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A solution of the cusp problem in virialized DM halos in standard cosmology

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Advanced study in cosmology

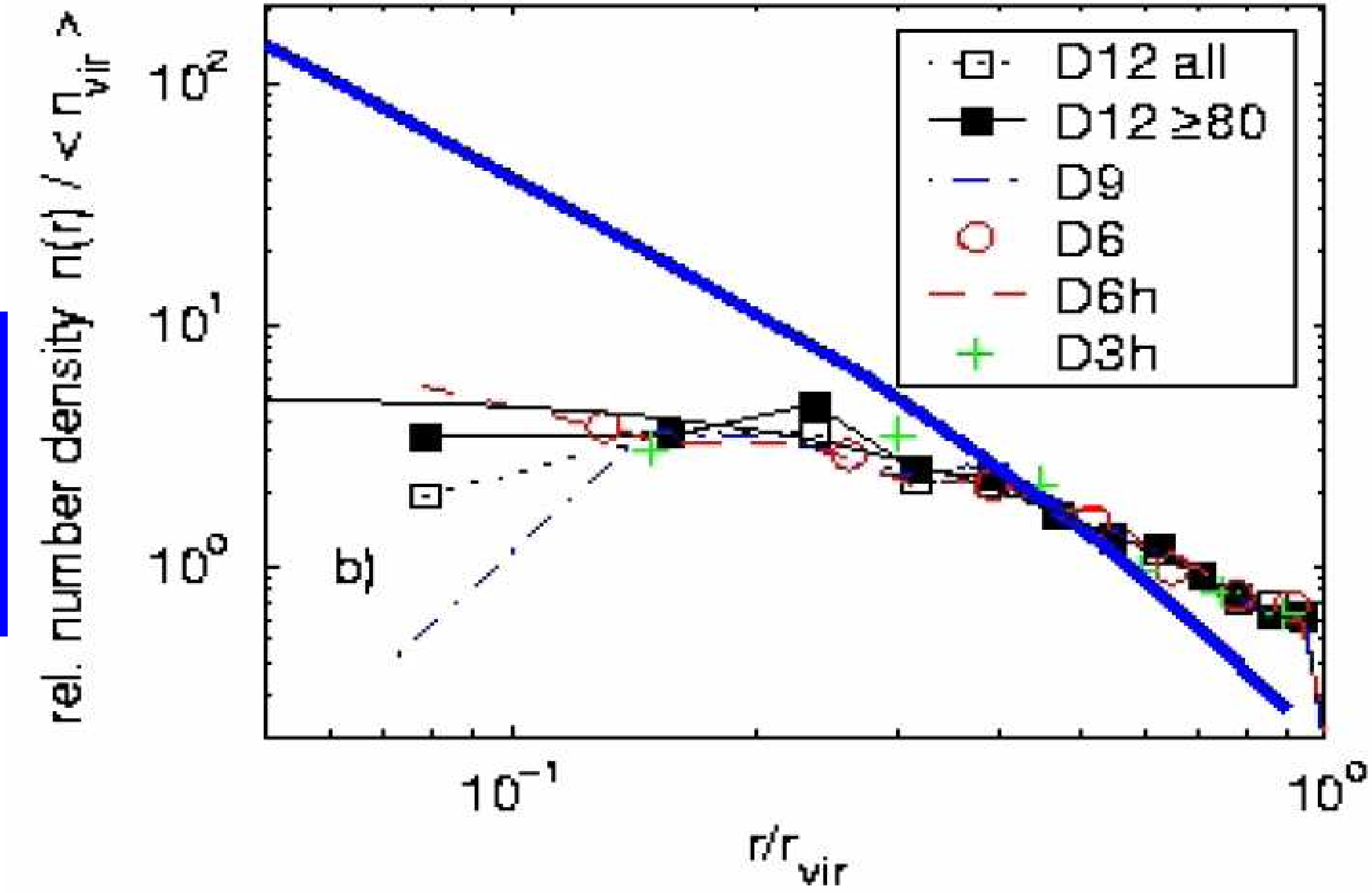
- Analysis of CMB and its polarization
- Investigation of Ly-α forest
- Properties of earlier galaxies and quasars at large redshifts
- Properties of dwarf galaxies and the internal structure of galaxies (black holes, rotation curves, etc)

Cusp problem in galaxy halos

(simulations)

$\rho \sim r^{-\alpha}$

$\alpha \in$ **(1, 3/2)**



Diemand et al. 2004

The cusp problem is being considered as main problem of the standard cosmology

(DM non-interacting to baryons)

We argue : it is solved within the framework of standard DM model

Internal structure of relaxed halos

(cores instead of cusps)

Idea : take into account the small scale part of initial cosmological perturbations that transforms into the thermal energy of DM particles during violent relaxation

Method : extra entropy of DM particles related to initial background perturbations

$\Rightarrow$ entropy profiles related to density profiles of DM halos

Standard cosmological model

- H_0 = 70 km/s/Mpc , h=0.7
- $\Omega_\Lambda = 0.7$
- $\Omega_m = 0.3$
- ΛCDM power spectrum:

$$P(k)=A\,\kappa T^2(\kappa)\exp(-R_f^2\,\kappa^2), \quad \kappa=k/k_0$$

$$k_0=0.2\ h/\mathrm{Mpc}, \quad A=240\sigma_8^2/k_0^3$$

$$\frac{H(z)}{H_0} = \sqrt{1 + \Omega_m (z^3 + 3z^2 + 3z)} \;\Rightarrow\; 0.5(1+z)^{3/2}\,, \quad z > 1$$

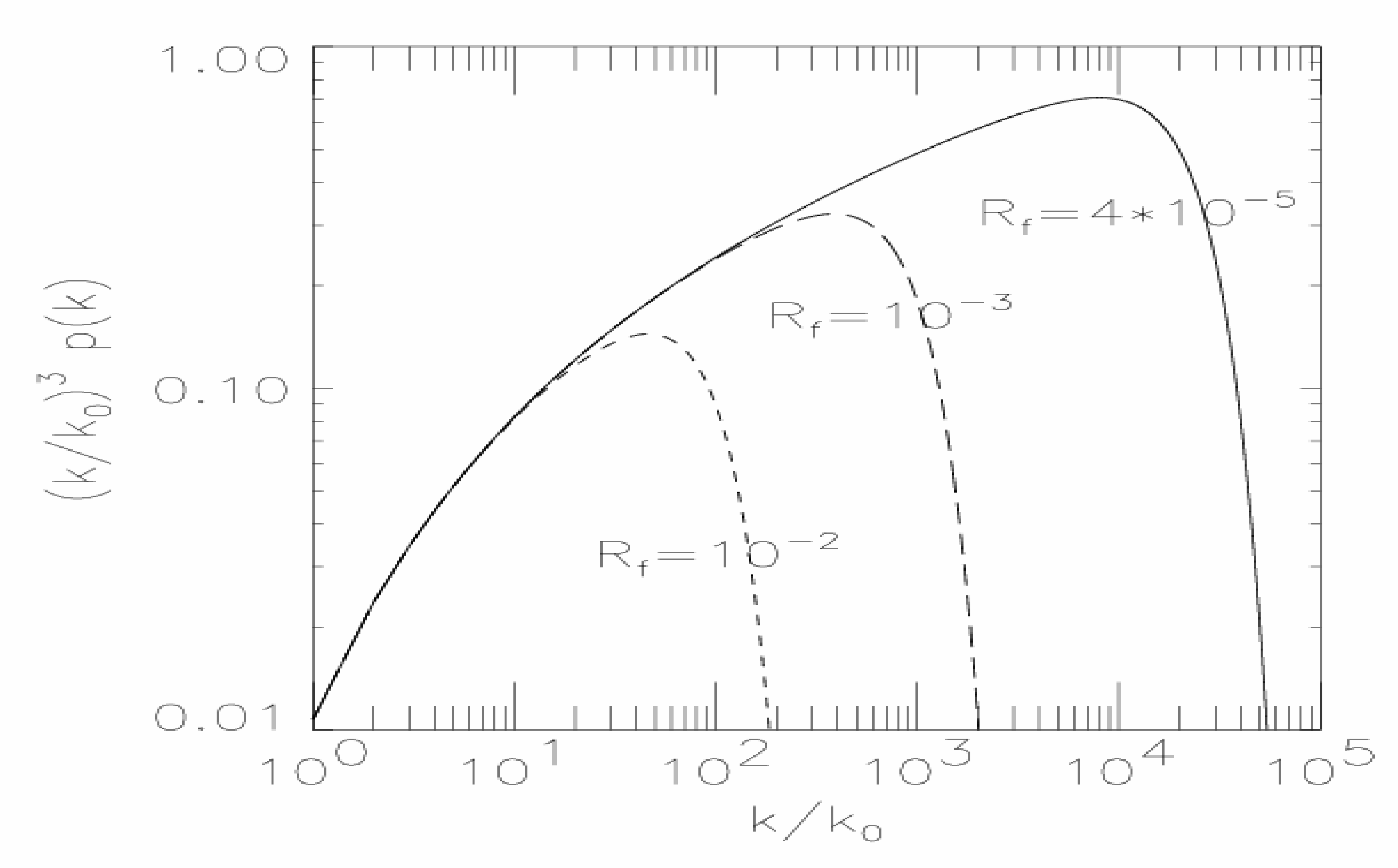
$$\rho_m = \frac{3H_0^2\Omega_m}{8\pi G}(1+z)^3 \cong 3\cdot 10^{-30}(1+z)^3\,\frac{g}{cm^3}$$

$$n = \frac{\rho_m}{m_{DM}} \cong 3\cdot 10^{-6}\mu^{-1}(1+z)^3\ cm^{-3}, \qquad \mu \equiv \frac{m_{DM}}{m_p}$$

R_f - **particle free path in the early Universe**

$m_{DM} > 1$ МэВ, $\quad R_f < 4\cdot 10^{-6} \Rightarrow$ **stellar halos**

Observational and model spectra



Equilibrium DM halos

- **Adiabatic and irreversible processes**
- **Entropy function:** $F = T/n^{2/3} = p/n^{5/3}$
- **Hydrostatic equilibrium:**

$$\boxed{\frac{1}{\rho}\frac{dp}{dr} = -\frac{GM(r)}{r^2}}$$

- **Initial (background) entropy in small scale:**

  $F \sim M^{1/3-2/3}$

- **Hierarchical and violent relaxation of compressed matter (large scale):**

  $F \sim M^{5/6}$, $F \sim M^{4/3}$

Entropy of collisionless particles for isotropic velocity distribution

$$p = \rho\langle v^2\rangle = nT = Fn^{5/3}$$

one-dimensional peculiar velocity

Power-law density profiles: $\alpha \in (0, 2)$

$$\rho(r) \propto r^{-\alpha}, \quad M \propto r^{3-\alpha}, \quad p = C_1 + C_2 r^{2(1-\alpha)}$$

$\alpha < 1$ - finite pressure in the centre (**core**)

$\alpha \geq 1$ - infinite pressure at $r \to 0$ (**cusp**)

$r \Rightarrow M \longrightarrow$ **conserving both for initial and relaxed matter fields**

$$\boxed{F(M) \propto C_1 M^{\beta_1} + C_2 M^{\beta_2} \propto M^{\beta}}$$

$$\beta \in (\beta_1, \beta_2), \qquad \beta_{1,2} = \frac{1 + 2\alpha/3}{3 - \alpha} \pm \frac{|\alpha - 1|}{\alpha - 3}$$

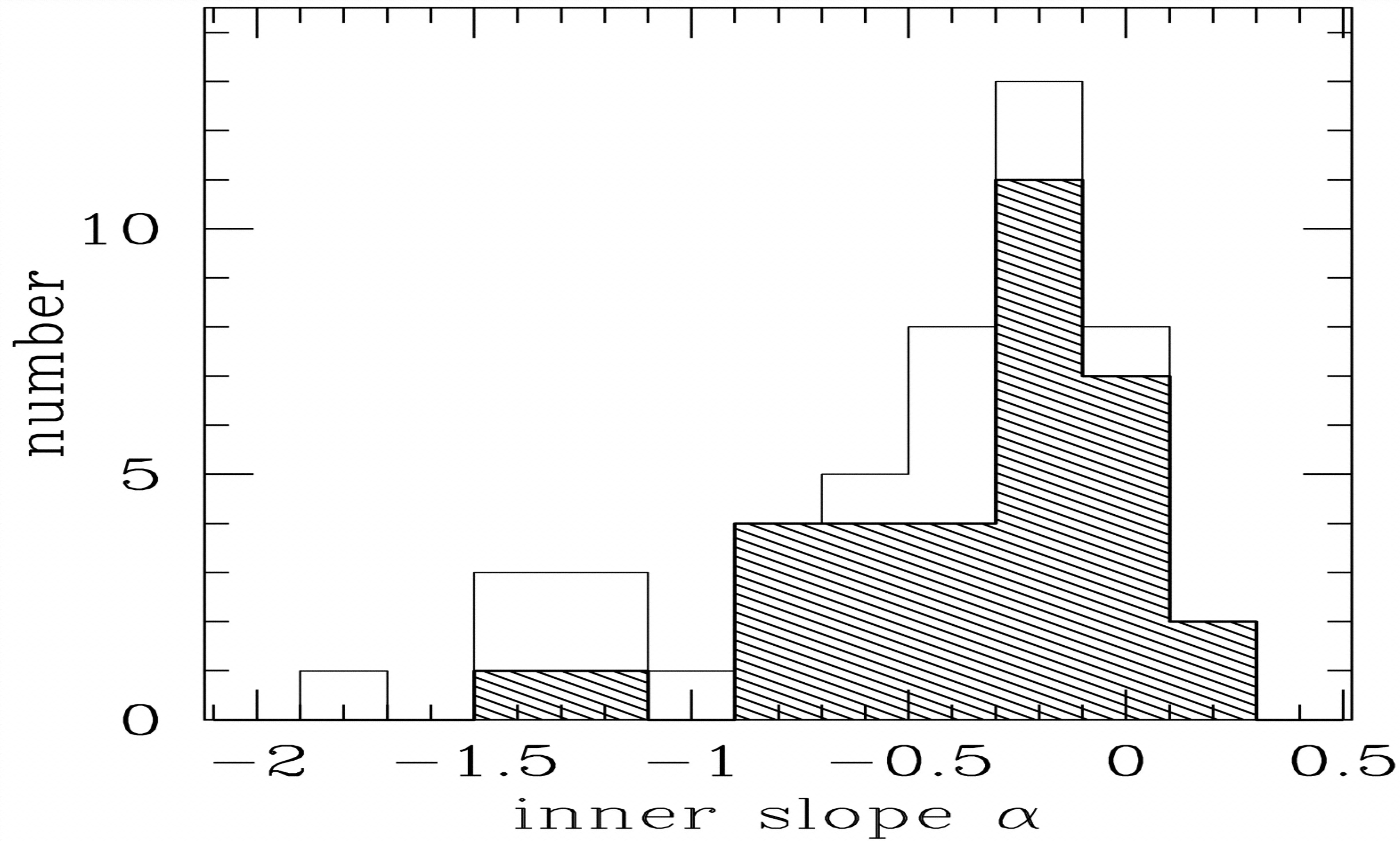
$$\beta_{cr} = \beta_1 = \beta_2 = \frac{5}{6} \quad \Rightarrow \quad \alpha_{cr} = 1$$

core ($\beta < 5/6$): $\beta_1 \in \left(0, \frac{5}{6}\right)$, $\beta_2 \in \left(\frac{2}{3}, \frac{5}{6}\right)$

cusp ($\beta \geq 5/6$): $\beta_1 \in \left(\frac{5}{6}, \frac{4}{3}\right)$, $\beta_2 \in \left(\frac{5}{6}, \frac{10}{3}\right)$

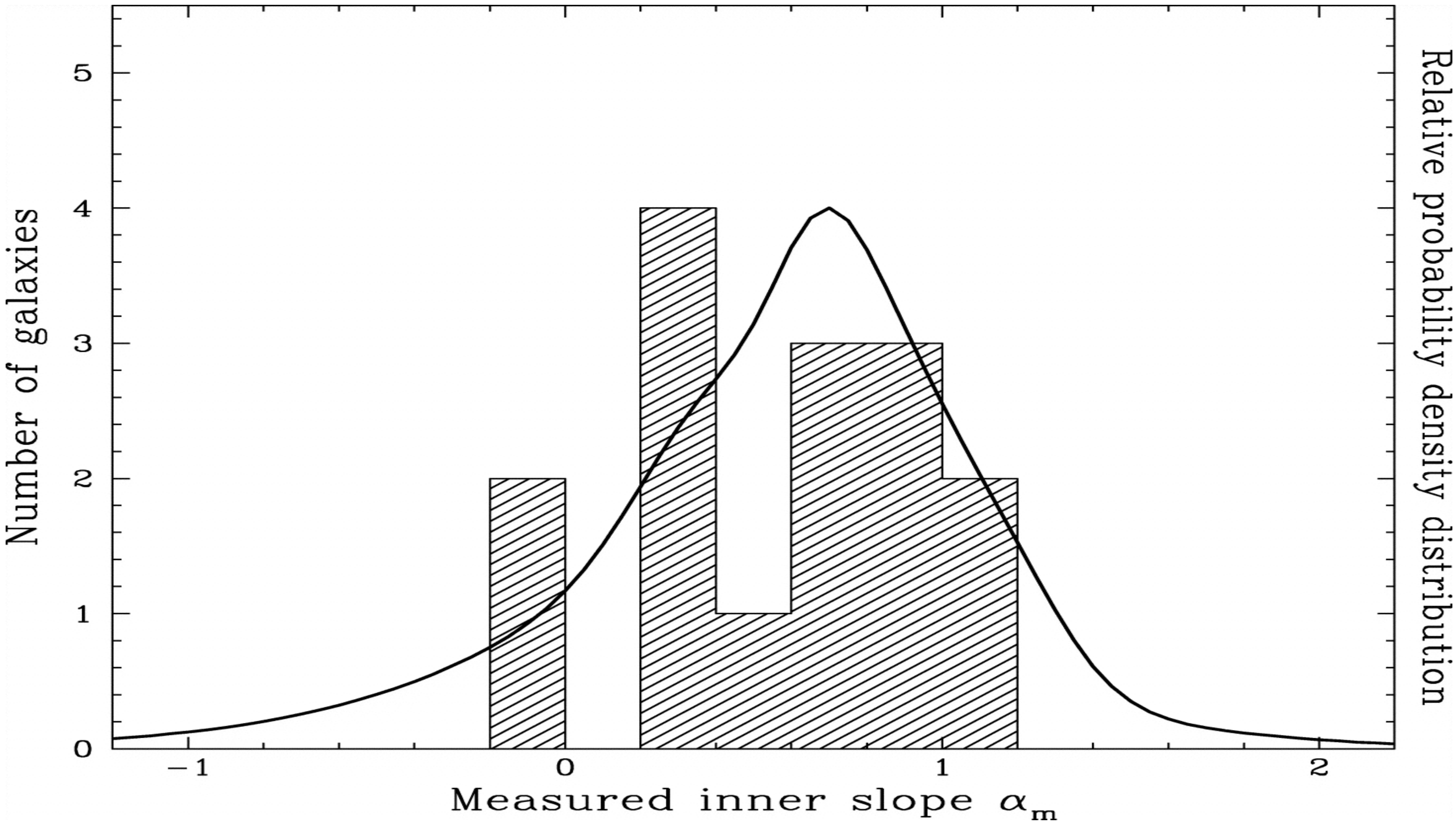
48 low brightness galaxies

(LBG, de Blok et al. 2001)



15 low brighness galaxies

(Swaters et al. 2003)



NB (in relation to A. Klypin talk):

we call cores by cores

and

cusps by cusps

Initial velocity field

$$\vec{r}(z,\vec{x}) = (1+z)^{-1}[\vec{x} - g(z)\vec{S}(\vec{x})] \qquad \delta(\vec{x}) \equiv \delta\rho/\rho = \mathbf{div}(\vec{S})$$

$$\vec{V}(z,\vec{x}) \equiv \dot{\vec{r}} = H(z)[\vec{r} + g'(z)\vec{S}(\vec{x})]$$

$$\Rightarrow \frac{H(z)}{1+z}[\vec{x} - 2g(z)\vec{S}(\vec{x})] , \quad z > 1$$

cosmological background non-conditional peculiar velocity

$$\langle \delta(\vec{x}) \rangle = \langle \vec{S}(\vec{x}) \rangle = 0 , \qquad \vec{x}_1 - \vec{x}_2 = x\vec{e} , \qquad q = x/\ell_v$$

$$\xi(x) \equiv \langle \delta(\vec{x}_1)\delta(\vec{x}_2) \rangle = \sigma_0^2 G_0(q) , \qquad G(0) = 1$$

$$\xi_{ij}(x) \equiv \langle S_i(\vec{x}_1) S_j(\vec{x}_2) \rangle = \frac{1}{3}\sigma_s{}^2[e_i e_j G_{12}(q) + (\delta_{ij} - e_i e_j)G_1(q)]$$

small scale density correlations

$$\ell_0 < 1\ \mathrm{kpc} \qquad (G_0 = 0.5)$$

large scale velocity correlations

$$\ell_1 \cong \sigma_S = 11\ h^{-1}\mathrm{Mpc} \quad (G_{12} = 0.5)$$

$$\ell_v \cong 31\ h^{-1}\mathrm{Mpc} \qquad (G_1 = 0.5)$$

DM halo formation

(Zel'dovich approximation)

$$\vec{r}(z,\vec{x}) = (1+z)^{-1}[\vec{x} - g\vec{S}(\vec{x})]\ ,\ g \cong 1.2(1+z)^{-1}$$

$$\vec{V}(z,\vec{x}) \cong H_0(1+z)^{1/2}[\vec{x}/2 - g(z)\vec{S}(\vec{x})]$$

$$\vec{S}(\vec{x}) = \vec{S}_R(\vec{x}) + \vec{S}_*(\vec{x})$$

local background – protohalo with scale/mass resolution R **(turns into relaxed halo via violent/hierarchical relaxation)**

conditional perturbations **(transform adiabatically to microscopic motion of particles inside the halo)**

halo formation time

$$\left\langle \vec{S}_R(\vec{x}) \right\rangle_* = (1+z_0)\vec{x}/2\ ,\quad \left\langle \vec{S}_*(\vec{x}) \right\rangle_* = 0\ ,\quad \sigma_S^* \equiv \sqrt{\left\langle \vec{S}_*^2 \right\rangle_*} \Rightarrow \sigma_S$$

Variance of one-dimensional peculiar velocities inside halo

$$\vec{v}_*(z,\vec{x}) \cong H_0(1+z)^{-1/2}\vec{S}_*(\vec{x})$$

$$v_{12} = \vec{e}\,[\vec{v}_*(z,\vec{x}_1) - \vec{v}_*(z,\vec{x}_2)]$$

$$\vec{x}_1 - \vec{x}_2 = \ell_v q\,\vec{e} \quad , \quad |\vec{e}| = 1$$

$$\sigma_v^2 \equiv \left\langle v_{12}^2 \right\rangle_* \cong H_0^2\sigma_S^2(1+z)^{-1}(1-G_{12}(q))$$

$$M \cong 10^{15} q^3 \; M_S$$

Analytical approximation

$$1 - G_{12}(q) = \frac{1.5q^2}{\sqrt{2.25q^4 + q^2 + q^2(p_0/q)^{1.4} + q_0^{\ 2}}}$$

$p_0 \cong 10^{-2}, \quad q_0 \leq 10^{-3}$ **Doroshkevich, Demianski 2005**

Background entropy at moment z_0

$$\langle F(M)\rangle = \frac{m_{DM}\sigma_v^2(z_0,q)}{n^{2/3}(z_0)} \approx F_0 z_5^{-3} \begin{cases} M_9^{0.33}, & M_9 > 1 \\ M_9^{0.56}, & M_9 < 1 \\ 10^{-5} M_0^{0.66}, & M_0 \le 1 \end{cases}$$

$$F_0 = \mu^{5/3}\,\mathrm{keV\ cm}^2\,, \quad z_5 \equiv \frac{1+z_0}{5}\,, \quad M_n = \frac{M(q)}{10^n M_S}$$

Probability distribution function

$$dW(f) = e^{-f/2}\frac{df}{\sqrt{2\pi f}}\,, \qquad f = \frac{F(M)}{\langle F(M)\rangle}$$

$$\langle f^2\rangle = 3\langle f\rangle^2 = 3$$

large variations of F from the mean value

Violent relaxation entropy

Isotermal shere

(Fillmore & Goldreich 1984)

$$\rho \sim r^{-2}, \quad M \sim r, \quad F \sim M^{4/3}$$

Collapse of ellipsoide

(Gurevich, Zybin 1988)

$$\alpha \sim 1.7 - 1.9$$

Generated entropy in the central region is negligible in comparison with background one!

Numerical simulation (N-body)

- Facilities and limitations
- Slopes of density and entropy in halo:

$$\rho \sim r^{-\alpha}, \quad F \sim M^{\beta}$$

core (α<1) or cusp (α >1) ?

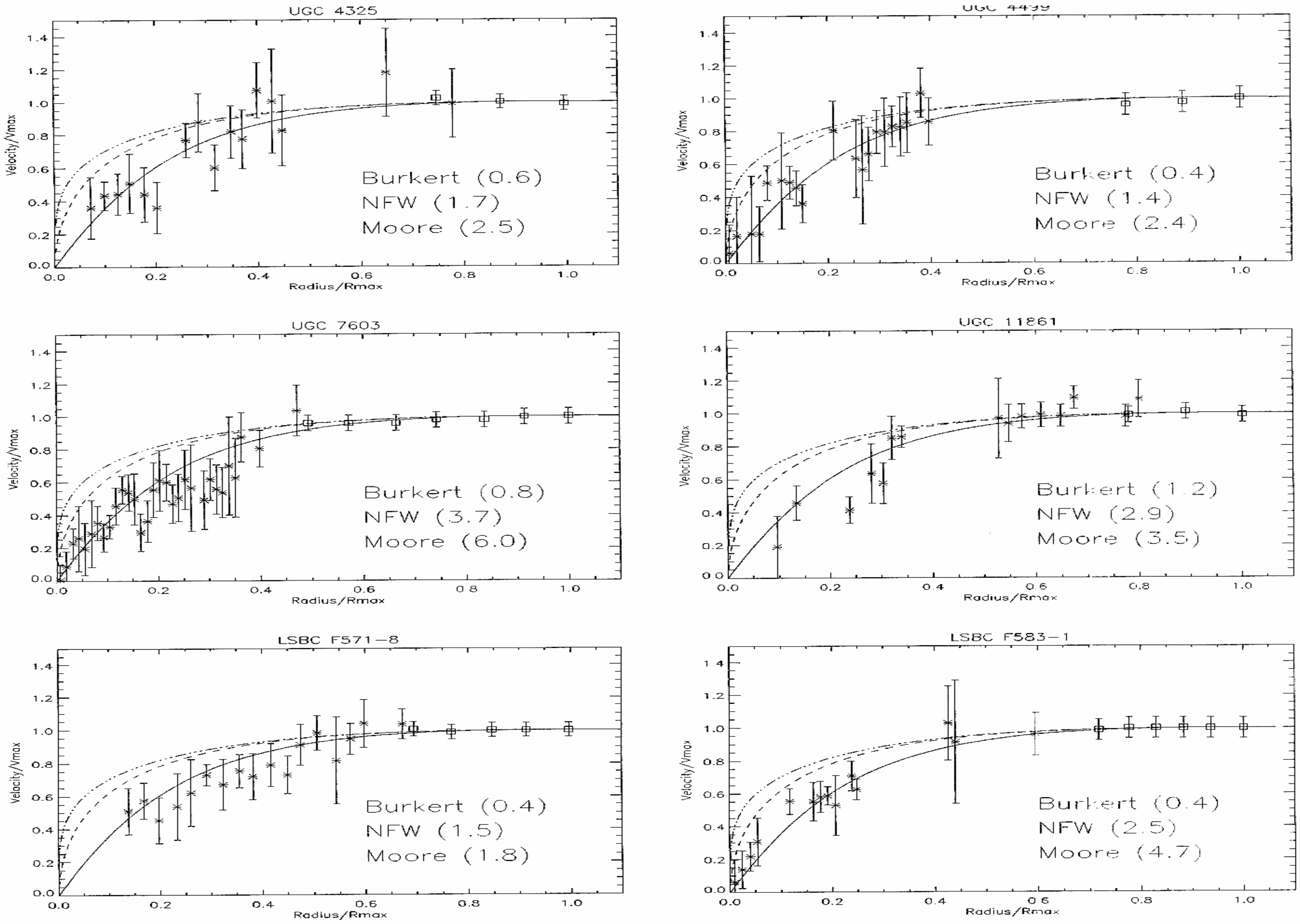
- Universal NFW profile:

$$x=r/r_s, \quad \rho \sim x^{-1}(1+x)^{-2}, \quad \alpha=1, \quad \beta=5/6$$

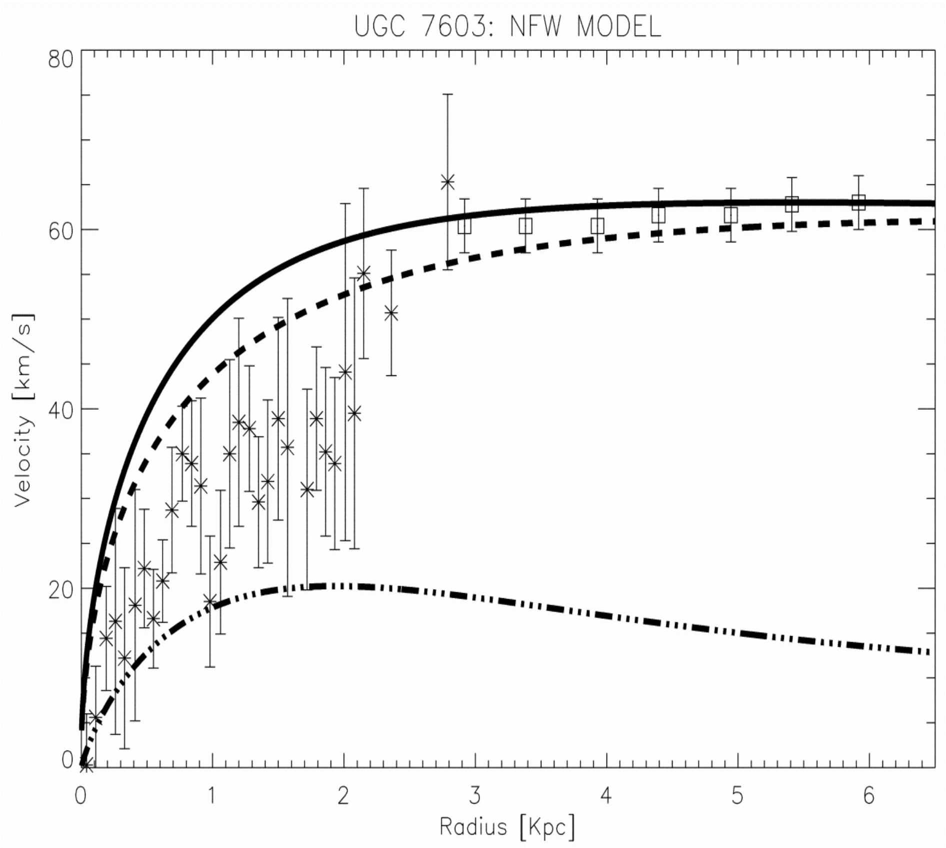
- Empirical Burkert profile:

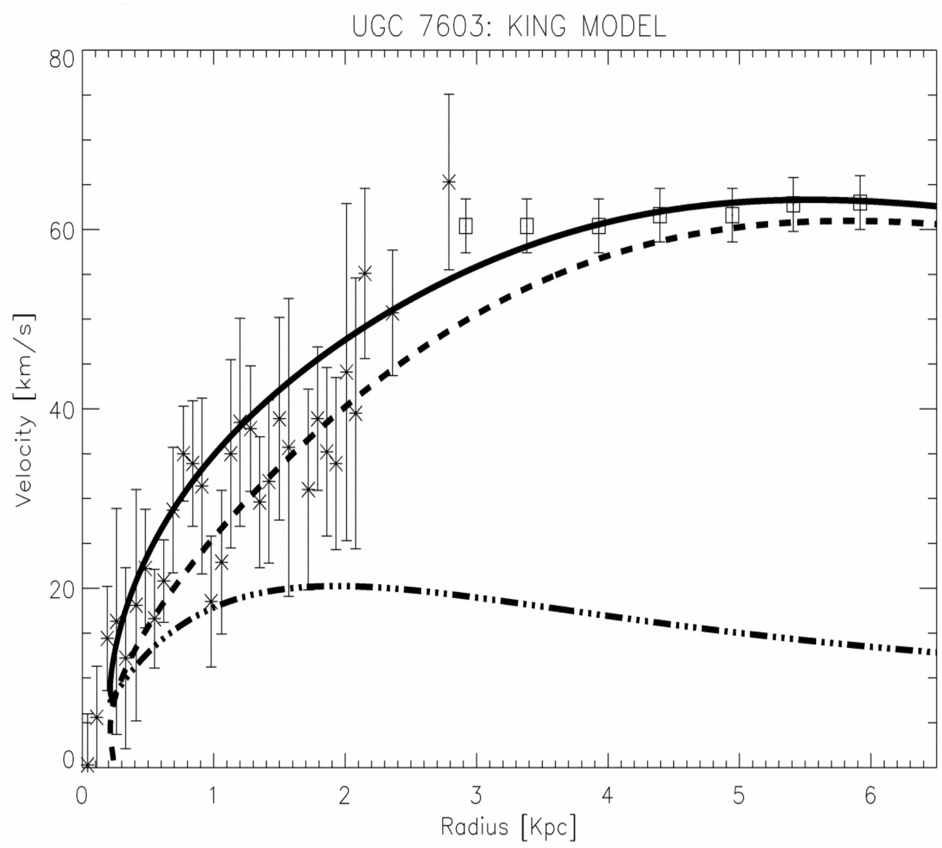
$$\rho \sim (1+x)^{-1}(1+x^2)^{-1}, \quad \alpha=0, \quad \beta=0$$

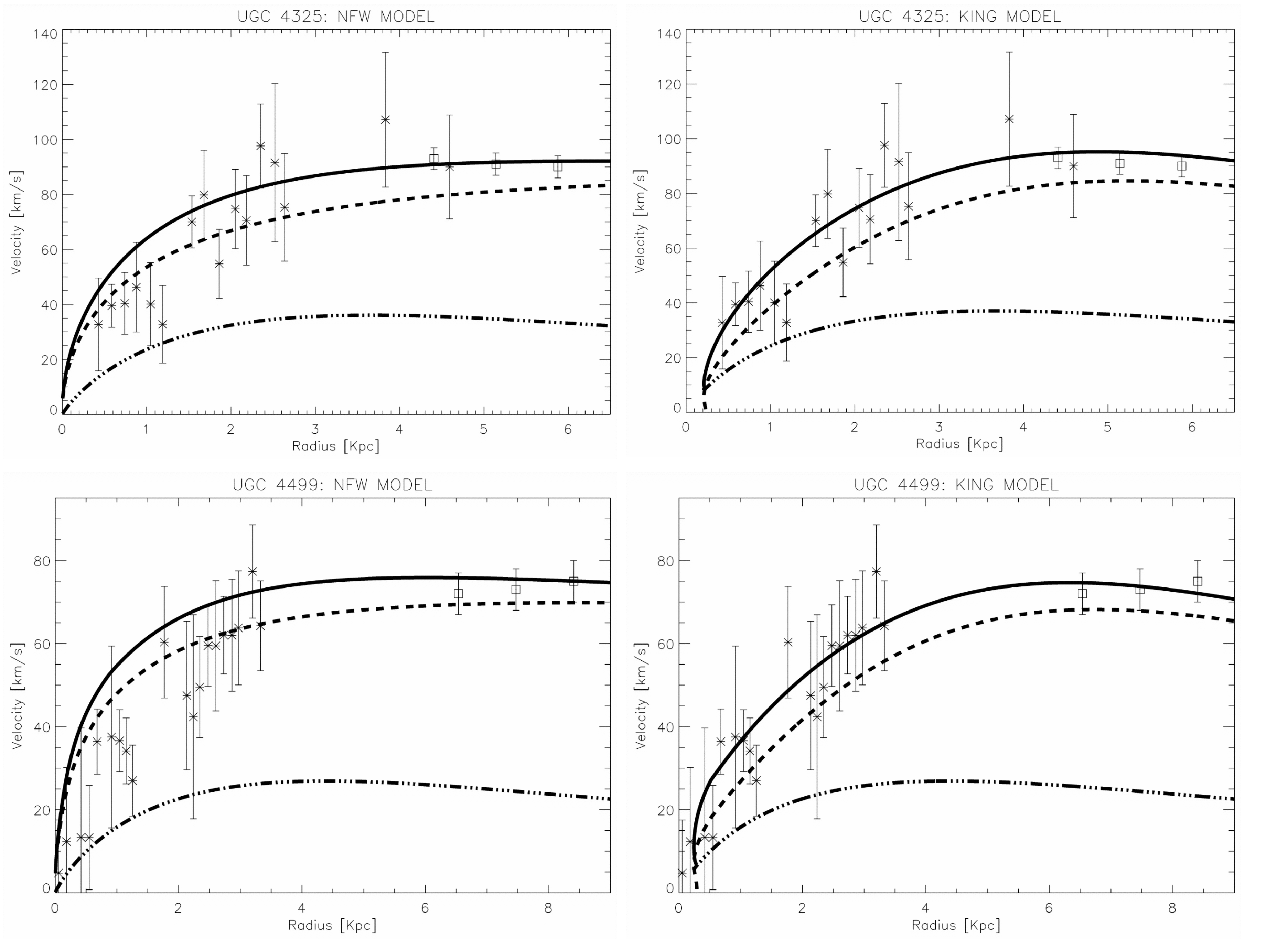
Rotation curves (Marchesini 2002)



Rotation curves (Marchesini 2002)



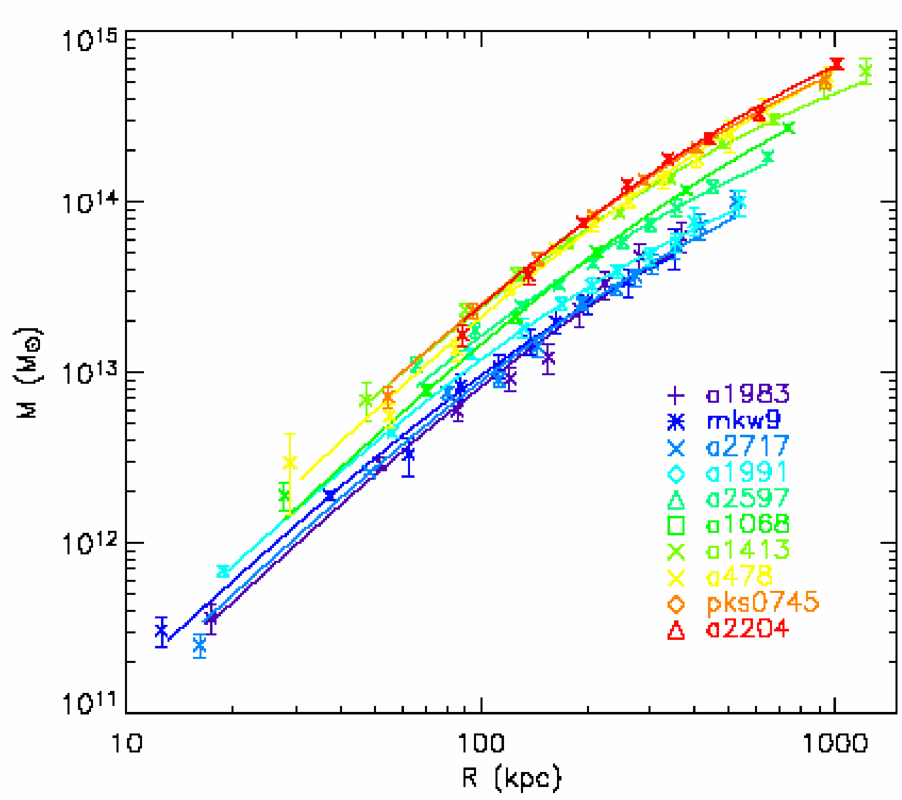


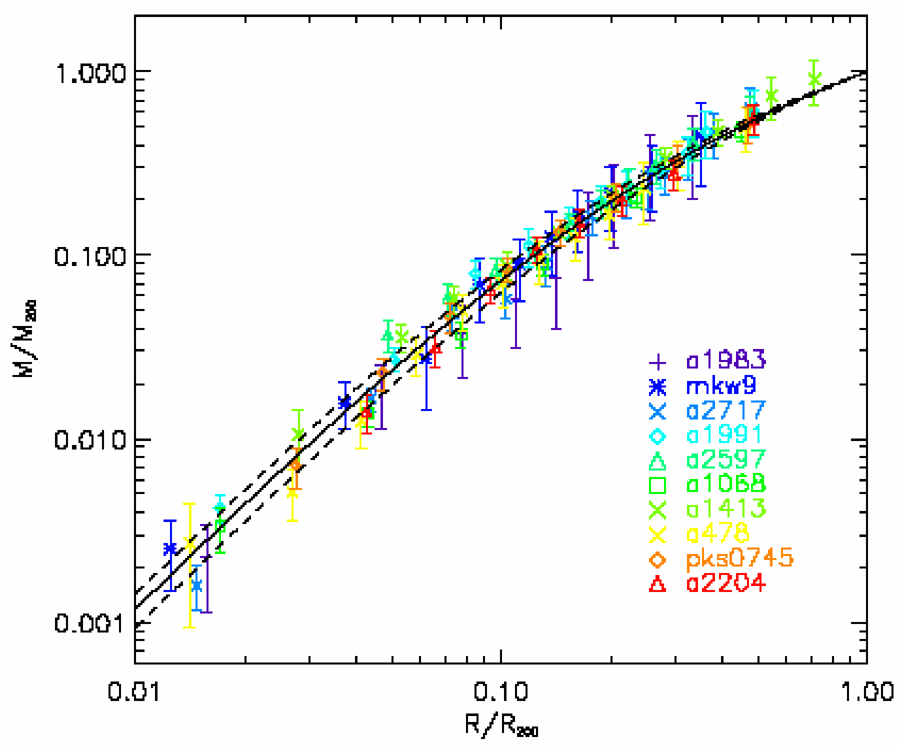


UGC 4325: NFW MODEL
UGC 4325: KING MODEL
UGC 4499: NFW MODEL
UGC 4499: KING MODEL
Velocity [km/s]
Radius [Kpc]

Galaxy clusters profiles

(Pointecouteau et al., 2005)





Observational rotation curves

$$\rho \cong \frac{\rho_0}{(x_m + x)(1 + x^2)}, \qquad x = \frac{r}{r_s}$$

$$M_0 = 4\pi\rho_0 r_s^3, \qquad x_m \in \quad (0,\ 1.3)$$

$$x_m << 1, \qquad x_{max} \cong 2, \qquad \frac{v_c^2(x)}{v_{max}^2(x)} \cong 2.5 \frac{M(x)}{xM_0}$$

$$x_m = 1.3, \qquad x_{max} \cong 3.5, \qquad \frac{v_c^2(x)}{v_{max}^2(x)} \cong 5.2 \frac{M(x)}{xM_0}$$

$$\frac{M(x)}{M_0} = \frac{x_m^2 \ln(1 + x / x_m) + 0.5 \ln(1 + x^2) - x_m \mathrm{arctg}(x)}{1 + x_m^2}$$

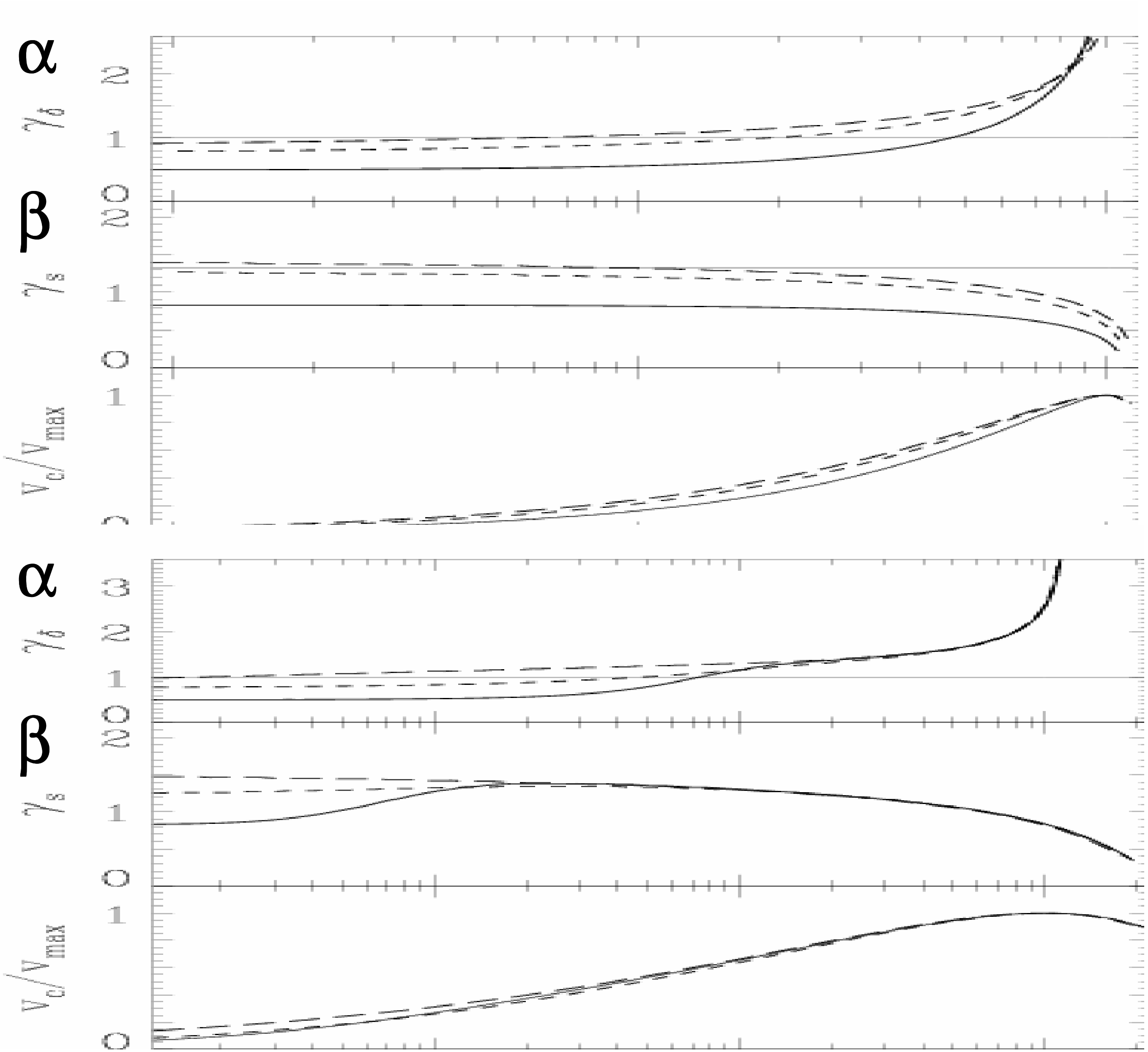
Analytically modeled halos

$$F_b(M) \sim M^{\beta_b}, \qquad \beta_b < 5/6$$

$$F_r(M) \sim M^{\beta_r}, \qquad \beta_r \geq 5/6$$

$$\boxed{F(M) = \sqrt{C_1 M^{2\beta_b} + C_2 M^{2\beta_r}}}$$

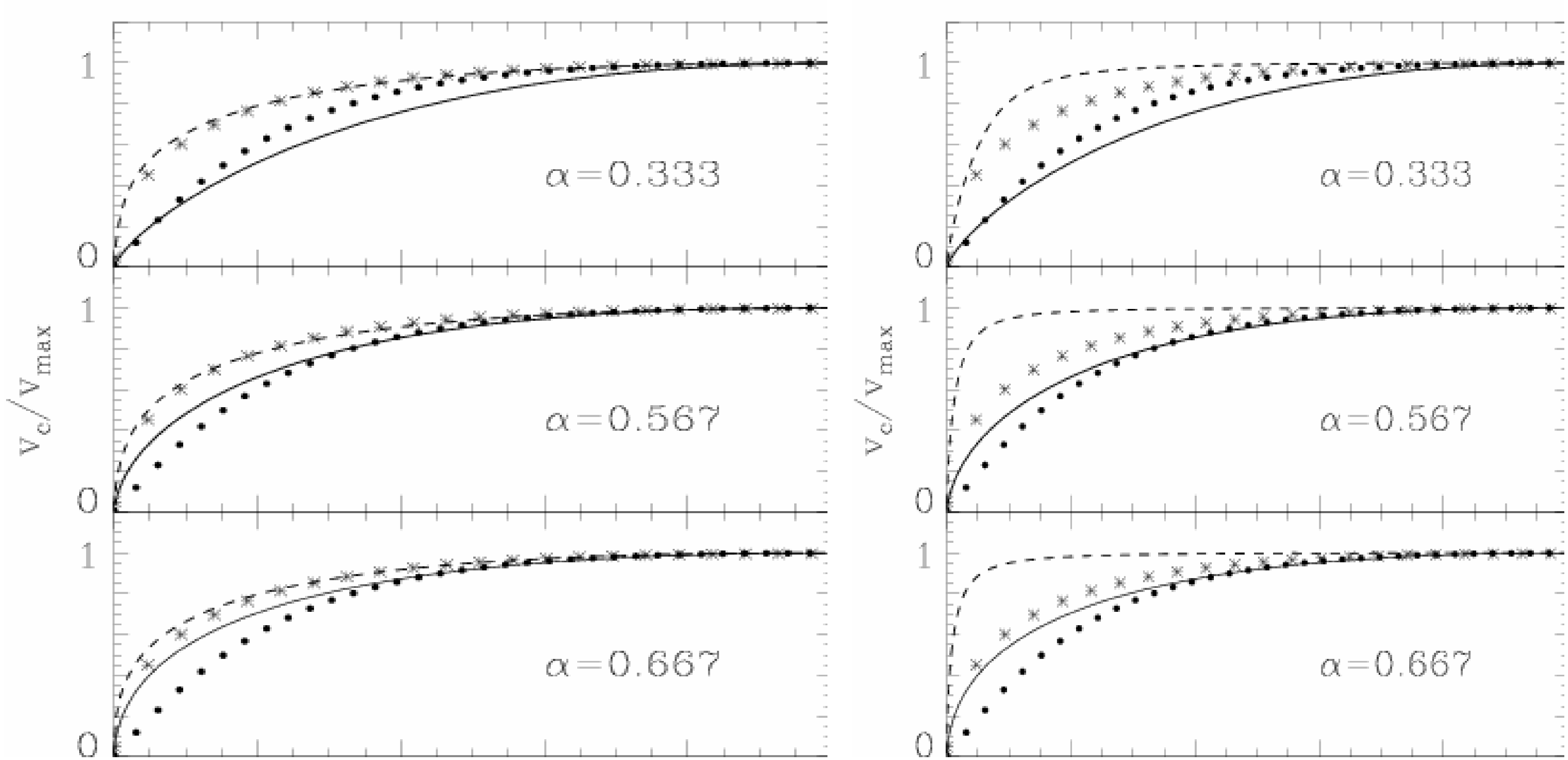
$$\kappa = \frac{\langle F_b(M) \rangle}{\langle F_g(M) \rangle} \in \quad (0,\ 1)$$



k ~ 1

k << 1

$(M/M_{max})^{1/3}$



α_r =5/6 **r/r_{max}** **α_r =4/3**

κ~1 – solid line, κ << 1 – dashed line, NFW – "stars", Burkert – "dots"

Lukash "A solution of the cusp problem", DSU2006, Madrid, 22 June

Conclusions

* The background entropy can prevent the cusp formation for halos with

$$10^6\, M_\odot < M < 10^{12}\, M_\odot$$

* For smaller and larger galaxies and for clusters of galaxies its impact is attenuated

* The impact of the background entropy allows to reproduce the observed rotation curves