NLO SUSY-QCD Corrections to the "EGRET Gamma-Ray Signal"

$\tilde{\chi}^0 \tilde{\chi}^0 \rightarrow A^0 \rightarrow b\bar{b}$

Björn Herrmann

in collaboration with Michael Klasen

LPSC Grenoble

Workshop "The Dark Side of the Universe"

Outline

1. Motivation
2. Model and Method
3. Analytical Discussion
4. Numerical Results
5. Conclusion
**Why Corrections at order $\alpha_s$...?**

**WMAP mission** has given precise range for Dark Matter relic density in our Universe [astro-ph/0302209]

$$0.0945 \leq \Omega_{CDM} h^2 \leq 0.1287 \quad \text{(at 2\sigma)}$$

Supersymmetry provides interesting **Dark Matter candidate** (lightest of the four neutralinos)

**Relic Density calculation** allows to constrain parameter space of supersymmetric models by comparing to WMAP region

$$\frac{dn}{dt} = -3Hn - \langle \sigma_{\text{eff}}v \rangle (n^2 - n_{eq}^2)$$

$$n \propto \Omega_{CDM} h^2$$

Higher precision in **cross sections** $\langle \sigma_{\text{eff}}v \rangle$ required to obtain better precision in relic density $\Omega_{CDM}$
Several public codes perform relic density calculation within supersymmetric models:

- DarkSUSY [hep-ph/0406204]
- micrOMEGAs [hep-ph/0602198]

At present most processes only implemented at leading order.

Corrections at next-to-leading order are supposed to be important, at least in certain regions of the parameter space...
Why $\tilde{\chi}^0\tilde{\chi}^0 \rightarrow A^0 \rightarrow b\bar{b}$...?

Annihilation channel into fermion/antifermion pairs is always open

Process claimed to be compatible with the EGRET gamma-ray excess by de Boer et al. for a neutralino mass of 50 - 100 GeV [astro-ph/0408272]

Process supposed to be important in the $A$-funnel parameter region, i.e. at large $\tan\beta$ and large $m_0$ (large $\tan\beta$ favoured by theory...)

Result can be carried over to neutralino-quark scattering cross section by $s$-$t$-crossing (interesting for direct detection...)

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Supersymmetric Model

Phenomenological MSSM with (only) seven free parameters

\[ m_0, \ M_2, \ m_A, \ \mu, \ \tan \beta, \ A_b, \ A_t \]

Spectrum calculation:

- SuSpect [hep-ph/0211331] (particle masses)
- HDecay [hep-ph/9704448] (Higgs decay width)

Assumptions:

- R-parity conservation (lightest supersymmetric particle stable)
- Dark Matter particle is the lightest neutralino \( \tilde{\chi}_1^0 \)
- No squark mixing (i.e. in particular \( m_{\tilde{b}_1} = m_{\tilde{b}_2} \))

Explore the different regions of the parameter space
Non-relativistic Limit

Cross sections have to be evaluated in the non-relativistic limit, i.e. expansion in powers of relative velocity

\[ s = 4 m_{\chi}^2 \left( 1 + \frac{v_{\text{rel}}^2}{4} \right) + O(v_{\text{rel}}^4) \quad \Longrightarrow \quad \sigma v_{\text{rel}} = a + b v_{\text{rel}}^2 + O(v_{\text{rel}}^4) \]

**Thermal averaged** cross section needed for relic density calculation

*[hep-ph/0301106]*

\[ \langle \sigma v_{\text{rel}} \rangle = \int dv_{\text{rel}} f(v_{\text{rel}}, T, m_{\chi}) \sigma v_{\text{rel}} \]

**Boltzmann equation** can be integrated numerically to obtain relic density...

\[ \frac{dn}{dt} = -3 H n - \langle \sigma_{\text{eff}} v \rangle (n^2 - n_{\text{eq}}^2) \]
The Born Calculation

Initial state of Majorana neutralinos has to be anti-symmetrized


Cross section important at large \( \tan \beta \)

\[
\sigma_{LO} \sim 2 \alpha_{ew}^2 N_C \tan^2 \beta \frac{\sqrt{s - 4m_b^2} \sqrt{s - 4m^2_\chi}}{|s - m^2_A + im_\Gamma_A|^2}
\]

Non-relativistic expansion in powers of relative velocity in agreement with Jungman et al. \[ \text{[hep-ph/9506380]} \]

Leading order cross section implemented in public codes

\( \tilde{\chi}^0 \tilde{\chi}^0 \rightarrow A^0 \rightarrow b\bar{b} \) at NLO
Virtual QCD Corrections at Next-to-Leading Order

Quark self-energy and vertex correction at order $\alpha_s$, Standard Model and Supersymmetry contributions

Virtual corrections factorize Born cross section

$$\sigma_V = 2 \Lambda_V \sigma_{LO} = 2 (2\Lambda_{bb} + \Lambda_{Abb}) \sigma_{LO}$$

Use dimensional regularization to handle singularities in loop integrals

$\tilde{\chi}^0 \tilde{\chi}^0 \rightarrow A^0 \rightarrow b\bar{b}$ at NLO
Handling UV-Singularities in QCD Correction

Unrenormalized virtual correction factor:

\[ \Lambda_V = \frac{\alpha_S C_F}{2\pi} \left[ \frac{2}{\epsilon_{uv}} - \frac{1 + \beta_b^2}{2\beta_b} \ln \frac{1 - \beta_b}{1 + \beta_b} \left( \frac{1}{\epsilon_{ir}} - \ln \frac{m_b^2}{\mu^2} \right) - 2 \ln \frac{m_b^2}{\mu^2} + \lambda(\beta_b) \right] \]

Counterterms from on-shell renormalization at scale \( \mu \)

\[ \delta Z_m + \delta Z_\psi = \frac{\alpha_S C_F}{2\pi} \left[ -\frac{2}{\epsilon_{uv}} - \frac{1}{\epsilon_{ir}} - 4 + 3 \ln \frac{m_b^2}{\mu^2} \right] \]

UV-singularities vanish in renormalized virtual correction factor, only IR-singularities remain

\[ \Lambda_V^{(\text{ren})} = \Lambda_V + (\delta Z_m + \delta Z_\psi) \]

Same procedure for Supersymmetry part, but no IR-singularities...
Handling UV-Singularities in SUSY-QCD Correction

Unrenormalized virtual correction factor:

\[ \Lambda_V = \frac{\alpha_S C_F}{2\pi} \frac{m_{\tilde{g}}(\mu + A_b \tan \beta)}{\tan \beta} C_0(m_b^2, s, m_{\tilde{b}}^2, m_{\tilde{g}}^2, m_b^2) \]

Counterterms from on-shell renormalization at scale \( \mu \)

\[
\delta Z_m + \delta Z_\psi = \frac{\alpha_S C_F}{2\pi} \left[ \frac{1}{\epsilon_{uv}} - \frac{1}{\epsilon_{uv}} + \frac{A_0^{(\text{fin})}(m_b^2)}{m_b^2} - \frac{A_0^{(\text{fin})}(m_{\tilde{g}}^2)}{m_b^2} - \frac{(m_b^2 - m_{\tilde{g}}^2)}{m_b^2} + B_0^{(\text{fin})}(m_b^2, m_{\tilde{g}}^2, m_b^2) + \frac{(m_b^2 + m_{\tilde{g}}^2 - m_{\tilde{b}}^2)}{m_b^2} B_0'(m_b^2, m_{\tilde{g}}^2, m_b^2) \right]
\]

Renormalized virtual correction factor is free of singularities

\[ \Lambda_V^{(\text{ren})} = \Lambda_V + (\delta Z_m + \delta Z_\psi) \]

Correction factor depends on supersymmetric parameters ...

Björn Herrmann (LPSC Grenoble) \[ \tilde{\chi}^0 \tilde{\chi}^0 \rightarrow A^0 \rightarrow b\bar{b} \] at NLO
Real Corrections at Next-to-Leading Order

Real gluon emission cancels remaining IR-singularities

\[ \frac{d\sigma_R}{\sigma_{LO}} = \frac{\alpha_S C_F}{2\pi} \left[ \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)} - \frac{2m_b^2}{s} \frac{(2 - x_1 - x_2)^2}{(1 - x_1)^2(1 - x_2)^2} \right] \frac{dx_1 dx_2}{\beta_b \beta_{\chi}^2} \]

Differential real emission cross section

Use Dipole Subtraction Method proposed by Catani et al. to compute virtual and real part numerically [hep-ph/0201036]

\[ \sigma_{NLO} = \left[ \sigma_V + \int d\sigma_{aux} \right]_{\epsilon=0} + \int \left[ d\sigma_R - d\sigma_{aux} \right]_{\epsilon=0} \]
Use Dipole Subtraction Method proposed by Catani et al. to compute virtual and real part numerically \[\text{[hep-ph/0201036]}\]

\[
\sigma_{\text{NLO}} = \left[ \sigma_{\text{V}} + \int d\sigma_{\text{aux}} \right]_{\epsilon=0} + \int \left[ d\sigma_{\text{R}} - d\sigma_{\text{aux}} \right]_{\epsilon=0}
\]

Auxiliary cross section will cancel seperately IR-singularities in both virtual and real part of the next-to-leading order cross section

\[
\frac{d\sigma_{\text{aux}}}{\sigma_{\text{LO}}} = \frac{\alpha_S C_F}{2\pi} \left[ \mathcal{D}_{31,2}(x_1, x_2) + \mathcal{D}_{32,1}(x_1, x_2) \right] \frac{dx_1 dx_2}{\beta_b \beta^2_{\chi}}
\]

\[
\int \frac{d\sigma_{\text{aux}}}{2\sigma_{\text{LO}}} = \frac{\alpha_S C_F}{2\pi} \left[ \left( 1 + \frac{1 + \beta^2_b}{2\beta_b} \ln \frac{1 - \beta_b}{1 + \beta_b} \right) \frac{1}{\epsilon_{\text{ir}}} - \frac{1}{2} \ln \frac{m^2_b}{\mu^2} + \lambda(\beta_b) \right]
\]

Both parts can be computed seperately
Final Analytical QCD Result

Cross section at next-to-leading order

$$\sigma_{\text{NLO}} = 2 \frac{\alpha_S C_F}{2\pi} \Lambda_{\text{NLO}} \sigma_{\text{LO}}$$

where

$$\Lambda_{\text{NLO}} = \left[ A(\beta_b) - \frac{1}{16\beta_b} \left( 19 + 2\beta_b^2 + 3\beta_b^4 \right) \ln \frac{1 - \beta_b}{1 + \beta_b} + \frac{3}{8} (7 - \beta_b^2) \right]$$

For large $s$ ($\beta_b \to 1$) correction becomes negative

$$\Lambda_{\text{NLO}} \approx \left[ \frac{3}{2} \ln \frac{m_b^2}{s} + \frac{9}{4} \right]$$

Resummation of leading logarithmic terms, i.e. use running quark mass

Process important at large $\tan \beta$

Parameters:

$m_0 = 1000$ GeV

$M_2 = 500$ GeV

$m_A = 1000$ GeV

$\mu = 300$ GeV

$A_b = A_t = 3000$ GeV
Parameters:
\[ m_0 = 1000 \text{ GeV} \]
\[ M_2 = 500 \text{ GeV} \]
\[ m_A = 1000 \text{ GeV} \]
\[ \mu = 300 \text{ GeV} \]
\[ \tan \beta = 10 \]
\[ A_b = A_t = 3000 \text{ GeV} \]

NLO corrections enhance cross section by about 10%
Dependence on Renormalization Scale

The only scale dependence is in $\alpha_S(\mu)$

Uncertainty of about 30% at "natural scale" $\mu = m_b$, lower uncertainty at higher scale (e.g. $\mu = m_A$)
Summary and Outlook

Relic density calculation allows to constrain supersymmetric models

Corrections at next-to-leading order important for better relic density accuracy

We present SUSY-QCD corrections at order $\alpha_S$ to the neutralino annihilation process $\tilde{\chi}^0 \tilde{\chi}^0 \rightarrow A^0 \rightarrow b\bar{b}$ [article in preparation]

The QCD corrections enhance the leading order cross section by about 10%

Next steps:

- Evaluate Supersymmetry corrections at order $\alpha_S$ numerically
- Evaluate thermal averaged cross section
- Evaluate effects on neutralino relic density
- ...

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