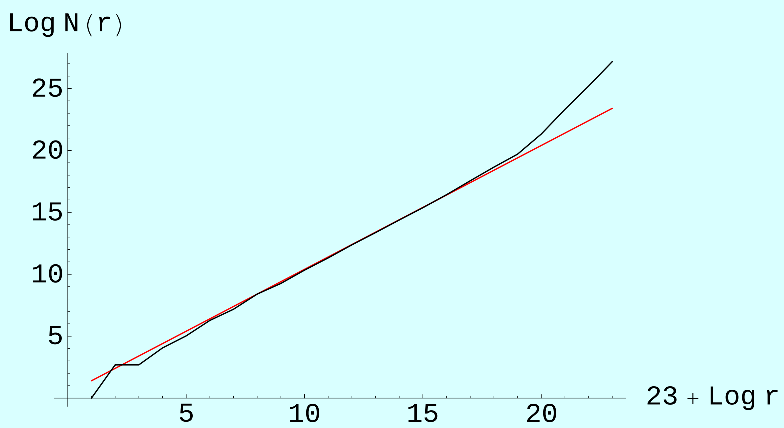
Note clusters and voids

Fractal voids

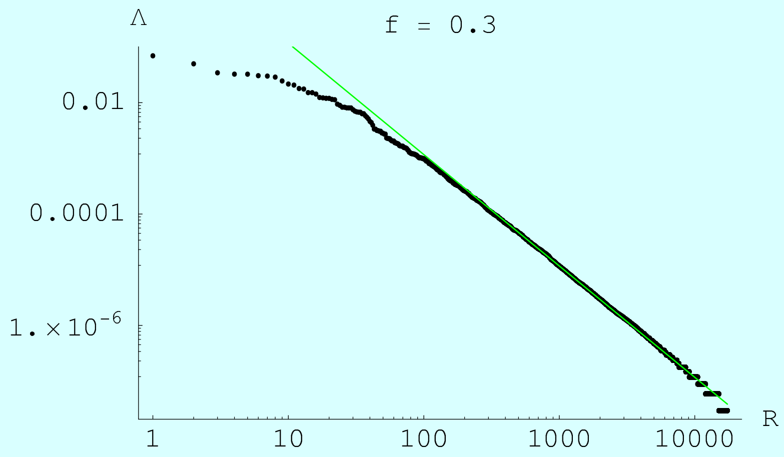
Void of size Λ in a fractal $\longrightarrow$ power law

$$N(\Lambda > \lambda) \propto \lambda^{-D/3} \Leftrightarrow \Lambda(R) \propto R^{-z}, \;\; z = \frac{3}{D},$$ Zipf's law

Transition to homogeneity $\Rightarrow$ flattening of the Zipf law



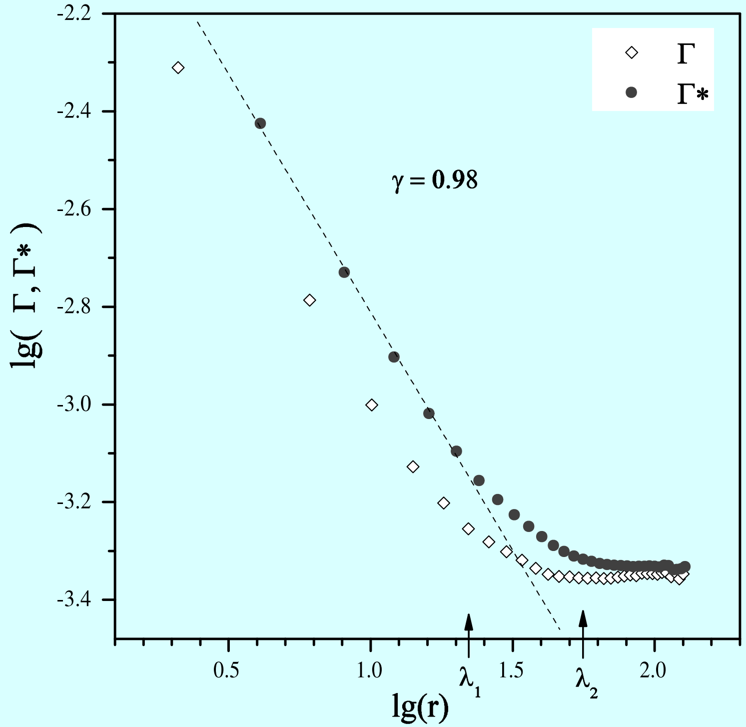
$N(r)$ for $D = 1$ fractal in two dimensions

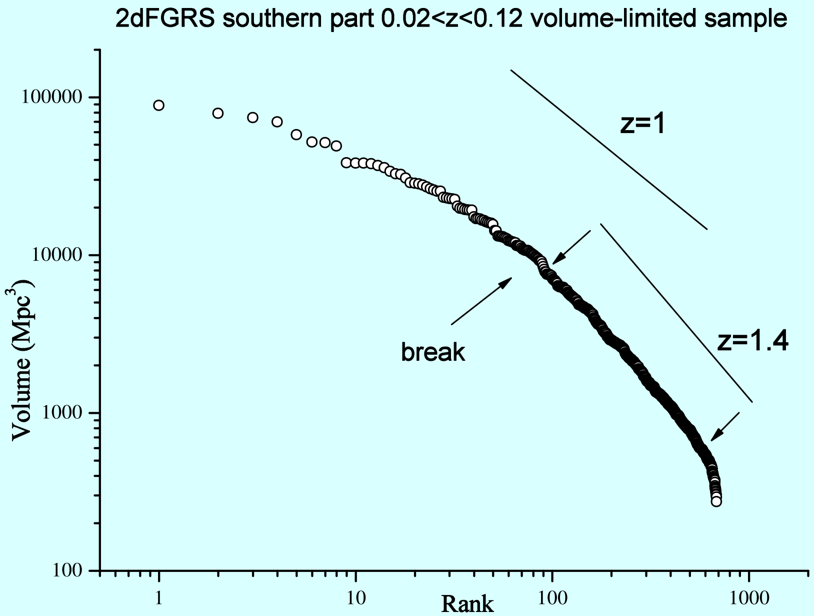


Zipf's law for voids with

$$z = 2/D$$

Scaling of galaxy clusters and voids



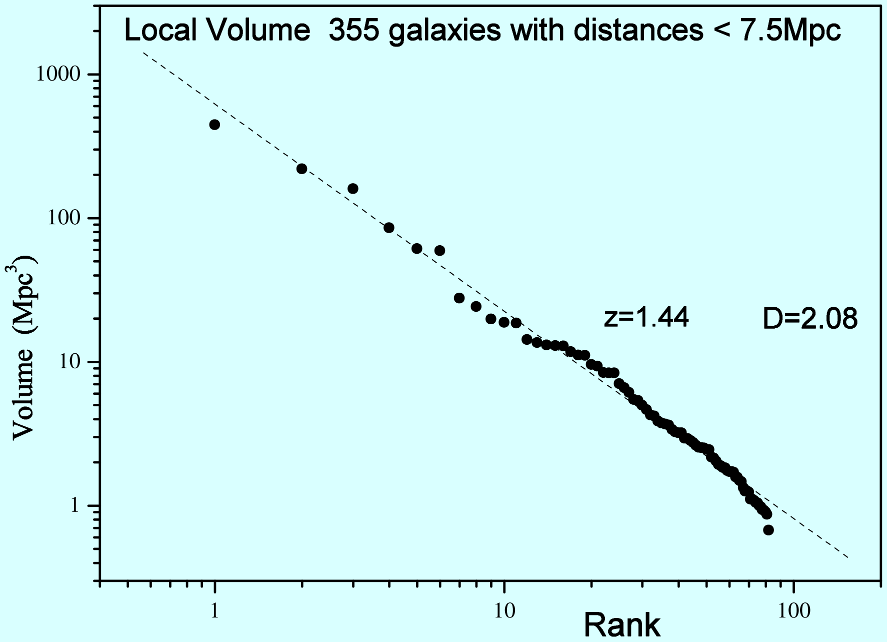


$\Gamma(r)$ of SDSS VL sample. $r_0 \simeq 15$ Mpc (Tikhonov). Slope $\gamma \Rightarrow D \simeq 2$

The Zipf law for a 2dF VL sample (Tikhonov). Slope $z \Rightarrow D \simeq 2$

Scaling of galaxy clusters and voids

Rank ordering of local voids (Karechentsev):

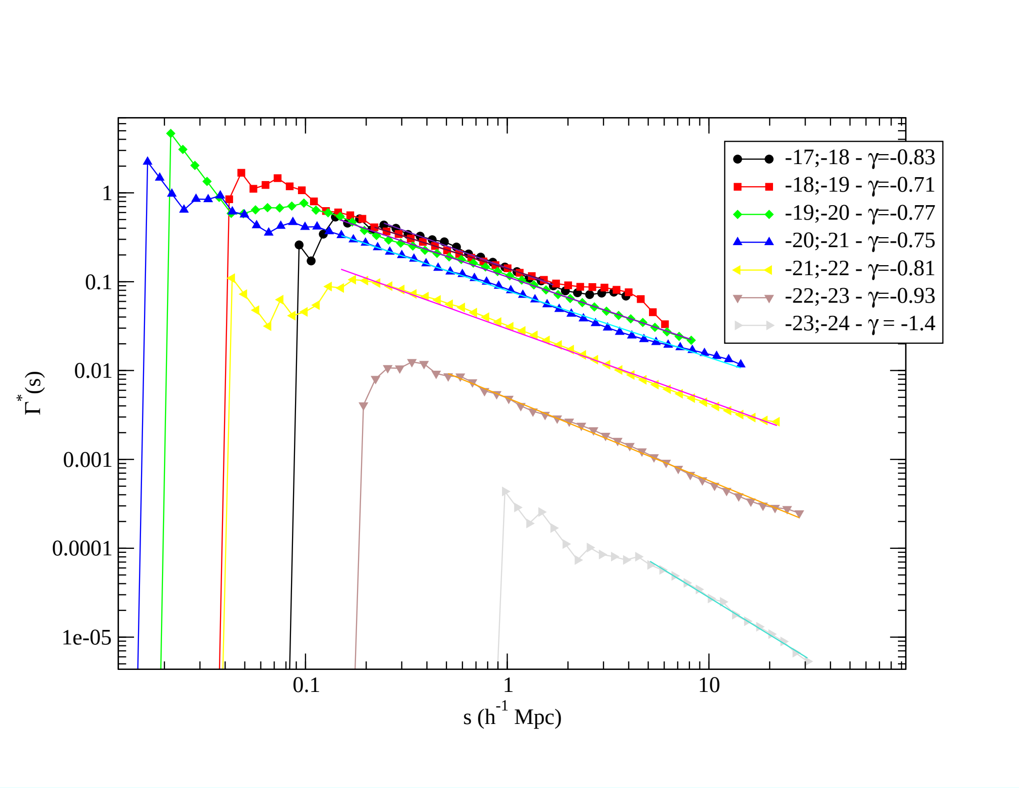


Slope $\gamma \Rightarrow D \simeq 2$. No transition to homogeneity

Analysis of the SDSS catalogue

- Correlation functions of galaxies from the SDSS catalogue according to their luminosity (SSDS Data Release 3, with 374,767 galaxies).

Selection of 7 *volumen limited* samples (150,067 galaxies altogether) with slope fitting $\Rightarrow$ slope changes



MULTIFRACTALS

- Scale invariance $\Rightarrow$ fractals
- Mass distributed irregularly + scale invariance $\Rightarrow$ multifractals
- Mass concentrations

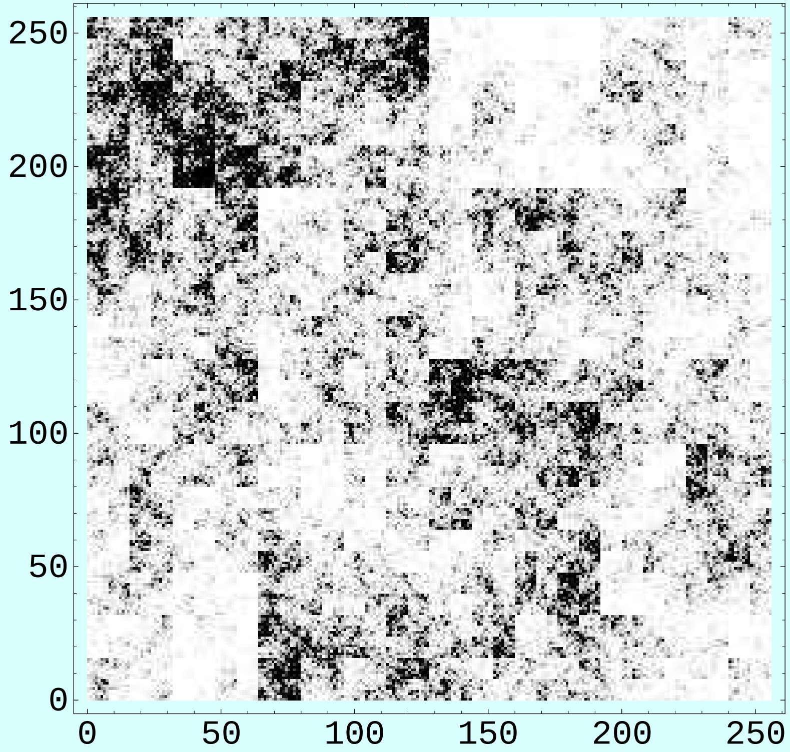
$$m[B(\boldsymbol{x}, r)] \sim r^{\alpha(\boldsymbol{x})}$$

- *Multifractal spectrum* $f(\alpha)$ is the function that gives the fractal dimension of the set of points with exponent α.

  Monofractal: constant $\alpha = f(\alpha)$.

Example: multinomial MF

Multinomial multifractals: the unit square is divided into (4) cells, the unit mass distributed among cells ($\{p_i\}$), and the process iterated.



Random multinomial measure with distribution $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{12}\}$.

MASS CONCENTRATIONS: HALOS

Mass concentrations of size ℓ with singular power-law profile $\rho(r) \propto r^{-\beta}, \; \beta = 3 - \alpha > 0$).

Natural value for ℓ is the lower cutoff to scaling. In N-body simulations, the larger of:

(i) The linear size of the volume per particle.

(ii) The gravitational softening length.

We identify mass concentrations with equal-size halos $\longrightarrow$
Halo *mass-function*: $N(m) \sim \ell^{-f(\alpha)}, \; \alpha = \log m / \log \ell.$

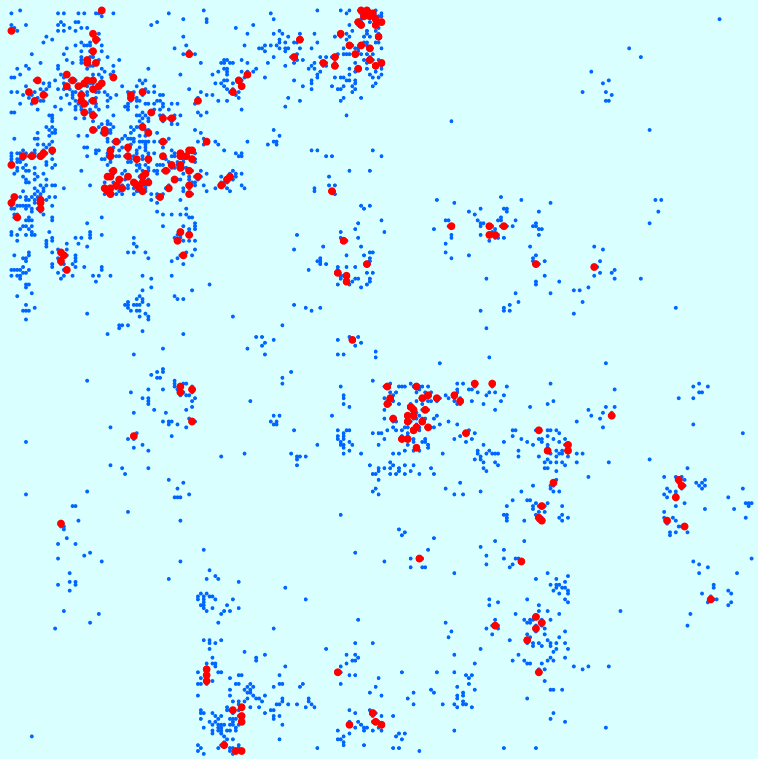
Multinomial bifractal

A bifractal can be extracted: select $\{\alpha_1, \alpha_2\} \Leftrightarrow \{m_1, m_2\}$.

Multifractal models support halo populations with different levels of clustering.

Note voids

(may be non-empty)

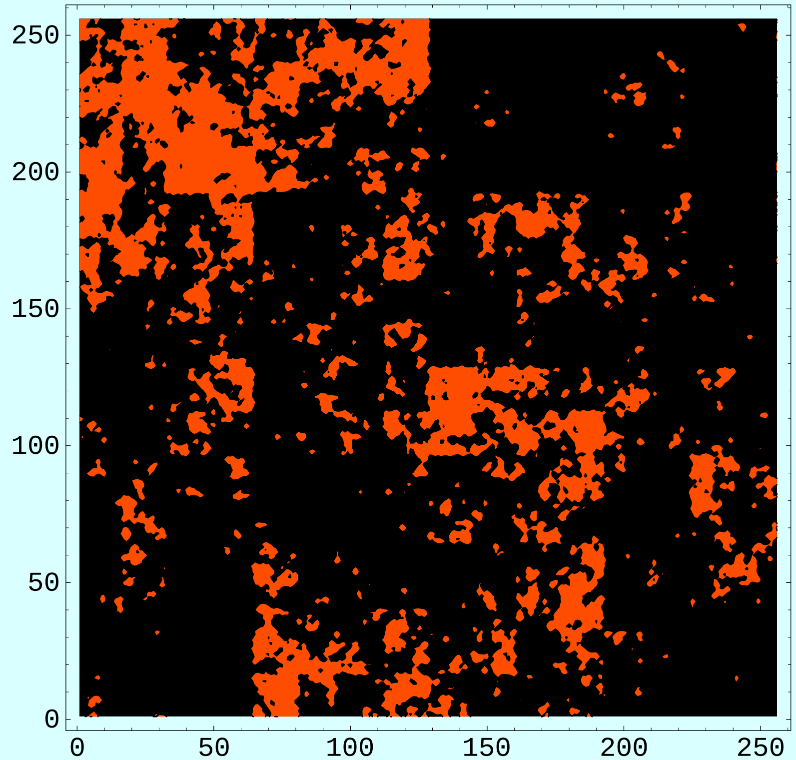


Two populations in a multinomial multifractal.

MASS DEPLETIONS: VOIDS

Voids have regular power-law profile $\rho(r) \propto r^{-\beta}$ $(\beta = 3 - \alpha)$. If $\alpha > 3 \Rightarrow \rho(0) = 0$.

Boundaries of voids: points with $\alpha = 3 \Rightarrow \rho(0) > 0$ and finite. Not regular but fractal surfaces with $D = f(3) > 2$.



Fractal boundary of voids in multinomial MF: $f(2) = 1.93$.

BIASING AND VOIDS

Biasing: peculiar distribution of certain set of objects with respect to the total DM distribution.

Bias from linear theory:

$$\frac{\delta\rho_g}{\rho_g} = b\,\frac{\delta\rho}{\rho} \Rightarrow \xi_{gg}(r) = b^2\xi(r),\ b > 1.$$

Constant b bias in the nonlinear regime $\Rightarrow$ similar voids for every population $\longrightarrow$ false in MF.

- Voids not empty but harbor faint galaxies $\leftrightarrow$ galaxy formation (Peebles, 2001).
- Distribution of dark matter inside voids (Gottlöber et al, 2003).

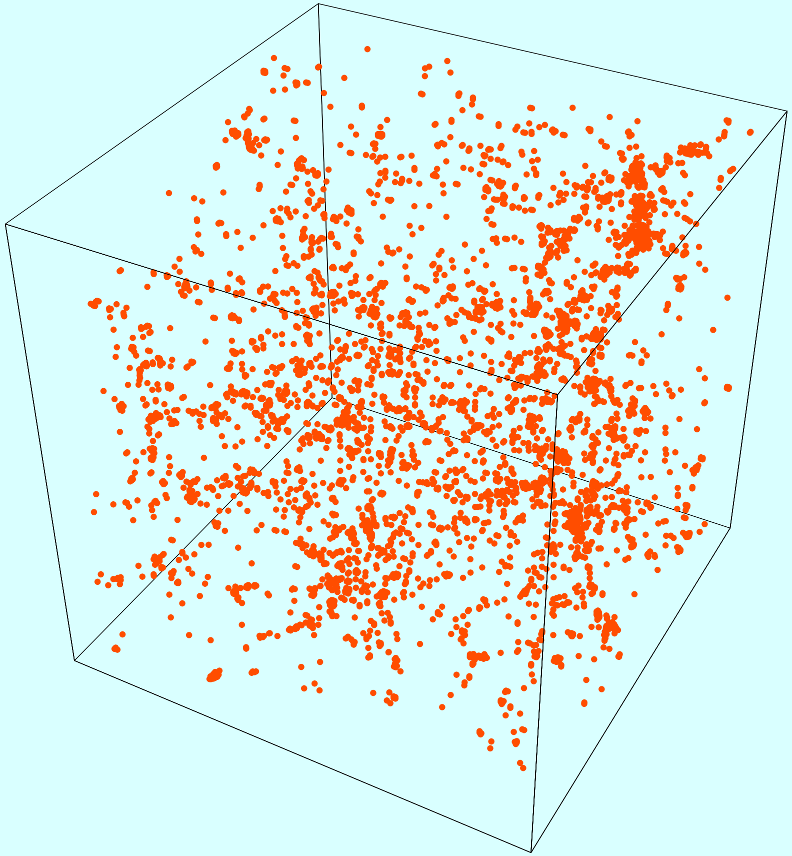
MULTIFRACTALITY IN N-BODY SIMULATIONS

$z = 0$ positions in Virgo Λ-CDM GIF2 simulation: 400^3 particles in a volume of $(110\ h^{-1}$ Mpc$)^3$ $\Rightarrow$ particle mass is $0.173\ 10^{10}\, h^{-1} M_\odot$.

Statistics by *counts-in-cells*:

$$M_q(\ell) = \sum_{m=1}^{\infty} N(m)\, m^q.$$

Halos: $\ell_H = 256^{-1} > 400^{-1}$ ($\ell_H = 0.43\ h^{-1}$ Mpc).

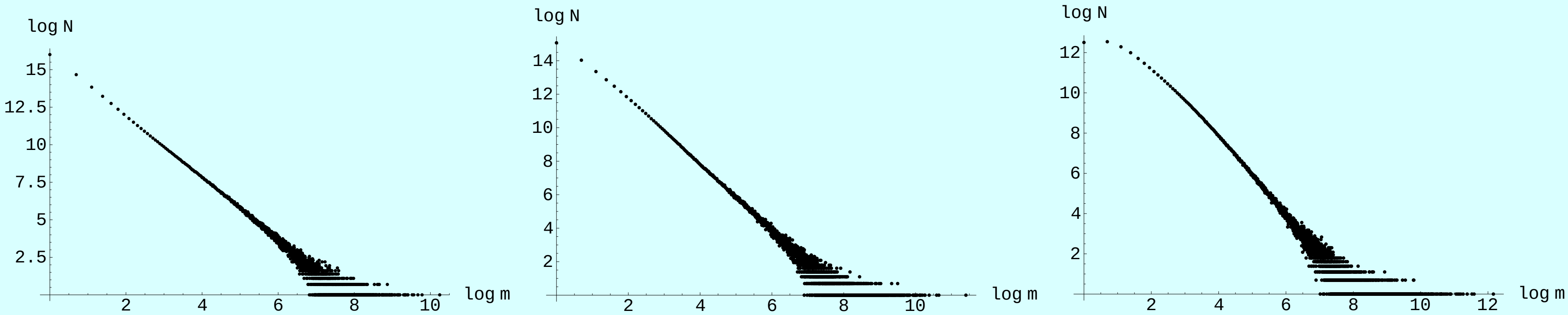


Distribution of 5515 haloes (cutoff 1000)

Mass-function variation with ℓ

If MF scaling holds $\Rightarrow$ stable MF spectrum.

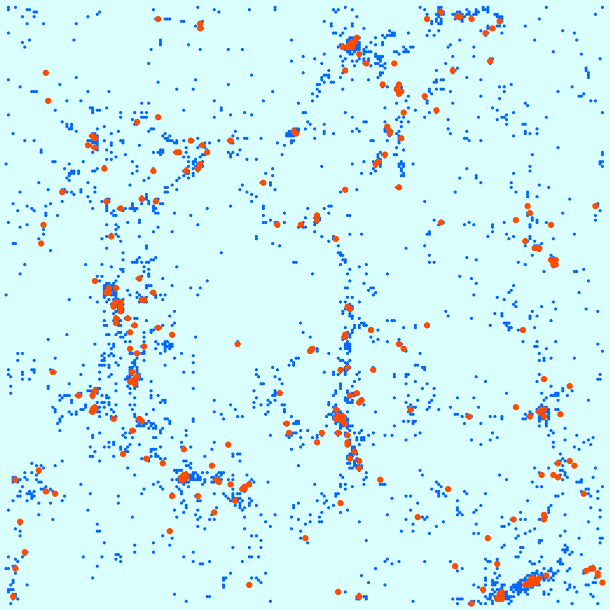
For $\ell > \ell_H$ we expect a stable MF spectrum. How does it change for $\ell < \ell_H$?

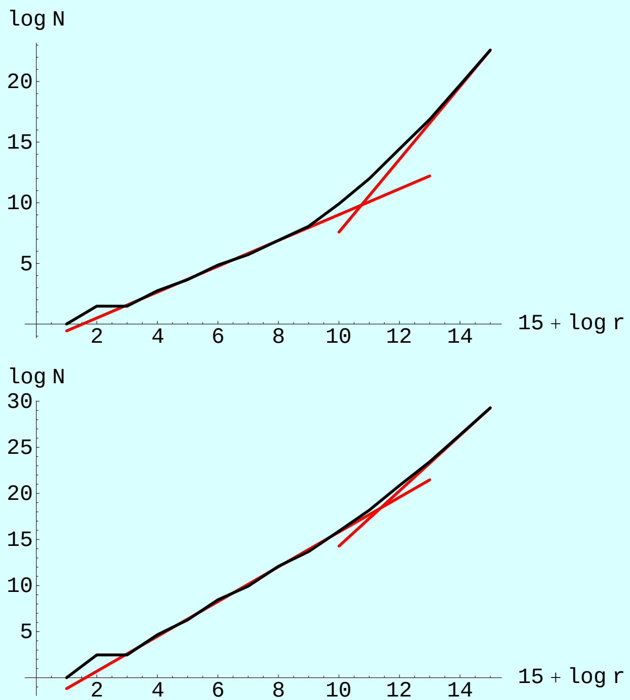


Log-log plots of number of halos $N(m)$ for $\ell = 512^{-1}$ (left), 256^{-1} (middle), and 128^{-1} (right).

Power-law (Press-Schechter) for $\ell < \ell_H$.

MF AS FRACTAL DISTRIBUTIONS OF HALOS



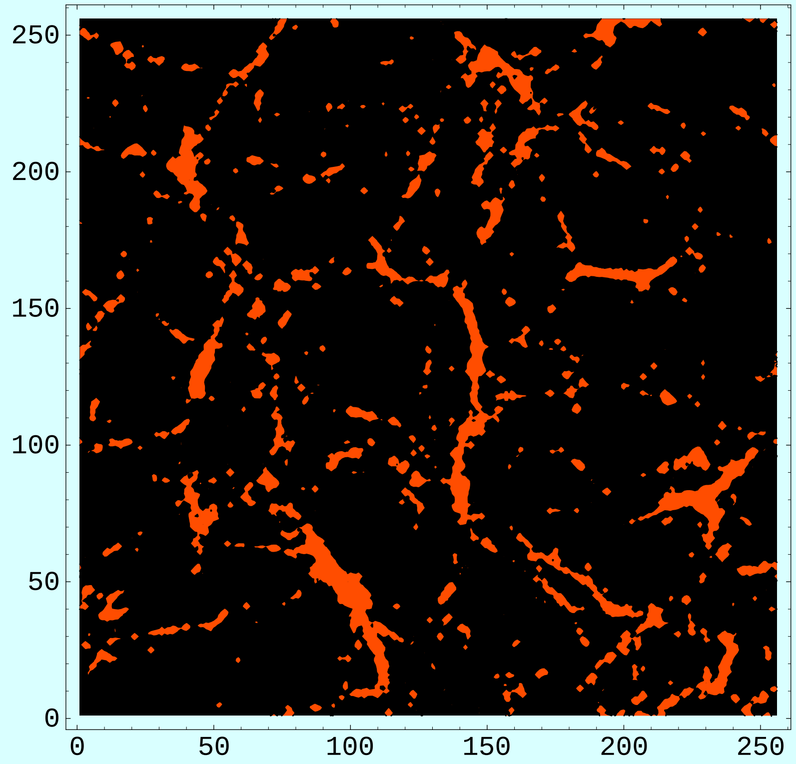


- GIF2 heavy haloes with 750 to 1000 particles (red);
- GIF2 light haloes with 100 to 150 particles (blue).

Number function $N[B(\boldsymbol{x}, r)] \sim r^D$: fractal dimensions $D = 1.1$ and $D = 1.9$. Transition to homogeneity starts at $\simeq 14\ h^{-1}$ Mpc.

Voids in the GIF2 simulation

MF spectrum shows that $f(3) \simeq 2.9 \Rightarrow$ boundary is a fractal surface with large dimension.



Fractal boundary of voids in a GIF2 slice

SUMMARY and CONCLUSIONS

- Adhesion model $\longrightarrow$ self-similar cosmic web (multifractal)
- Observations of galaxies $\longrightarrow$ scaling with variable γ
- Multifractals: most general scaling mass distributions, support halos (concentrations) and voids (depletions).
- MF spectrum $\longrightarrow$ halo mass function.
- Halos have *nonlinear bias*
- GIF2: fractal populations of haloes $\longrightarrow$ good scaling.