NLO SUSY-QCD Corrections to the "EGRET Gamma-Ray Signal" $\tilde{\chi}^0\tilde{\chi}^0 \longrightarrow A^0 \longrightarrow b\bar{b}$

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Outline

Why Corrections at order α_s...?

WMAP mission has given precise range for Dark Matter relic density in our Universe [astro-ph/0302209]

$$0.0945 \leq \Omega_{\mathrm{CDM}} h^2 \leq 0.1287 \qquad \text{(at } 2\sigma\text{)}$$

Supersymmetry provides interesting Dark Matter candidate (lightest of the four neutralinos)

Relic Density calculation allows to constrain parameter space of supersymmetric models by comparing to WMAP region

$$\frac{dn}{dt} = -3Hn - \langle \sigma_{\mathrm{eff}} v \rangle \left(n^2 - n_{\mathrm{eq}}^2 \right) \qquad n \propto \Omega_{\mathrm{CDM}} h^2$$

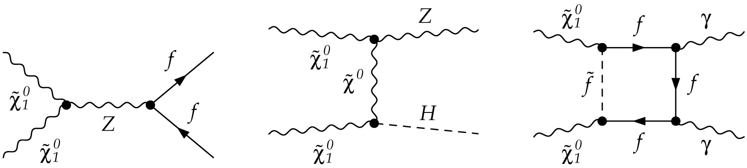
Higher precision in cross sections $\langle \sigma_{\mathrm{eff}} v \rangle$ required to obtain better precision in relic density Ω_{CDM}

Why Corrections at order α_s...?

Several public codes perform relic density calculation within supersymmetric models:

- DarkSUSY [hep-ph/0406204]
- micrOMEGAs [hep-ph/0602198]

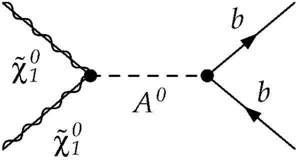
At present most processes only implemented at leading order



Corrections at next-to-leading order are supposed to be important, at least in certain regions of the parameter space...

Why $\tilde{\chi}^0\tilde{\chi}^0 \to A^0 \to b\bar{b}$...?

Annihilation channel into fermion/antifermion pairs is always open



Process claimed to be compatible with the EGRET gamma-ray excess by de Boer et al. for a neutralino mass of 50 - 100 GeV [astro-ph/0408272]

Process supposed to be important in the *A-funnel* parameter region , i.e. at large $\tan\beta$ and large m_0 (large $\tan\beta$ favoured by theory...)

Result can be carried over to neutralino-quark scattering cross section by s-t-crossing (interesting for direct detection...)

Supersymmetric Model

Phenomenological MSSM with (only) seven free parameters

$$m_0, \quad M_2, \quad m_A, \quad \mu, \quad \tan\beta, \quad A_b, \quad A_t$$

Spectrum calculation:

- SuSpect [hep-ph/0211331] (particle masses)
- HDecay [hep-ph/9704448] (Higgs decay width)

Assumptions:

- R-parity conservation (lightest supersymmetric particle stable)
- Dark Matter particle is the lightest neutralino $\tilde{\chi}_1^0$
- No squark mixing (i.e. in particular $m_{\tilde{b}_1} = m_{\tilde{b}_2}$)

Explore the different regions of the parameter space

Non-relativistic Limit

Cross sections have to be evaluated in the non-relativistic limit, i.e. expansion in powers of relative velocity

$$s \doteq 4m_\chi^2\left(1+\frac{v_{\rm rel}^2}{4}\right)+\mathcal{O}(v_{\rm rel}^4) \quad \Longrightarrow \quad \sigma v_{\rm rel} \doteq a + b v_{\rm rel}^2 + \mathcal{O}(v_{\rm rel}^4)$$

Thermal averaged cross section needed for relic density calculation

[hep-ph/0301106]

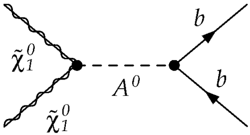
$$\langle \sigma v_{\rm rel}\rangle = \int dv_{\rm rel}\; f(v_{\rm rel}, T, m_\chi)\; \sigma v_{\rm rel}$$

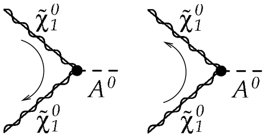
Boltzmann equation can be integrated numerically to obtain relic density...

$$\frac{dn}{dt} = -3Hn - \langle \sigma_{\rm eff} v\rangle \left(n^2 - n_{\rm eq}^2\right)$$

The Born Calculation

Initial state of Majorana neutralinos has to be anti-symmetrized
[Phys.Lett.B291:278-280,1992]





Cross section important at large $\tan\beta$

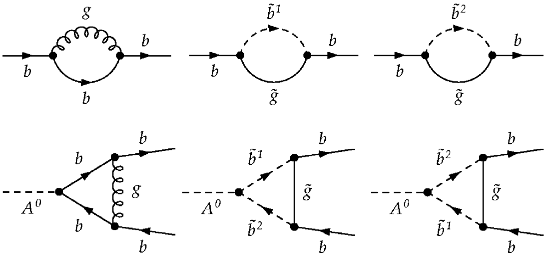
$$\sigma_{\rm LO} \sim 2\,\alpha_{\rm ew}^2 N_{\rm C} \tan^2\beta \frac{\sqrt{s-4m_{\rm b}^2}\sqrt{s-4m_\chi^2}}{\left|s-m_{\rm A}^2+im_{\rm A}\Gamma_{\rm A}\right|^2}$$

Non-relativistic expansion in powers of relative velocity in agreement with Jungman et al. [hep-ph/9506380]

Leading order cross section implemented in public codes

Virtual QCD Corrections at Next-to-Leading Order

Quark self-energy and vertex correction at order α_s,
Standard Model and Supersymmetry contributions



Virtual corrections factorize Born cross section

$$\sigma_V = 2\,\Lambda_V\,\sigma_{LO} = 2\,(2\Lambda_{bb} + \Lambda_{Abb})\,\sigma_{LO}$$

Use dimensional regularization to handle singularities in loop integrals

Handling UV-Singularities in QCD Correction

Unrenormalized virtual correction factor:

$$\Lambda_V = \frac{\alpha_S C_F}{2\pi}\left[\frac{2}{\epsilon_{uv}} - \frac{1+\beta_b^2}{2\beta_b}\ln\frac{1-\beta_b}{1+\beta_b}\left(\frac{1}{\epsilon_{ir}} - \ln\frac{m_b^2}{\mu^2}\right) - 2\ln\frac{m_b^2}{\mu^2} + \lambda(\beta_b)\right]$$

Counterterms from on-shell renormalization at scale μ

$$\delta Z_m + \delta Z_\psi = \frac{\alpha_S C_F}{2\pi}\left[-\frac{2}{\epsilon_{uv}} - \frac{1}{\epsilon_{ir}} - 4 + 3\ln\frac{m_b^2}{\mu^2}\right]$$

UV-singularities vanish in renormalized virtual correction factor, only IR-singularities remain

$$\Lambda_V^{(\mathrm{ren})} = \Lambda_V + (\delta Z_m + \delta Z_\psi)$$

Same procedure for Supersymmetry part, but no IR-singularities...

Handling UV-Singularities in SUSY-QCD Correction

Unrenormalized virtual correction factor:

$$\Lambda_{\rm V} = \frac{\alpha_S C_F}{2\pi} \frac{m_{\tilde{g}}(\mu + A_b \tan\beta)}{\tan\beta} C_0(m_b^2, s, m_b^2; m_{\tilde{b}}^2, m_{\tilde{g}}^2, m_{\tilde{b}}^2)$$

Counterterms from on-shell renormalization at scale μ

$$\delta Z_m + \delta Z_\psi = \frac{\alpha_S C_F}{2\pi} \left[\frac{1}{\epsilon_{\rm uv}} - \frac{1}{\epsilon_{\rm uv}} + \frac{A_0^{(fin)}(m_{\tilde{b}}^2)}{m_b^2} - \frac{A_0^{(fin)}(m_{\tilde{g}}^2)}{m_b^2} \right.$$
$$\left. - \frac{(m_{\tilde{b}}^2 - m_{\tilde{g}}^2)}{m_b^2} + B_0^{(fin)}(m_b^2, m_{\tilde{g}}^2, m_{\tilde{b}}^2) + \frac{(m_b^2 + m_{\tilde{g}}^2 - m_{\tilde{b}}^2)}{m_b^2} B_0'(m_b^2, m_{\tilde{g}}^2, m_{\tilde{b}}^2) \right]$$

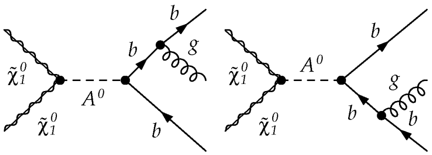
Renormalized virtual correction factor is free of singularities

$$\Lambda_{\rm V}^{\rm (ren)} = \Lambda_{\rm V} + (\delta Z_m + \delta Z_\psi)$$

Correction factor depends on supersymmetric parameters ...

Real Corrections at Next-to-Leading Order

Real gluon emission cancels remaining IR-singularities



Differential real emission cross section

$$\frac{d\sigma_{\rm R}}{\sigma_{\rm LO}} = \frac{\alpha_S C_F}{2\pi}\left[\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)} - \frac{2m_b^2}{s}\frac{(2-x_1-x_2)^2}{(1-x_1)^2(1-x_2)^2}\right]\frac{dx_1 dx_2}{\beta_b\beta_\chi^2}$$

Use Dipole Subtraction Method proposed by Catani et al. to compute virtual and real part numerically [hep-ph/0201036]

$$\sigma_{\rm NLO} = \left[\sigma_{\rm V} + \int d\sigma_{\rm aux}\right]_{\epsilon=0} + \int\left[d\sigma_{\rm R} - d\sigma_{\rm aux}\right]_{\epsilon=0}$$

Handling IR-Singularities

Use Dipole Subtraction Method proposed by Catani et al. to compute virtual and real part numerically [hep-ph/0201036]

$$\sigma_{\mathrm{NLO}} = \left[\sigma_{\mathrm{V}} + \int d\sigma_{\mathrm{aux}}\right]_{\epsilon=0} + \int \left[d\sigma_{\mathrm{R}} - d\sigma_{\mathrm{aux}}\right]_{\epsilon=0}$$

Auxiliary cross section will cancel seperately IR-singularities in both virtual and real part of the next-to-leading order cross section

$$\frac{d\sigma_{\mathrm{aux}}}{\sigma_{\mathrm{LO}}} = \frac{\alpha_S C_F}{2\pi}\left[\mathcal{D}_{31,2}(x_1, x_2) + \mathcal{D}_{32,1}(x_1, x_2)\right]\frac{dx_1\, dx_2}{\beta_b \beta_\chi^2}$$

$$\int \frac{d\sigma_{\mathrm{aux}}}{2\sigma_{\mathrm{LO}}} = \frac{\alpha_S C_F}{2\pi}\left[\left(1 + \frac{1+\beta_b^2}{2\beta_b}\ln\frac{1-\beta_b}{1+\beta_b}\right)\frac{1}{\epsilon_{\mathrm{ir}}} - \frac{1}{2}\ln\frac{m_b^2}{\mu^2} + \lambda(\beta_b)\right]$$

Both parts can be computed seperately

Final Analytical QCD Result

Cross section at next-to-leading order

$$\sigma_{\rm NLO} = 2\,\frac{\alpha_S\,C_F}{2\pi}\Lambda_{\rm NLO}\;\sigma_{\rm LO}$$

where

$$\Lambda_{\rm NLO} = \left[A(\beta_b) - \frac{1}{16\beta_b}\left(19 + 2\beta_b^2 + 3\beta_b^4\right)\ln\frac{1-\beta_b}{1+\beta_b} + \frac{3}{8}\left(7-\beta_b^2\right)\right]$$

For large s ($\beta_b \to 1$) correction becomes negative

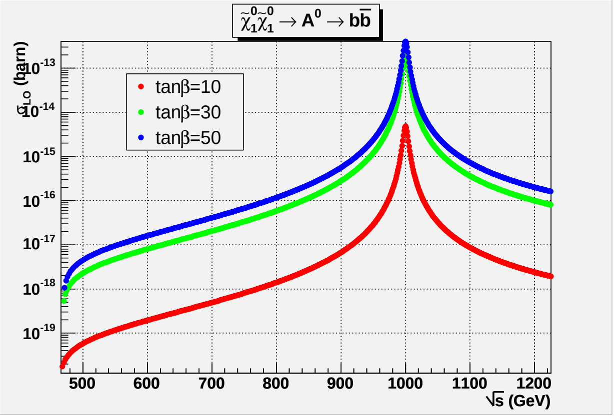
$$\Lambda_{\rm NLO} \simeq \left[\frac{3}{2}\ln\frac{m_b^2}{s} + \frac{9}{4}\right]$$

Resummation of leading logarithmic terms, i.e. use running quark mass

[Phys. Lett. B240: 455, 1990]

$$\sigma_{\rm NLO} = 2\,\frac{\alpha_S\,C_F}{2\pi}\left[\frac{\ln(4m_b^2/\Lambda_{\rm QCD})}{\ln(s/\Lambda_{\rm QCD})}\right]^{\frac{24}{33-2N_f}}\left[\Lambda_{\rm NLO} - \frac{3}{2}\ln\frac{m_b^2}{s}\right]\sigma_{\rm LO}$$

Born Cross Section

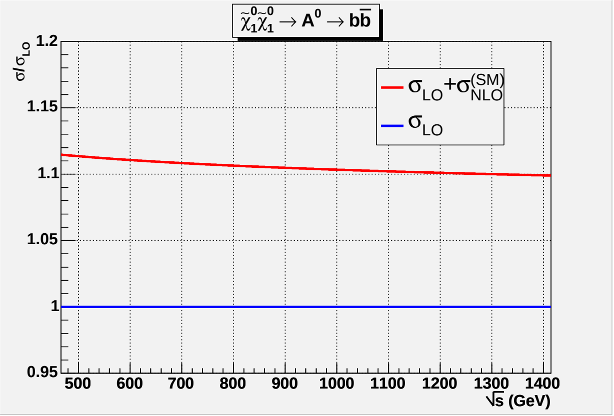


Parameters:

$m_0 = 1000$ GeV
$M_2 = 500$ GeV
$m_A = 1000$ GeV
$\mu = 300$ GeV
$A_b = A_t = 3000$ GeV

Process important at large $\tan\beta$

QCD Cross Section at Next-To-Leading Order



Parameters:

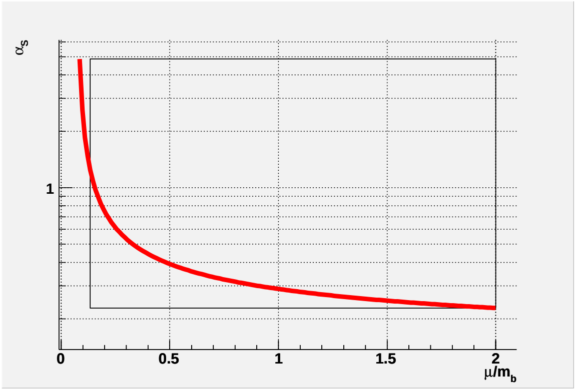
$m_0 = 1000$ GeV
$M_2 = 500$ GeV
$m_A = 1000$ GeV
$\mu = 300$ GeV
$\tan\beta = 10$
$A_b = A_t = 3000$ GeV

QCD correction independant from SUSY parameters

NLO corrections enhance cross section by about 10%

Dependance on Renormalization Scale

The only scale dependance is in $\alpha_S(\mu)$



Uncertainty of about 30% at "natural scale" $\mu = m_b$,
lower uncertainty at higher scale (e.g. $\mu = m_A$)...

Summary and Outlook

Relic density calculation allows to constrain supersymmetric models

Corrections at next-to-leading order important for better relic density accuracy

We present SUSY-QCD corrections at order α_S to the neutralino annihilation process $\tilde{\chi}^0\tilde{\chi}^0 \longrightarrow A^0 \longrightarrow b\bar{b}$ [article in preparation]

The QCD corrections enhance the leading order cross section by about 10%

Next steps:

- Evaluate Supersymmetry corrections at order α_S numerically
- Evaluate thermal averaged cross section
- Evaluate effects on neutralino relic density
- ...