The first WIMPy halos

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- Why?
- WIMP micro-physics
- The first WIMPy halos
- Evolution
- Open questions

Green, Hofmann & Schwarz, astro-ph/030962; astro-ph/0503387
building on earlier work by Hofmann, Schwarz & Stöcker
Brax, van de Bruck, Davis & Green, astro-ph/0509878
Green & Goodwin, astro-ph/0604142

Why?

In CDM cosmologies structure forms heirarchically; small halos form first, with larger halos forming from mergers and accretion.

Simulation of the formation of a Galaxy Cluster by Juerg Diemand, Joakim Stadel, Ben Moore (University of Zurich) on the zBox Supercomputer at the University of Zurich.

Simulations produce halos containing large amounts of substructure: [Klypin et al.; Moore et al.]

$$\frac{\mathrm{d}n}{\mathrm{d}m} \propto m^{-\alpha} \qquad\qquad \alpha \sim 2$$

down to the resolution limit ($\sim 10^6 M_\odot$ for a Milky Way-like halo), with ~10% of the total mass in (resolved) sub-structure.

What happens on smaller scales?

Does this mass function carry on down to infinitesimally small scales?/How big are the first DM halos to form?

(n.b. there must be a cut-off at some point, otherwise the contribution of the density perturbations to the local energy density would diverge)

What fraction of the total mass is in substructure?

Also interesting/important for practical reasons:

- ## Indirect detection

Event rates proportional to ρ^2, enhanced by small scale sub-structure. [Silk & Stebbins; Bergström et al.; Calcaneo-Roldan & Moore; Ullio et al.; Taylor & Silk]

Nearby mini-halos easier to detect than nearest larger subhalo (smaller distance outweighs smaller mass).

- ## Direct detection

Signals depends on the dark matter distribution on sub-milli-pc scales. [Silk & Stebbins; Moore et al.; Green]

n.b. The 'Halo models' which are often used in direct detection calculations are solutions of the collisionless Boltzmann equation- this applies to the coarse grained (i.e. spatially averaged) distribution function and assumes the dark matter distribution has reached a steady state.

WIMP microphysics

Kinetic decoupling

[Schmid, Schwarz & Widerin; Boehm, Fayet & Schaeffer; Chen, Kamionkowski & Zhang; **Hofmann, Schwarz & Stöcker**; Schwarz, Hofmann & Stöcker; Berezinsky, Dokuchaev & Eroshenko; Green, Hofmann & Schwarz x2; Loeb & Zaldarriaga]

After freeze-out (chemical decoupling) WIMPS carry on interacting kinetically with radiation:

$$\chi + \chi \not\Leftrightarrow X + X \qquad\qquad \chi + X \Leftrightarrow \chi + X$$

The WIMPs kinetically decouple when

$$\tau_{\rm relax} = H^{-1}$$

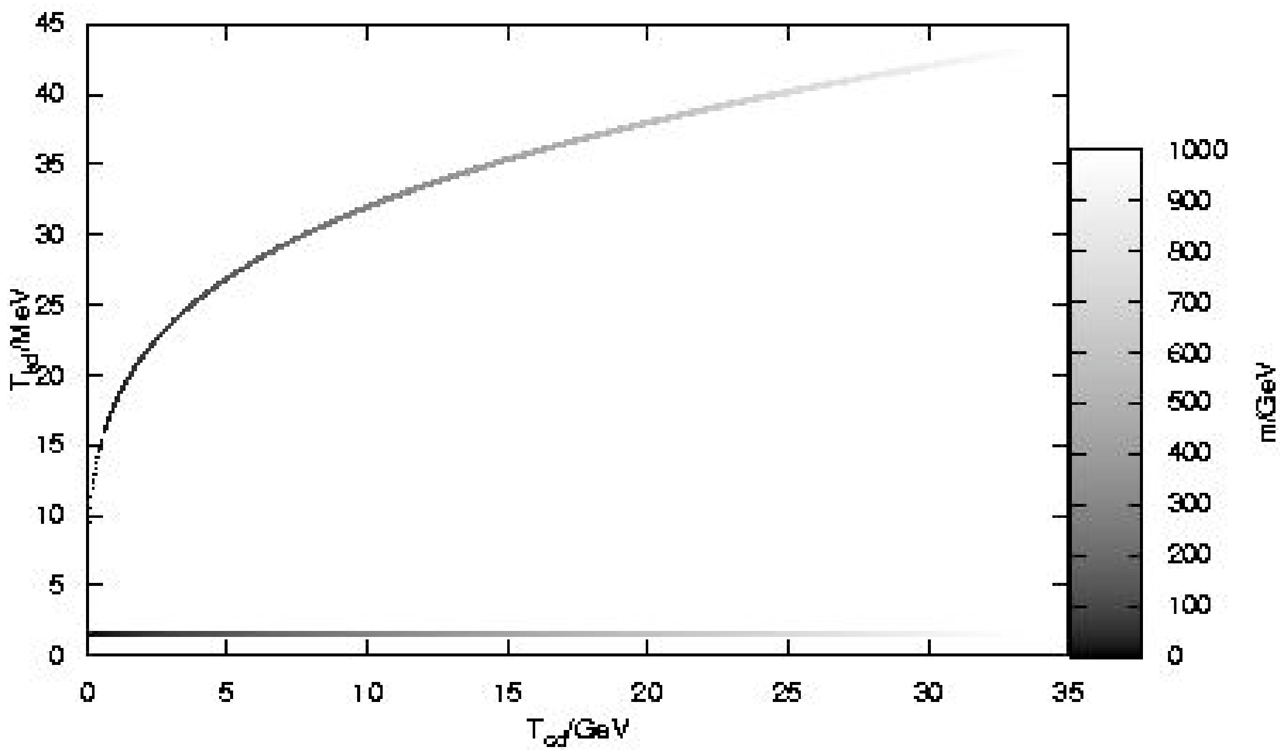
n.b. the momentum transfer per scattering (~T) is small compared with the WIMP momentum (~M), therefore a very large number of collisions are required to keep or establish thermal equilibrium.

$$\tau_{\rm relax} \gg \tau_{\rm col}$$

Dependence of decoupling temperatures on WIMP mass, for WIMPs with present day density compatible with WMAP measurements, for *l* =1 for Majorana particles (i.e. neutralinos interacting via sfermion exchange) and *l*=0 Dirac particles (i.e. standard modelesque particles interacting via Z0 exchange).

$$< \sigma_{\rm el} >= \sigma_0^{\rm el} \left(\frac{T}{m} \right)^{l+1} \qquad \sigma_0^{\rm el} \approx \frac{(G_{\rm F} m_{\rm W}^2)^2 m^2}{m_{\rm Z}^2}$$

Kinetic decoupling temperature in MeV



mass in GeV

Chemical decoupling temperature in GeV

Collisional damping

Energy transfer between radiation and WIMP fluids (due to bulk and shear viscosity) leads to collisional damping of density perturbations.

Free-streaming

After kinetic decoupling WIMPs free-stream, leading to further (collision-less) damping.

$$\frac{1}{k_{\rm fs}} \sim l_{\rm fs}(\eta) = \bar{v}_{\rm kd} a_{\rm kd} \int_{\eta_{\rm kd}}^{\eta} \frac{{\rm d}\eta'}{a(\eta')}$$

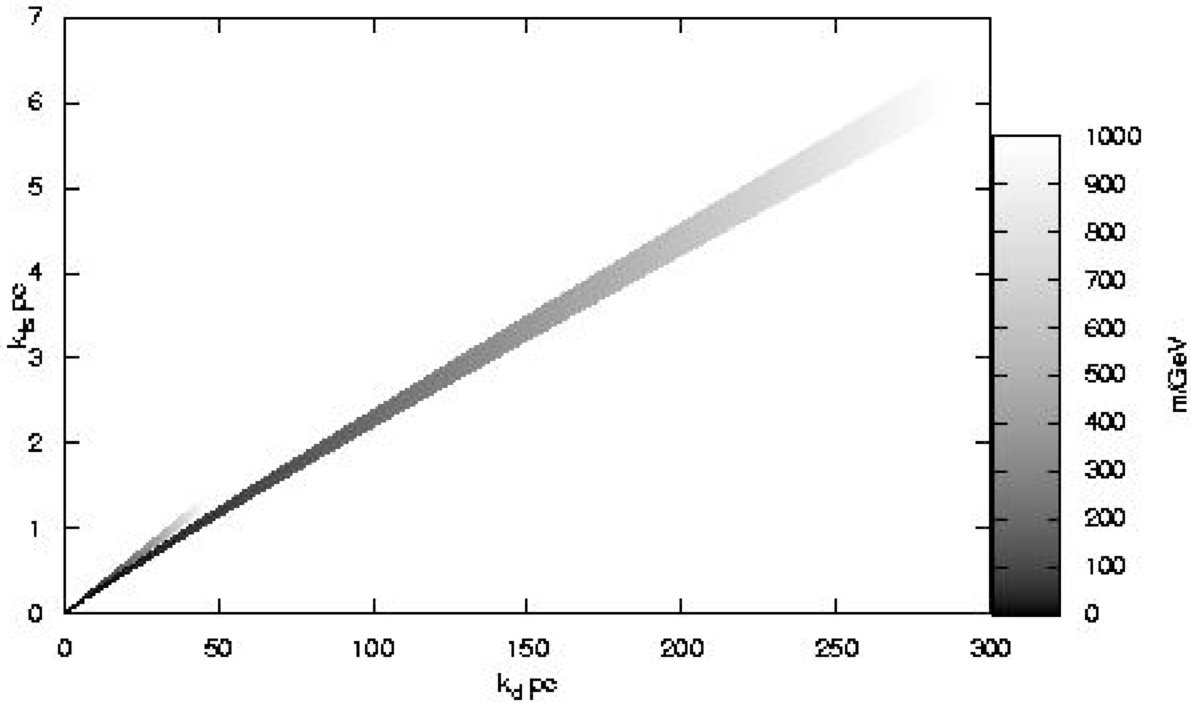
Calculate free-streaming length by solving the collisionless Boltzmann equation, taking into account perturbations present at kinetic decoupling.

Net damping factor:

$$D(k) \equiv \frac{\delta_{\rm WIMP}(k,\eta)}{\delta_{\rm WIMP}(k,\eta_{\rm i})} = D_{\rm cd}(k) D_{\rm fs}(k) = \left[1 - \frac{2}{3}\left(\frac{k}{k_{\rm fs}}\right)^2\right] \exp\left[-\left(\frac{k}{k_{\rm fs}}\right)^2 - \left(\frac{k}{k_{\rm d}}\right)^2\right]$$

Dependence of damping scales on WIMP mass, for WIMPs with present day density compatible with WMAP measurements, and *l* =0/1 (top and bottom).

Free-streaming comoving wavenumber (pc)



mass in GeV

Collisional damping comoving wavenumber (pc)

WIMP micro-physics summary

☆ $T > T_{cd}$ [O(1–10) GeV]

In chemical and thermal equilibrium

☆ $T = T_{cd}$

Chemical decoupling/freeze-out, comoving number density becomes fixed.

☆ $T_{kd} < T < T_{cd}$

Interact kinetically with radiation. Perturbations collisionally damped due to bulk and shear viscosity.

☆ $T = T_{kd}$ [O(1–10) MeV]

Kinetic decoupling, free-streaming regime commences.

☆ $T_{eq} < T < T_{kd}$

Free-streaming erases further perturbations.

Two more ingredients needed to calculate the (processed) density perturbation power spectrum:

- Primordial power spectrum

  Simplest possibility: scale invariant (n=1), WMAP normalised.

- Gravitational growth of fluctuations

  Solved perturbation equations for $k \gg k_{\rm eq} \sim 0.01/{\rm Mpc}$ for 2 overlapping regimes:

  i) radiation domination $\rho_{\rm rad} \gg \rho_{\rm mat}$

  ii) $\rho_{\rm mat}\delta_{\rm mat} \gg \rho_{\rm rad}\delta_{\rm rad}$ (Meszaros equation)

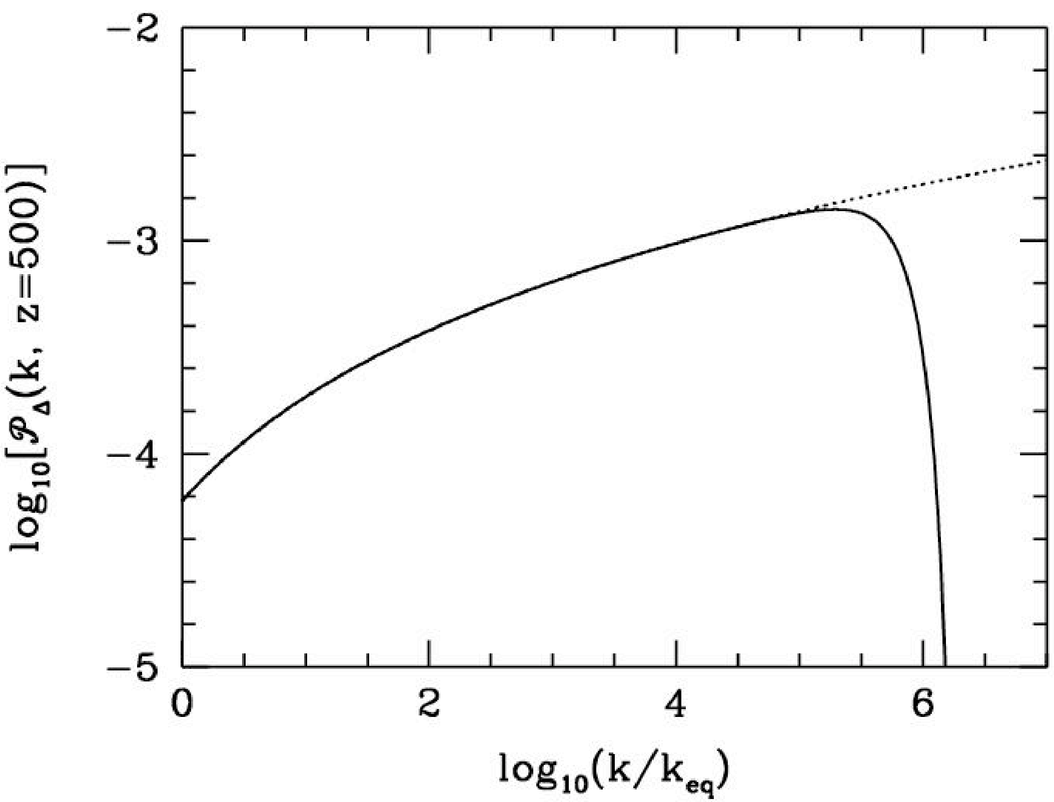
  (included growth supression due to baryons and verified accuracy of solutions using COSMICs package [Bertschinger])

[Loeb & Zaldarriaga numerical treatment:

Memory of coupling to radiation fluid leads to accoustic oscillations of CDM fluid and additional damping therefore <10% accurate calculation of the cut-off scale and the detailed shape of the processed power spectrum requires numerical calculations.]

Power spectrum

For a 100 GeV bino-like WIMP and a scale invariant, WMAP normalised, primordial power spectrum at z=500:

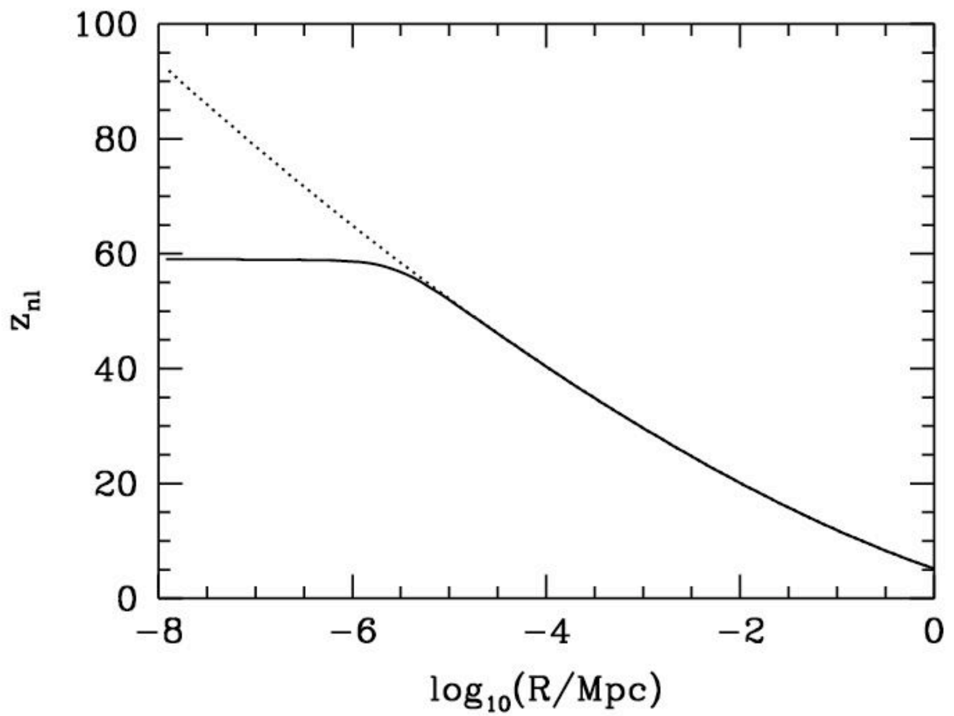


$$\mathcal{P}_\delta(k) = \frac{k^3}{2\pi^2}\langle|\delta^2|\rangle$$

Sharp cut-off at $k = k_{fs} \sim 1/\text{pc}$

z_{nl}

The red-shift at which typical fluctuations on co-moving physical scale R go non-linear can be estimated via the mass variance:



$$\sigma(R, z_{\rm nl}) = 1$$

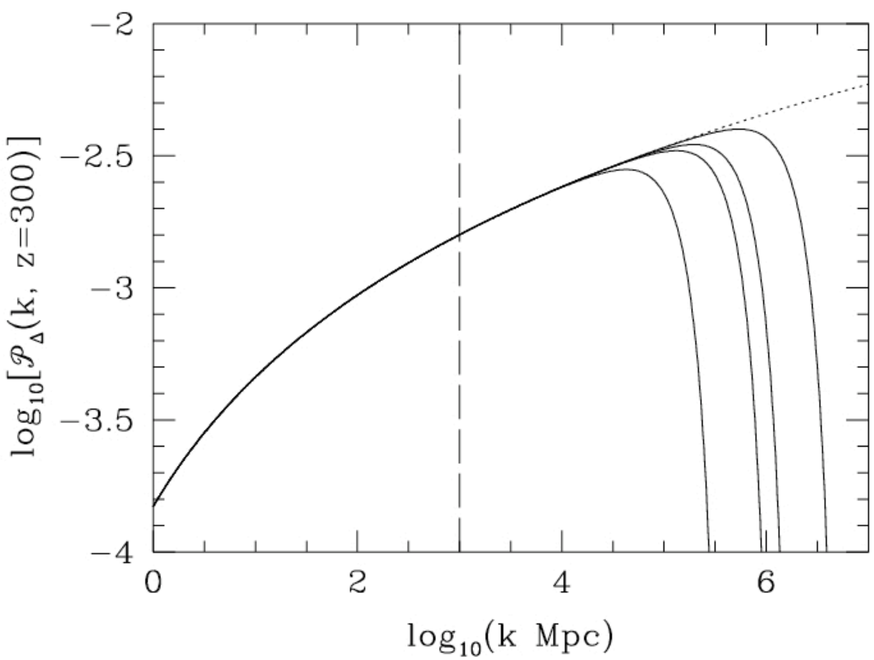
$$\sigma^2(R, z) = \int_0^\infty W^2(kR)\mathcal{P}_\delta(k, z)\frac{\mathrm{d}k}{k}$$

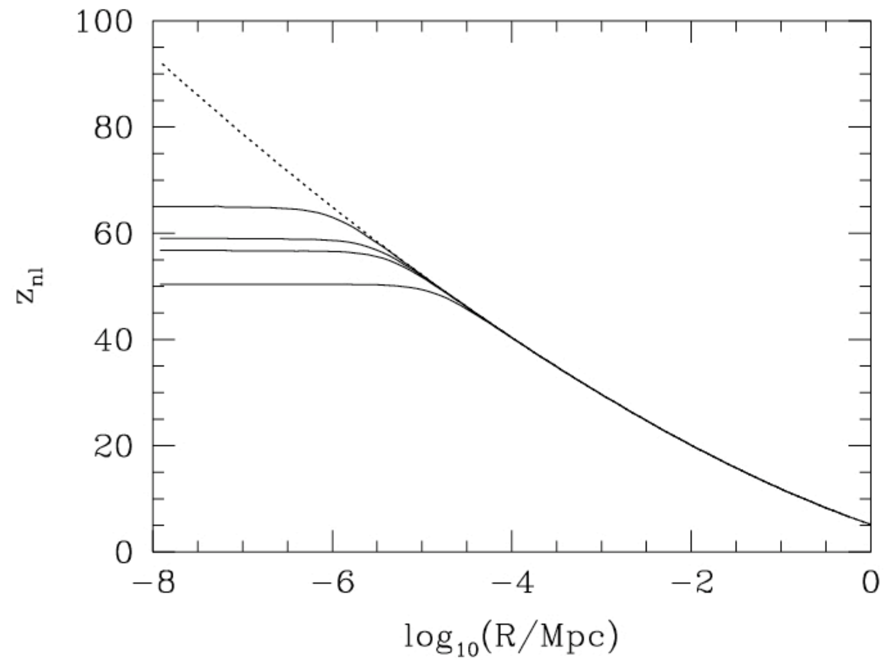
Typical one-sigma fluctuations collapse at z_{nl}~60.

(N-sigma fluctuations collapse at z_{nl}~ 60N)

Effect of varying:

i) WIMP properties





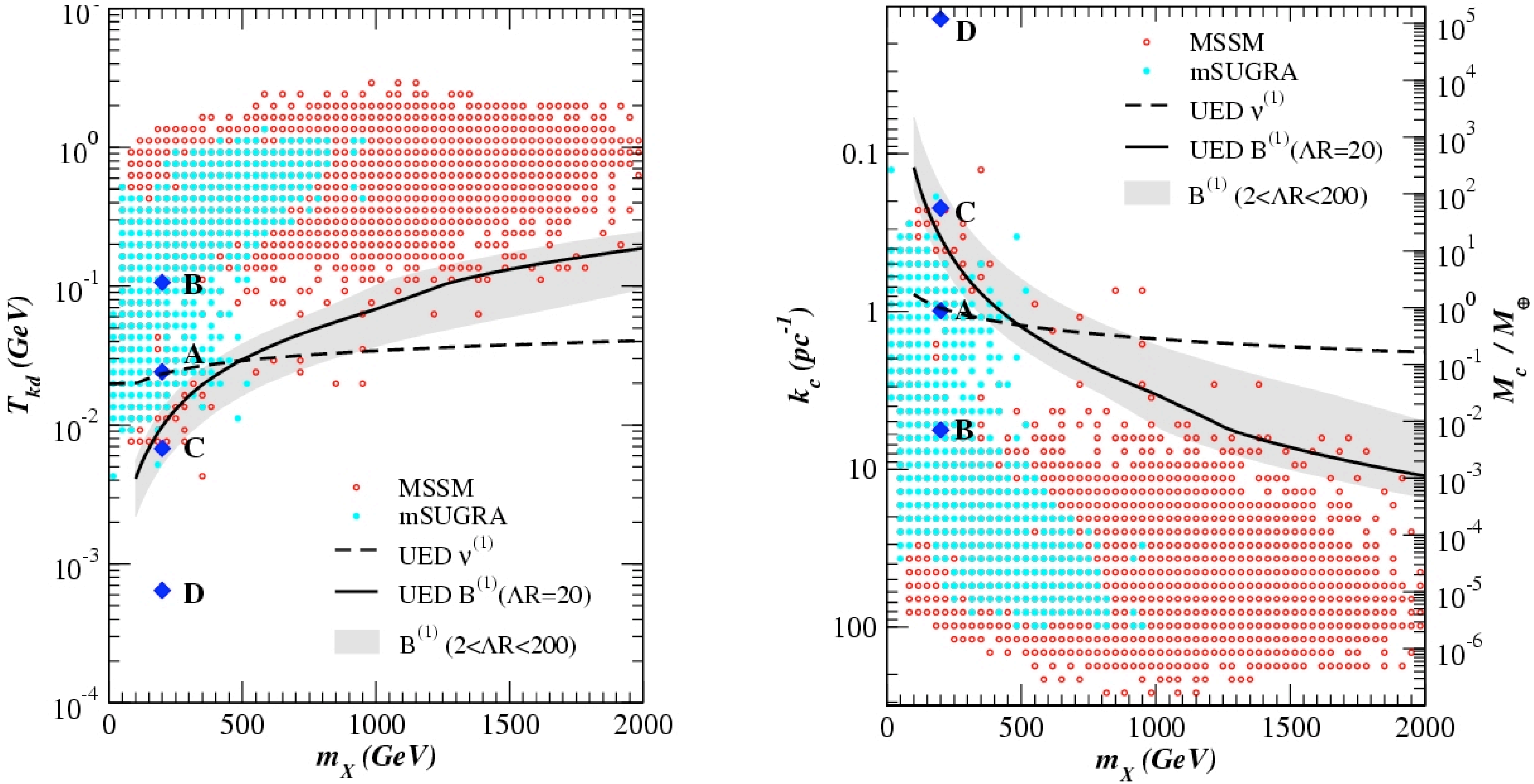
left to right/bottom to top:

Dirac (elastic scattering mediated by Z_0 exchange) m = 100 GeV

Majorana (Z_0 exchange supressed) m = 50, 100, 500 GeV

Profumo, Sigurdson & Kamionkowski:

Scan MSSM and also consider Universal Extra Dimensions and heavy neutrino like dark matter.



A: coannihilation region, light scalar sparticles, (quasi-degenerate) NLSP is stau

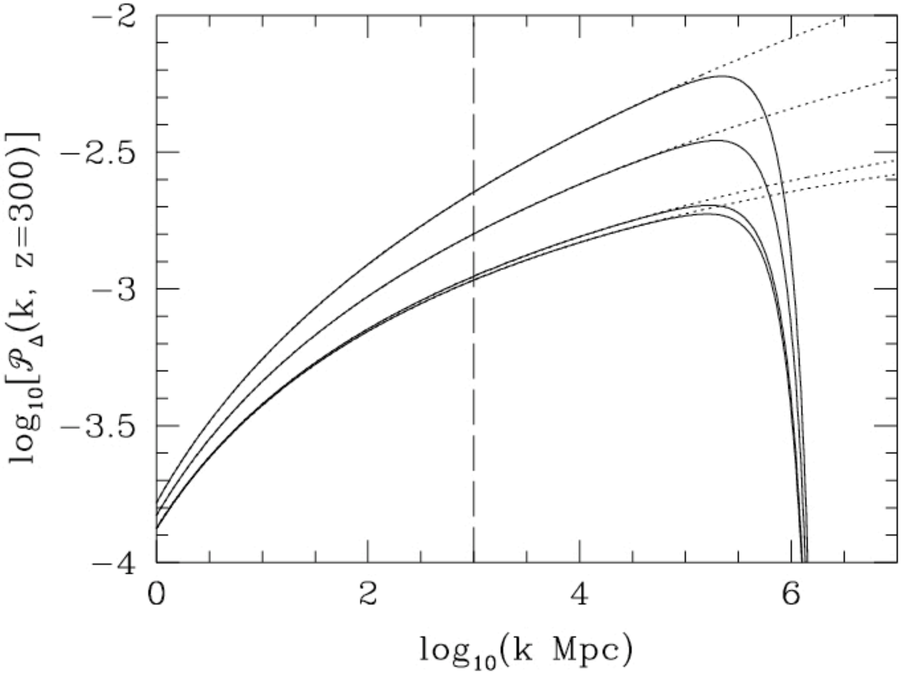
B: focus point region, heavy scalars, scattering from light fermions is via Z0 exchange

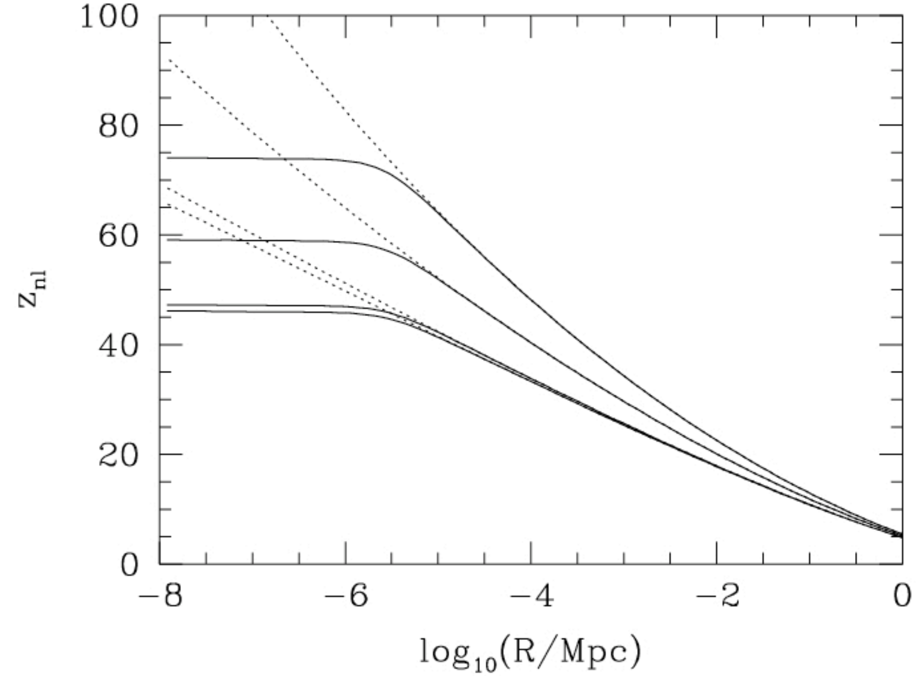
C: $\Delta m_{\tilde{\nu}_{e,\mu}} \equiv m_{\tilde{\nu}_{e,\mu}} - m_\chi = 1\,\mathrm{GeV}$

D: $\Delta m_{\tilde{\nu}_{e,\mu}} \equiv m_{\tilde{\nu}_{e,\mu}} - m_\chi = 0.01\,\mathrm{GeV}$

} Sfermion resonances. At high T scattering from light fermions energy independent.

ii) primordial power spectrum





top to bottom:

false vacuum dominated hybrid inflation	n=1.036,	α =0
scale invariant	n=1.000,	α =0
power law inflation	n=0.964,	α =0
$m^2 \phi^2$ chaotic inflation	n=0.964,	α =-0.0006

$$\alpha = \frac{\mathrm{d}n}{\mathrm{d}k}$$

iii) gravity

Chameleon cosmology

[Khoury & Weltman x2; Brax, van de Bruck, Davis, Khoury & Weltman; Brax, van de Bruck, Davis & Green]

Scalar field with gravitational strength coupling to matter.

Mass depends on local background density (high on Earth, tiny on cosmological scales).

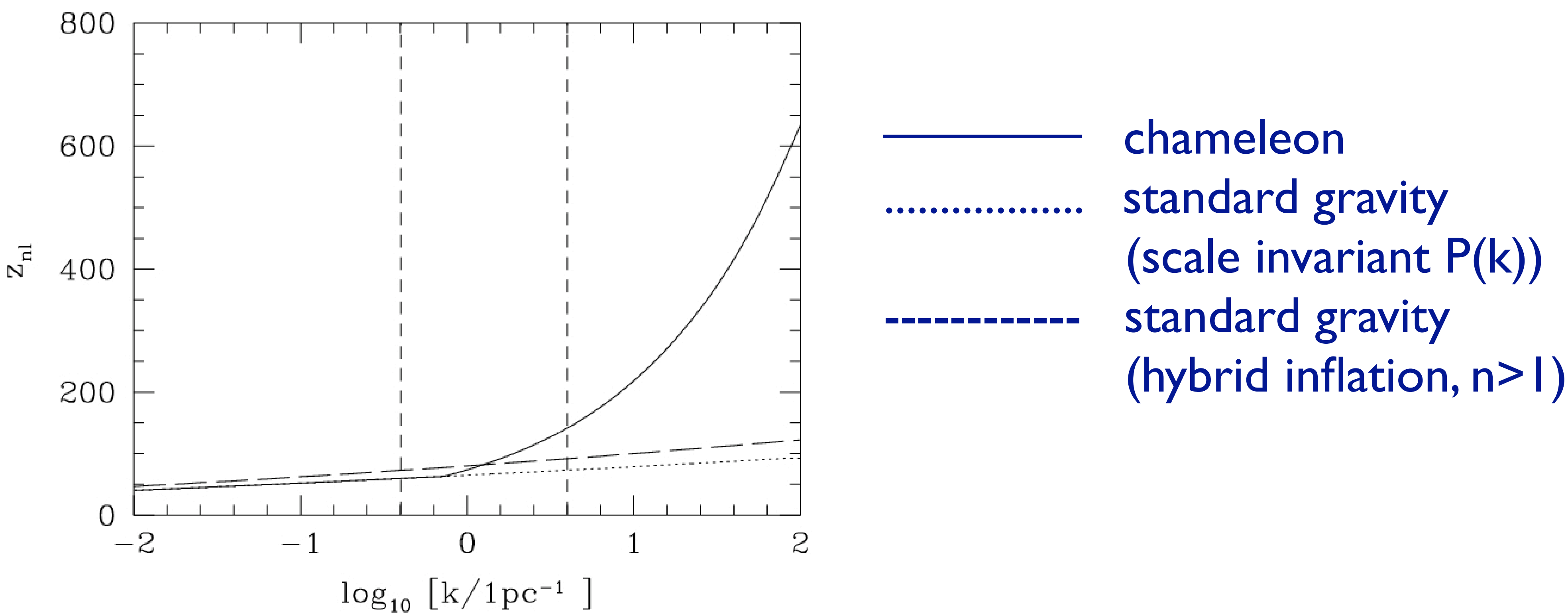
Can evolve on Hubble timescale and generate the observed present day acceleration while evading tests of gravity.

On small scales ($k > k_{\rm cham}(t)$) density perturbation growth law during matter domination ($\delta \propto a^{\nu/2}$) is modified:

$$\nu = \frac{\sqrt{1 + 24(1+2\beta^2)(1-f_{\rm b})} - 1}{2} \qquad f_{\rm b} = \frac{\Omega_{\rm b}}{\Omega_{\rm m}}$$

$\beta \sim \mathcal{O}(1)$ is the coupling (relating excitations of the field to the Einstein metric).

The red-shift at which typical fluctuations go non-linear:



In chameleon cosmology mini-halos form earlier and are hence denser and more resilient to disruption.

The first WIMPy halos

Spherical collapse model

Estimates of properties: $M \sim 10^{-6} M_{\odot}$

$$r \sim \frac{0.02}{N} \,\mathrm{pc}$$

present day density contrast: $\Delta \sim 10^6 N^3$

Simulations

Current state of the art: particle mass ~ $10^5 M_{\odot}$ for a Milky Way mass halo ('resolution' is an order of magnitude or so bigger than this).

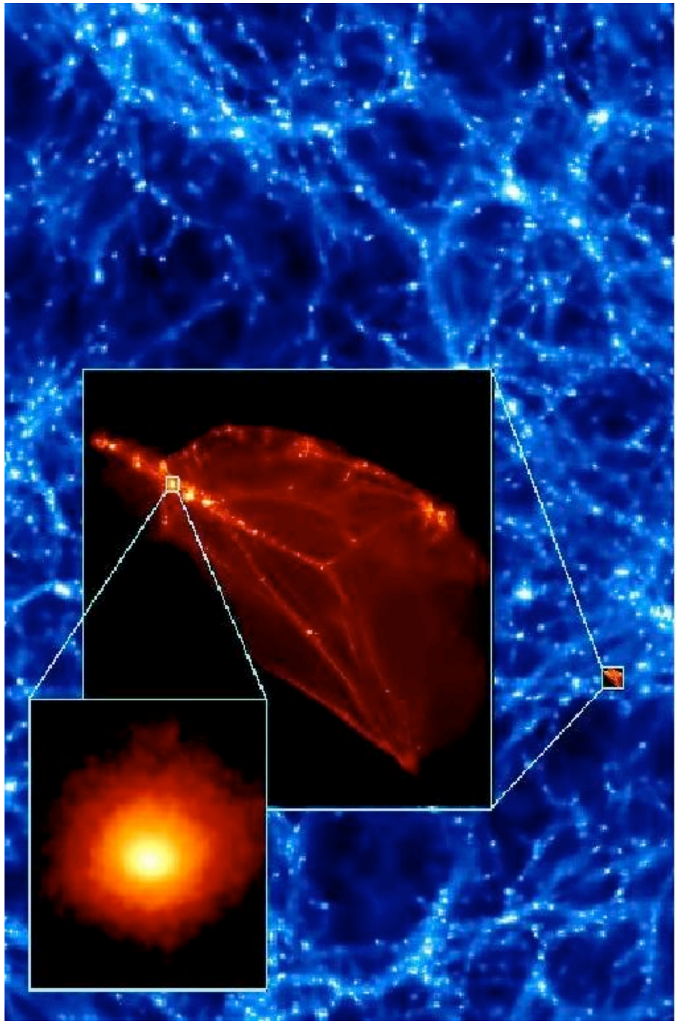
Re-simulation technique:

- Extract a region of interest from a cosmological simulation.
- Trace particles back to initial time.
- Re-simulate at higher resolution (smaller particle mass) with surrounding high mass particles to reproduce the tidal forces from the surrounding region.

[Diemand, Moore and Stadel]

Re-simulate a small 'typical' region starting at z=350 (when the fluctuations are still linear) up until z=26 (when the high resolution region begins to merge with surrounding low resolution regions).

Input: Our power spectrum with cut-off at k=0.6pc.
Cosmological parameters as measured by WMAP.

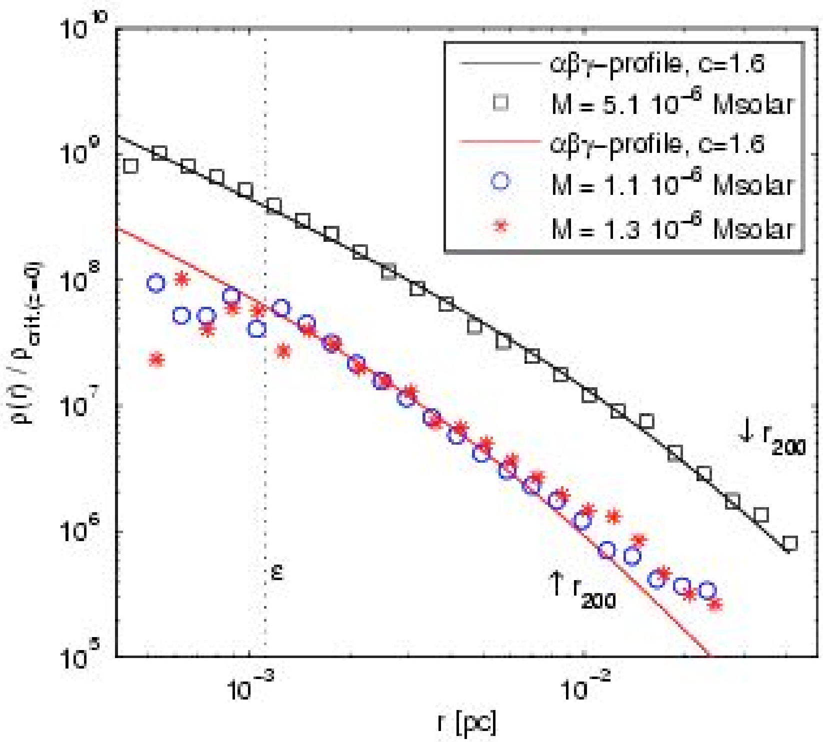


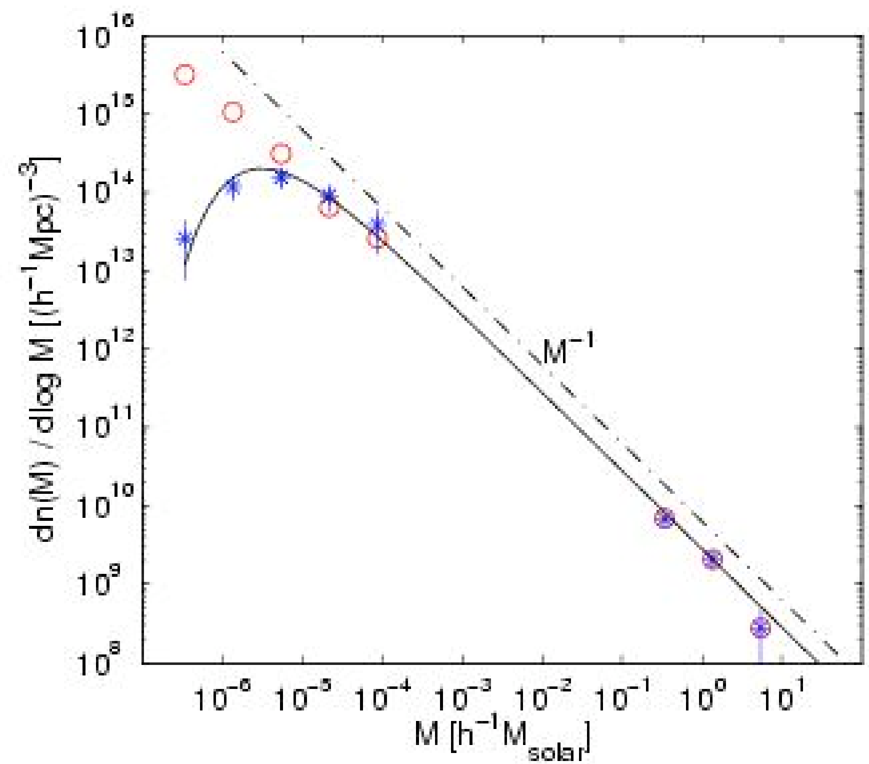
Initial box size $(3\ \mathrm{kpc})^3$
both zooms are x100.

First non-linear structures form at z~60 and have $M \sim 10^{-6} M_{\odot}$.

Properties of halos at z=26:

$$\rho \propto r^{-\gamma} \qquad \gamma \approx 1.5 - 2 \qquad \frac{\mathrm{d}n}{\mathrm{dlog}M} \propto M^{-1}$$





Extrapolating from galaxy simulations, claim $n(R_{\odot}) \sim 500\mathrm{pc}^{-3}$

Subsequent evolution

Disruption in similar mass mergers

Far more common than on Galactic scales (power spectrum is weak function of scale).

Berezinsky, Dokuchaev & Eroshenko analytic calculations: most mini-halos destroyed.

Tidal striping

Matter stripped from outer-regions if gravitational field of parent halo exceeds field of mini-halo.

Diemand, Moore & Stadel 'back of the envelope' calculation: halos in the Milky Way at $R < 3\,\mathrm{kpc}$ should survive intact.

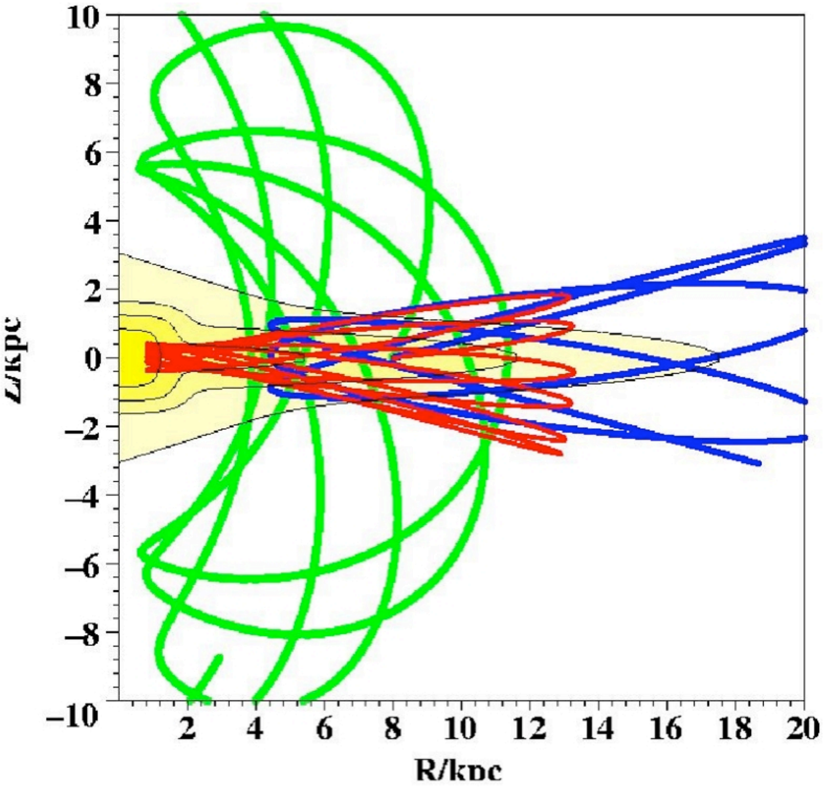
Encounters with stars

[Zhao, Taylor, Silk & Hooper; Moore, Diemand, Stadel & Quinn; Zhao, Taylor, Silk & Hooper;Berezinsky, Dokuchaev & Eroshenko]

Will heat the mini-halos and destroy (some fraction of) them.

More on mini-halo star encounters

Total mass loss (and disruption probability) depends on the number of stars the mini-halo encounters which depends on the amount of time the mini-halo spends in the Milky Way disk, which depends on its orbit.



[Zhao et al.] Orbits launched from solar neighbourhood. Red, blue and green orbits encounter mean stellar densities (calculated used Bescancon star counts model)

$< n >= 0.075,\ 0.045,\ 0.002 M_{\odot} \mathrm{pc}^{-3}$

Need to calculate orbits in a realistic (evolving, triaxial, barred) potential.

How much energy does a mini-halos lose in an encounter with impact parameter p, with a star with mass $M_\star$ and relative velocity v?

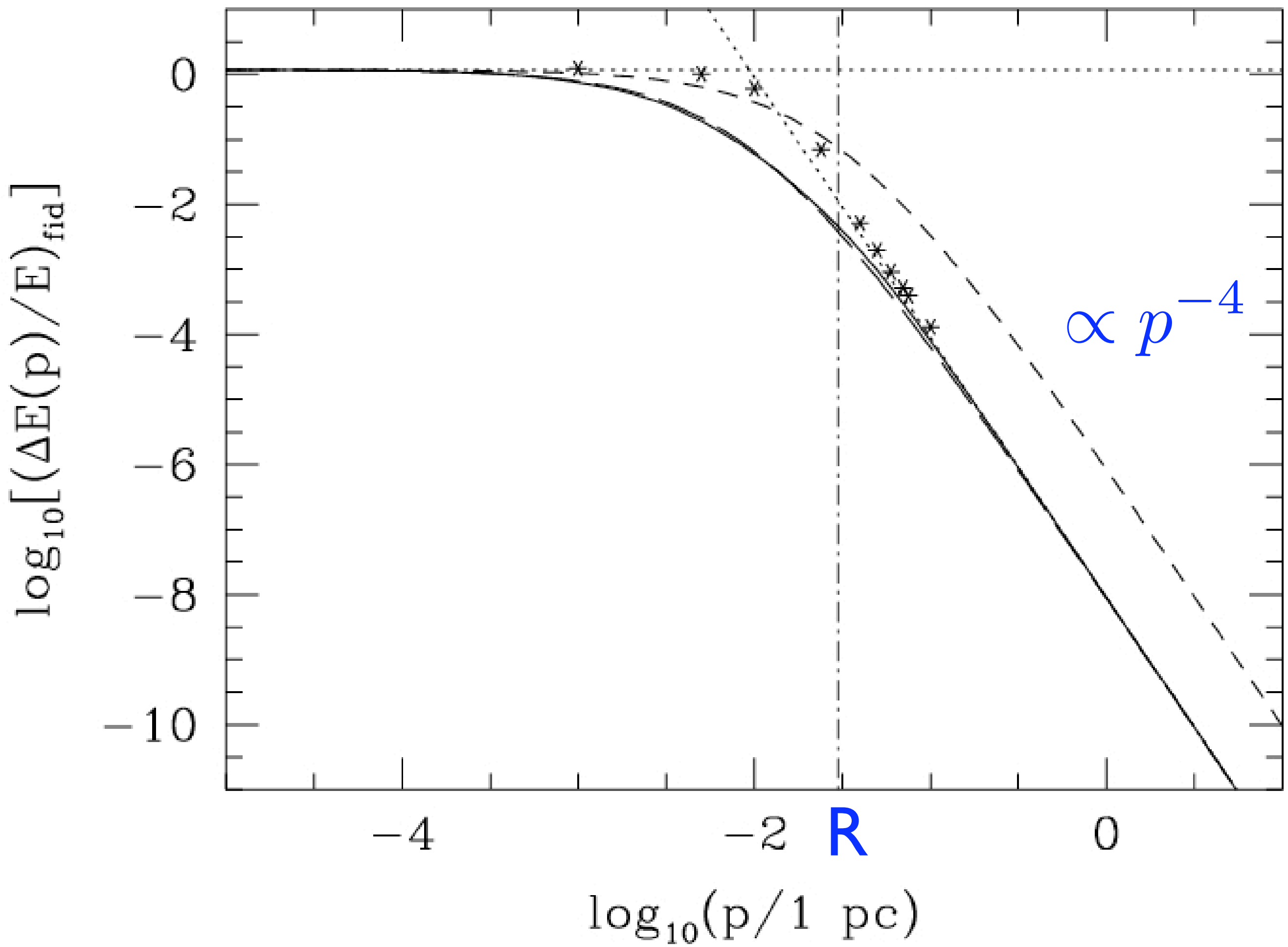
And how does this depend on the mini-halo mass?

Impulse approximation:

Duration of interaction is much less than dynamical time-scale of mini-halo (relative velocity ~100 km/s >> mini-halo velocity dispersion ~1 m/s) therefore can treat interaction as being instantaneous with particles in mini-halo effectively at rest.

Given density profile can calculate energy loss as a function of p (semi)-analytically, proportional to $(M_\star/v)^2$ [Spitzer, Gerhard & Fall, Carr & Sakellariadou]

Fractional energy loss in an interaction with a fiducial $M = M_\odot,\ v = 300\mathrm{km\,s^{-1}}$ star for a Plummer sphere:



——— impulse approx

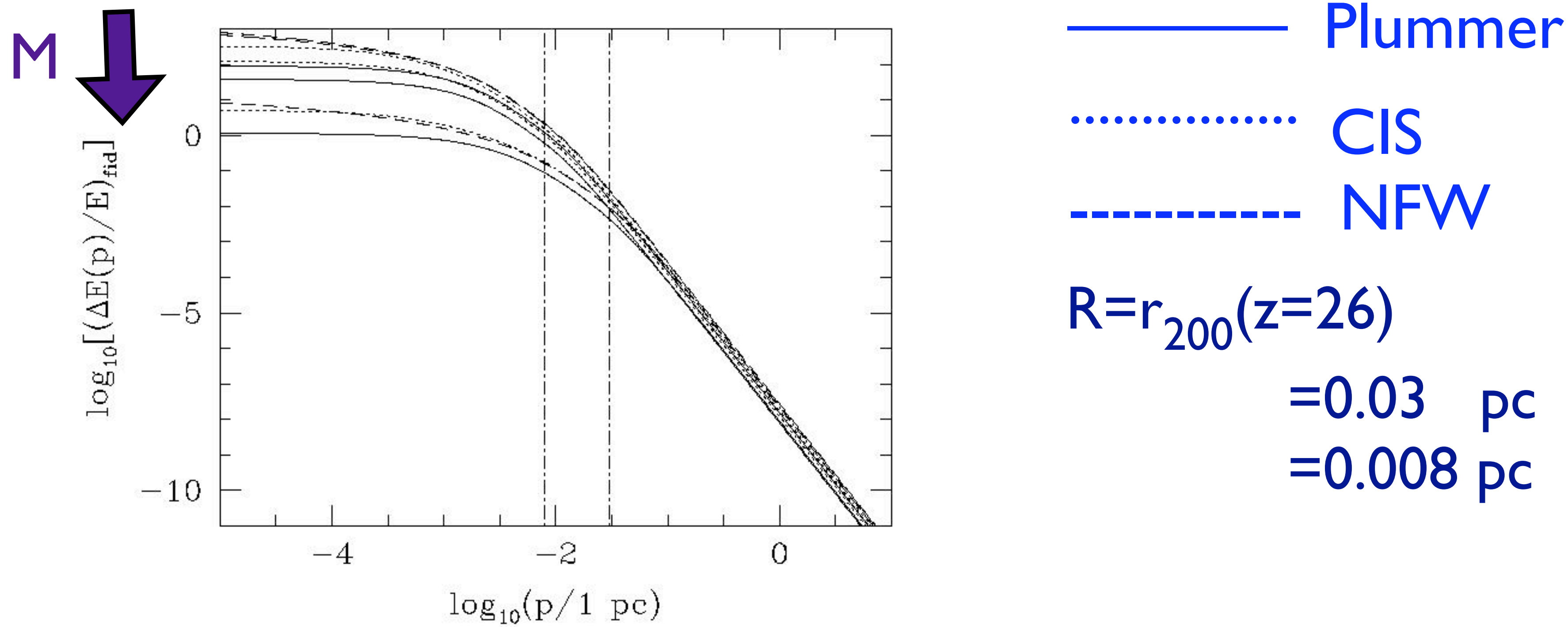
* * * * * simulations

- - - - - - old fitting function

— — — — — new fitting function

.... asymptotic limits

Dependence of fractional energy loss from impulse approximation on density profile for 3 sample halos from Diemand et al.:



$R=r_{200}(z=26)$
=0.03 pc halo 1
=0.008 pc halos 2&3

Energy loss for p<< and ~ R depends on density profile.

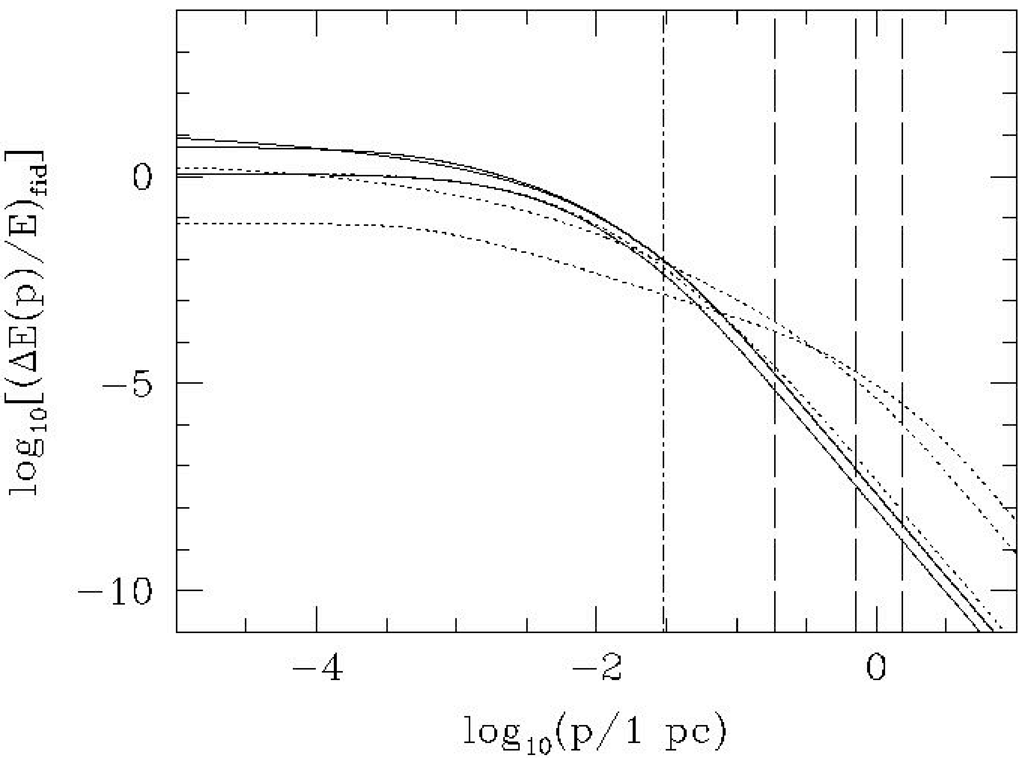
More massive halos lose a smaller fraction of their energy in close (p<<R) encounters (and close encounters are rarer since R is larger). (Scaling with M can be understood by considering uniform density sphere.)

Mini-halos with $M > \mathcal{O}(10^{-4}) M_\odot$ (exact threshold depends on scaling of small p energy loss with mass) will not be destroyed regardless of their orbit.

A problem: How to define/fix mini-halo radius?

NFW profile has $\rho(r) \propto r^{-3}$ at large radii. Formally infinite....

Radius is usually taken as radius at which density is ~200 times background density. This is time dependent....



Fractional energy loss in an interaction with a fiducial star for halo 1.

—— $R = r_{200}(z{=}26) = 0.03$ pc

.... $R = r_{200}(z{=}0)$

$= 0.19, 0.72, 1.5$ pc

(for Plummer, NFW, CIS)

Possible solution: radius=tidal radius imposed by parent halo

depends on parent and mini-halo profiles and mini-halo orbit (specifically its pericenter)

Other outstanding questions/issues:

What density profile should be used to model the mini-halos?
(quite possibly not NFW as they form monolithically, rather then heirarchically.....)

How many/which mini-halos survive the rapid merger phase?

Need an accurate determination of the fractional energy loss for p~R (taking into account energy carried away by unbound stars).

Can a mini-halo retain a (small, dense) bound core even if it loses most of its energy?

How much less susceptible to disruption are mini-halos formed from larger over-densities?

Still lots of work to be done before there is a reliable calculation/model of the present day mini-halo distribution........

Summary

* WIMP direct and indirect detection probe the dark matter distribution on small scales.

* Collisional damping and free-streaming erase density perturbations on small scales and set the scale of the first halos to form.

* Do (a significant fraction of) these halos survive to the present day??

* If WIMPs are detected can we probe:
  - WIMP properties (mass & interactions)?
  - small scale primordial power spectrum?
  - small scale modifications of gravity?