A solution of the cusp problem in virialized DM halos in standard cosmology

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Advanced study in cosmology

- Analysis of CMB and its polarization
- Investigation of Ly-α forest
- Properties of earlier galaxies and quasars at large redshifts
- Properties of dwarf galaxies and the internal structure of galaxies (black holes, rotation curves, etc.)

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Cusp problem in galaxy halos
(simulations)

\[ \rho \sim r^{-\alpha} \]
\[ \alpha \in (1, 3/2) \]

Diemand et al. 2004
The cusp problem is being considered as main problem of the standard cosmology (DM non-interacting to baryons)

We argue: it is solved within the framework of standard DM model

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Internal structure of relaxed halos
(cores instead of cusps)

Idea: take into account the small scale part of initial cosmological perturbations that transforms into the thermal energy of DM particles during violent relaxation

Method: extra entropy of DM particles related to initial background perturbations

⇒ entropy profiles related to density profiles of DM halos

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Standard cosmological model

- $H_0 = 70 \text{ km/s/Mpc, } h=0.7$
- $\Omega_\Lambda = 0.7$
- $\Omega_m = 0.3$
- $\Lambda$CDM power spectrum:

$$P(k) = A \, \kappa T^2(\kappa) \exp(-R_f^2 \kappa^2), \quad \kappa = k/k_0$$

$$k_0 = 0.2 \text{ h/Mpc, } A = 240\sigma_8^2/k_0^3$$

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\[ \frac{H(z)}{H_0} = \sqrt{1 + \Omega_m (z^3 + 3z^2 + 3z)} \Rightarrow 0.5(1+z)^{3/2}, \quad z > 1 \]

\[ \rho_m = \frac{3H_0^2 \Omega_m}{8\pi G} (1 + z)^3 \approx 3 \cdot 10^{-30} (1 + z)^3 \frac{g}{cm^3} \]

\[ n = \frac{\rho_m}{m_{DM}} \approx 3 \cdot 10^{-6} \mu^{-1} (1+z)^3 \text{ cm}^{-3}, \quad \mu \equiv \frac{m_{DM}}{m_p} \]

- **R_f** - particle free path in the early Universe

\[ m_{DM} > 1 \text{ M}\ell, \quad R_f < 4 \cdot 10^{-6} \Rightarrow \text{stellar halos} \]

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Observational and model spectra

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Equilibrium DM halos

- Adiabatic and irreversible processes

- Entropy function: \( F = \frac{T}{n^{2/3}} = \frac{p}{n^{5/3}} \)

- Hydrostatic equilibrium:
  \[
  \frac{1}{\rho} \frac{dp}{dr} = -\frac{GM(r)}{r^2}
  \]

- Initial (background) entropy in small scale:
  \( F \sim M^{1/3-2/3} \)

- Hierarchical and violent relaxation of compressed matter (large scale):
  \( F \sim M^{5/6}, \quad F \sim M^{4/3} \)

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Entropy of collisionless particles for isotropic velocity distribution

\[ p = \rho \langle v^2 \rangle = nT = F n^{5/3} \]

one-dimensional peculiar velocity

Power-law density profiles: \( \alpha \in (0, 2) \)

\[ \rho(r) \propto r^{-\alpha}, \quad M \propto r^{3-\alpha}, \quad p = C_1 + C_2 r^{2(1-\alpha)} \]

\( \alpha < 1 \) - finite pressure in the centre (core)

\( \alpha \geq 1 \) - infinite pressure at \( r \to 0 \) (cusp)

\( r \Rightarrow M \) -> conserving both for initial and relaxed matter fields

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\[ F(M) \propto C_1 M^{\beta_1} + C_2 M^{\beta_2} \propto M^\beta \]

\[ \beta \in (\beta_1, \beta_2), \quad \beta_{1,2} = \frac{1 + 2\alpha / 3}{3 - \alpha} \pm \frac{|\alpha - 1|}{\alpha - 3} \]

\[ \beta_{cr} = \beta_1 = \beta_2 = \frac{5}{6} \quad \Rightarrow \quad \alpha_{cr} = 1 \]

**core (\( \beta < 5/6 \)):** \[ \beta_1 \in \left(0, \frac{5}{6}\right), \quad \beta_2 \in \left(\frac{2}{3}, \frac{5}{6}\right) \]

**cusp (\( \beta \geq 5/6 \)):** \[ \beta_1 \in \left(\frac{5}{6}, \frac{4}{3}\right), \quad \beta_2 \in \left(\frac{5}{6}, \frac{10}{3}\right) \]

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48 low brightness galaxies (LBG, de Blok et al. 2001)
15 low brightness galaxies

(Swaters et al. 2003)
NB (in relation to A. Klypin talk):

we call cores by cores and cusps by cusps

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Initial velocity field

\[ \vec{r}(z, \vec{x}) = (1 + z)^{-1}[\vec{x} - g(z)\vec{S}(\vec{x})] \quad \delta(\vec{x}) \equiv \delta \rho / \rho = \text{div}(\vec{S}) \]

\[ \vec{V}(z, \vec{x}) \equiv \dot{\vec{r}} = H(z)[\vec{r} + g'(z)\vec{S}(\vec{x})] \]

\[ \Rightarrow \frac{H(z)}{1 + z}[\vec{x} - 2g(z)\vec{S}(\vec{x})] , \quad z > 1 \]

**cosmological background**

\[ \langle \delta(\vec{x}) \rangle = \langle \vec{S}(\vec{x}) \rangle = 0 , \quad \vec{x}_1 - \vec{x}_2 = x \vec{e} , \quad q = x / \ell_v \]

\[ \xi(x) \equiv \langle \delta(\vec{x}_1)\delta(\vec{x}_2) \rangle = \sigma^2_0 G_0(q) , \quad G(0) = 1 \]

\[ \xi_{ij}(x) \equiv \langle S_i(\vec{x}_1)S_j(\vec{x}_2) \rangle = \frac{1}{3} \sigma^2_s [e_i e_j G_{12}(q) + (\delta_{ij} - e_i e_j)G_1(q)] \]

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small scale density correlations
\[ \ell_0 < 1 \text{ kpc} \quad (G_0 = 0.5) \]

large scale velocity correlations
\[ \ell_1 \equiv \sigma_S = 11 \text{ } h^{-1}\text{Mpc} \quad (G_{12} = 0.5) \]
\[ \ell_v \equiv 31 \text{ } h^{-1}\text{Mpc} \quad (G_1 = 0.5) \]

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DM halo formation
(Zel’dovich approximation)

\[ \tilde{r}(z, \tilde{x}) = (1 + z)^{-1}[\tilde{x} - g\tilde{S}(\tilde{x})] \]
\[ \tilde{V}(z, \tilde{x}) \equiv H_0 (1 + z)^{1/2}[\tilde{x}/2 - g(z)\tilde{S}(\tilde{x})] \]

local background – protohalo with scale/mass resolution \( R \)
(turns into relaxed halo via violent/hierarchical relaxation)

conditional perturbations
(transform adiabatically to microscopic motion of particles inside the halo)

halo formation time

\[ \langle \tilde{S}_R(\tilde{x}) \rangle_* = (1 + z_0)\tilde{x}/2 \]
\[ \langle \tilde{S}_*(\tilde{x}) \rangle_* = 0 \]
\[ \sigma_* \equiv \sqrt{\langle \tilde{S}_*^2 \rangle_*} \Rightarrow \sigma_S \]

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Variance of one-dimensional peculiar velocities inside halo

\[ \vec{v}_* (z, \vec{x}) \equiv H_0 (1 + z)^{-1/2} \tilde{S}_* (\vec{x}) \]

\[ v_{12} = \bar{e} \left[ \vec{v}_* (z, \vec{x}_1) - \vec{v}_* (z, \vec{x}_2) \right] \]

\[ \vec{x}_1 - \vec{x}_2 = \ell_v q \bar{e}, \quad |\bar{e}| = 1 \]

\[ \sigma_v^2 \equiv \langle v_{12}^2 \rangle_* \equiv H_0^2 \sigma_S^2 (1 + z)^{-1} (1 - G_{12}(q)) \]

\[ M \approx 10^{15} q^3 M_S \]

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Analytical approximation

\[ 1 - G_{12}(q) = \frac{1.5q^2}{\sqrt{2.25q^4 + q^2 + q^2 (p_0 / q)^{1.4} + q_0^2}} \]

\[ p_0 \approx 10^{-2}, \quad q_0 \leq 10^{-3} \]

Doroshkevich, Demianski 2005

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Background entropy at moment $z_0$

$$\langle F(M) \rangle = \frac{m_{DM}\sigma_v^2(z_0,q)}{n^{2/3}(z_0)} \approx F_0 z_5^{-3} \begin{cases} M_9^{0.33}, & M_9 > 1 \\ M_9^{0.56}, & M_9 < 1 \\ 10^{-5}M_0^{0.66}, & M_0 \leq 1 \end{cases}$$

$$F_0 = \mu^{5/3}\text{keV cm}^2, \quad z_5 \equiv \frac{1+z_0}{5}, \quad M_n = \frac{M(q)}{10^n M_S}$$

**Probability distribution function**

$$dW(f) = e^{-f^2/2} \frac{df}{\sqrt{2\pi f}}, \quad f = \frac{F(M)}{\langle F(M) \rangle}$$

$$\langle f^2 \rangle = 3\langle f \rangle^2 = 3$$ large variations of $F$ from the mean value

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Violent relaxation entropy

**Isotermal shere**
(Fillmore & Goldreich 1984)

\[ \rho \sim r^{-2}, \quad M \sim r, \quad F \sim M^{4/3} \]

**Collapse of ellipsoide**
(Gurevich, Zybin 1988)

\[ \alpha \sim 1.7 - 1.9 \]

**Generated entropy in the central region is negligible in comparison with background one!**

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Numerical simulation (N-body)

- Facilities and limitations
- Slopes of density and entropy in halo:
  \[ \rho \sim r^{-\alpha}, \quad F \sim M^\beta \]
  core (\(\alpha < 1\)) or cusp (\(\alpha > 1\))? 
- Universal NFW profile:
  \[
  x = r/r_s, \quad \rho \sim x^{-1}(1+x)^{-2}, \quad \alpha = 1, \quad \beta = 5/6
  \]
- Empirical Burkert profile:
  \[
  \rho \sim (1+x)^{-1}(1+x^2)^{-1}, \quad \alpha = 0, \quad \beta = 0
  \]

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Rotation curves (Marchesini 2002)
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Galaxy clusters profiles
(Pointecouteau et al., 2005)
Observational rotation curves

\[ \rho \approx \frac{\rho_0}{(x_m + x)(1 + x^2)}, \quad x = \frac{r}{r_s} \]

\[ M_0 = 4\pi\rho_0 r_s^3, \quad x_m \in (0, 1.3) \]

\[ x_m \ll 1, \quad x_{\text{max}} \approx 2, \quad \frac{v_c^2(x)}{v_{\text{max}}^2(x)} \approx 2.5 \frac{M(x)}{xM_0} \]

\[ x_m = 1.3, \quad x_{\text{max}} \approx 3.5, \quad \frac{v_c^2(x)}{v_{\text{max}}^2(x)} \approx 5.2 \frac{M(x)}{xM_0} \]

\[ \frac{M(x)}{M_0} = \frac{x_m^2 \ln(1 + x/x_m) + 0.5 \ln(1 + x^2) - x_m \arctg(x)}{1 + x_m^2} \]

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Analytically modeled halos

\[ F_b(M) \sim M^{\beta_b}, \quad \beta_b < \frac{5}{6} \]

\[ F_r(M) \sim M^{\beta_r}, \quad \beta_r \geq \frac{5}{6} \]

\[ F(M) = \sqrt{C_1 M^{2\beta_b} + C_2 M^{2\beta_r}} \]

\[ \kappa = \frac{\langle F_b(M) \rangle}{\langle F_g(M) \rangle} \in (0, 1) \]

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\( \alpha_r = \frac{5}{6} \quad r/r_{\text{max}} \quad \alpha_r = \frac{4}{3} \)

\( \kappa \sim 1 \) – solid line, \( \kappa \ll 1 \) – dashed line,

NFW – “stars”, Burkert – “dots”

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Conclusions

* The background entropy can prevent the cusp formation for halos with

\[ 10^6 M_\odot < M < 10^{12} M_\odot \]

* For smaller and larger galaxies and for clusters of galaxies its impact is attenuated

* The impact of the background entropy allows to reproduce the observed rotation curves

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