The Dark Side of the Universe
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Dark Matter and Dark Energy from the solution of the strong CP problem

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Outline

1 - Uncoupled and coupled Dark Energy: motivation and features

2 - From axion model to dual-axion model:

- Strong CP problem , Peccei-Quinn solution and the axion

- Generalizing the PQ model…..axion and dark energy from a single scalar field

- Dual-axion cosmology: background and density fluctuations

- Dual-axion and WMAP data

3 - Conclusions

Mainini & Bonometto Phys.Rev.Lett. 93 (2004) 121301
Mainini, Colombo & Bonometto Apj 632 (2005) 691

Introduction

Two dark components required:

1. **Dark Matter (DM):** non-baryonic component to account for dynamics in galaxy and galaxy cluster
2. **Dark Energy (DE):** smooth component with $p<0$ to account for cosmic acceleration

Most cosmological data (CMB, SNIa, LSS) fitted by '*concordance model*' (ΛCDM)

$$H \approx 72 \text{ km/s/Mpc}, \quad n \approx 0.95$$

$$\Omega_{DM} \approx 0.22, \quad \Omega_{DE} \approx 0.74, \quad \Omega_{bar} \approx 0.04$$

DE from cosmological constant Λ but….

…. $|\Lambda| < 10^{-56}$ cm^{-2} why Λ so small? (fine-tuning problem)
…. why $\Omega_{DM} \approx \Omega_{DE}$? (coincidence problem)

Dynamical DE models to ease ΛCDM problems:

Tracking quintessence

DE from a scalar field ϕ self-interacting through an effective potential $V(\phi)$

Independence of Ω_{DE} from initial conditions → no fine tuning

at least **1 more parameter required**

Ex. : RP model $V(\phi) = \frac{\Lambda^{4+\alpha}}{\phi^{\alpha}}$ we have to set the energetic scale Λ

More complex models to try to unify DM and DE….

Coupled DE models

Similar values of ρ_{DM} and ρ_{DE} for a long period → no coincidence problem

coupling parameter also required

Cosmologies with scalar field

Basic equations

Spatially flat FRW universe with:
baryons, radiation, cold DM and DE (scalar field ϕ with self-interaction potential $V(\phi)$)

Friedmann eq.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\left(\rho_r + \rho_b + \rho_c + \rho_{DE}\right) a^2$$

Continuity equations:

uncoupled case

coupled case

$$\dot{\rho}_b + 3\frac{\dot{a}}{a}\rho_b = 0$$

$$\dot{\rho}_r + 4\frac{\dot{a}}{a}\rho_r = 0$$

Usual eqs. for baryons and radiation

$$\dot{\rho}_c + 3\frac{\dot{a}}{a}\rho_c = 0$$

$$\dot{\rho}_{DE} + 3\frac{\dot{a}}{a}(1 + 3w_{DE})\rho_{DE} = 0$$

Interaction DM-DE parametrized by

$$C(\phi) = \sqrt{16\pi G/3}\,\beta(\phi)$$

Coupled Dark Energy

Coupling effects: modified DM dynamics

- Variable mass for DM particles

$$\rho_c \propto a^{-3} e^{-\int C d\phi} \Rightarrow m_c \propto e^{-\int C d\phi}$$

- Violation of equivalence principle

$$\dot{p} = 0 \Rightarrow \ddot{x} + \frac{\dot{m}}{m}\dot{x} = 0$$

- Newtonian interactions:

DM-DM particles: ***effective gravitational constant***

$$G^* = G\left(1 + \frac{4}{3}\beta^2\right) = G\gamma$$

DM-baryons or baryons-baryons: ordinary gravitational constant *a particular coupled DE model:*

dual-axion model

Axion: DM candidate from Peccei-Quinn (PQ) solution to the strong CP problem

modifying PQ model :

- Strong CP problem solved

- DM and DE from single complex scalar field

- no fine-tuning

- number of parameter as SCDM (even 1 less than ΛCDM…)
DM an DE in fair proportion from one parameter

- DM and DE coupled but …..
no extra parameter (coupling strength set by theory)

Strong CP problem

QCD Lagrangian includes a CP violanting term:

From electroweak sector

$$\mathcal{L}_\theta = \frac{\alpha_s}{2\pi}\theta_{eff}\; G\cdot\tilde{G}$$

where $\theta_{eff} = \theta_{QCD} + \arg\det M_q$

from QCD sector

→ Prediction of an electric dipole moment for the neutron

CP violation incompatible with experimental limits on electric dipole moment unless:

$$\theta_{eff} < 10^{-10}$$

← **Why θ is so small?**

Peccei-Quinn solution and the axion

Peccei & Quinn 1977

PQ idea: **CP-violating term is suppressed making θ a dynamical variable driven to zero by the action of its potential**

- Imposing an additional global $U(1)_{PQ}$ symmetry in SM spontaneously broken at a scale F_{PQ}

→ **Existence of a massless Nambu-Golstone boson *A***

- Assigning $U(1)_{PQ}$ charges to quarks so to have an interaction with gluon field

$$\mathcal{L}_A = \frac{1}{2}\partial^\mu A\,\partial_\mu A + \frac{\alpha_s}{2\pi}\frac{A}{F_{PQ}} G\cdot\tilde{G}$$

θ_{eff} re-interpreted as dynamical variable (the axion): $\theta_{eff} \longrightarrow \theta = \theta_{eff} + A/F_{PQ}$

At low energies CP-violating terms generate a potential for the θ with minimun at $\theta = 0$ restoring dynamically CP-symmetry and θ (or A) acquires a mass $m(T)$.

$$V(\theta) = m^2 F_{PQ}^2 (1-\cos\theta) \approx \frac{1}{2} m^2 F_{PQ}^2 \theta^2$$

$$m(T \gg \Lambda_{QCD}) = 0$$

$$m(T \ll \Lambda_{QCD}) \propto F_{PQ}^{-1}$$

Axion cosmology

Equation of motion ($\theta << 1$):

$$\ddot{\theta} + 2\frac{\dot{a}}{a}\dot{\theta} + a^2 m^2(T)\theta = 0$$

Harmonic oscillations for $m(T) > 2H$ damped by cosmic expansion

Averaging over cosmological time → $<E_{kin}> = <E_{pot}>$ so….

Pressure: $$p_\theta = <E_{kin}> - <E_{pot}> = 0$$

→ **Axions as Dark Matter candidate**

Energy density:

$$\dot{\rho}_\theta = \left(\frac{\dot{m}}{m} - 3\frac{\dot{a}}{a}\right)\rho_\theta \Rightarrow \rho \propto a^{-3} m(T)$$

→ $$T << \Lambda_{QCD} \Rightarrow \dot{m} = 0 \Rightarrow \rho_\theta \propto a^{-3}$$

Kolb & Turner 1990

In PQ model:

Additional U(1) symmetry introducing:

Complex scalar field $\Phi = \frac{\phi}{\sqrt{2}} e^{i\theta}$ **with NG potential** $V(|\Phi|) = \lambda(|\Phi|^2 - F_{PQ}^2)^2$

After symmetry break $<\phi> = F_{PQ}$ = *const.* The phase θ is the axion.

Can a single scalar field account for both DM and DE?

The dual-axion model

Replacing NG potential with a tracker quintessence potential…..

….$|\Phi| = \phi$ no longer constant but evolves over cosmological times

→ ϕ yields DE

Dual-axion model: *Lagrangian theory*

$$\mathcal{L} = \sqrt{-g}\left\{\frac{1}{2}\left[\partial^\mu\phi\partial_\mu\phi + \phi^2\partial^\mu\theta\partial_\mu\theta\right] - V(\phi) - m^2(T,\phi)\phi^2(1-\cos\theta)\right\}$$

PQ model

Phase (axion):

$$\ddot{\theta} + 2\frac{\dot{a}}{a}\dot{\theta} + a^2 m^2\theta = 0$$

Modulus:

$$\phi = \mathrm{F}_{\mathrm{PQ}} = \mathrm{cost}$$

Dual-axion model

Phase (axion):

$$\ddot{\theta} + 2\left(\frac{\dot{a}}{a} + \frac{\dot{\phi}}{\phi}\right)\dot{\theta} + a^2 m^2\theta = 0$$

Friction term modified: more damping

Modulus (Dark Energy):

$$\ddot{\phi} + 2\frac{\dot{a}}{a}\dot{\phi} + a^2 V'(\phi) = \phi\dot{\theta}^2$$

ϕ evolves over cosmological time

Averaging on Hubble time, at low energies ($T \ll \Lambda_{QCD}$).....

PQ model

Energy density:

$$\dot{\rho}_\theta + 3\frac{\dot{a}}{a}\rho_\theta = 0 \longrightarrow \rho_\theta \propto a^{-3}$$

As ordinary matter

Axion mass:

$$m \propto F_{PQ}^{-1} = \text{cost}$$

dual-axion model

Energy density:

$$\dot{\rho}_\phi + 3\frac{\dot{a}}{a}(\rho_\phi + p_\phi) = C(\phi)\dot{\phi}\rho_\theta$$

$$\dot{\rho}_\theta + 3\frac{\dot{a}}{a}\rho_\theta = -C(\phi)\,\dot{\phi}\,\rho_\theta$$

$$\longrightarrow \boxed{\rho_\theta \propto a^{-3}\phi^{-1}}$$

Axion mass:

$$m \propto \phi^{-1}$$

ϕ variation causes mass to vary with time

equivalent to:

COUPLED QUINTESSENCE MODEL
with a time dependent coupling *$C(\phi)=1/\phi$*

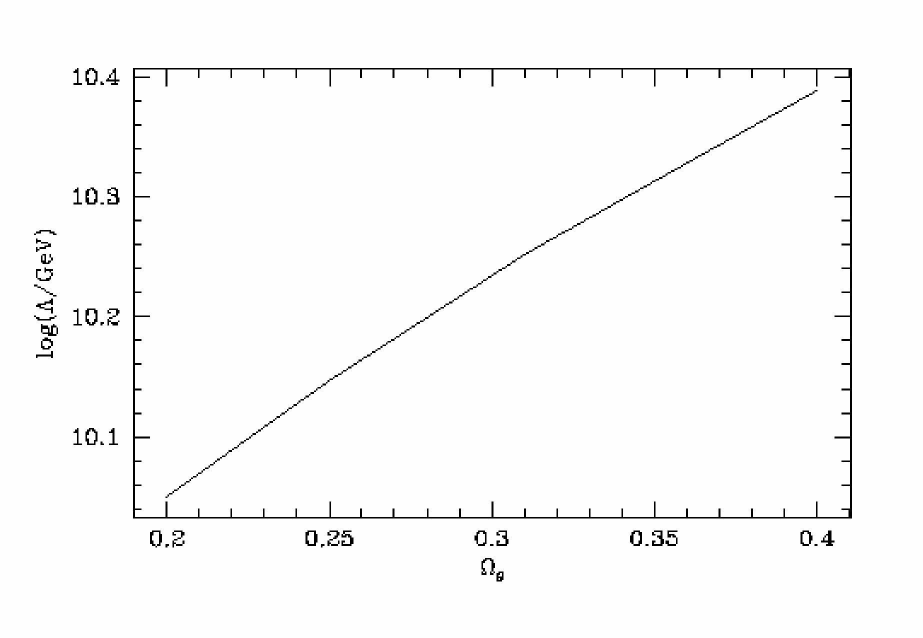
($\longrightarrow$ Coupling set by the theory)

Dual-axion model: *background evolution*

We use a SUGRA potential

$$V(\phi) = \frac{\Lambda^{4+\alpha}}{\phi^{\alpha}} e^{4\pi G\phi^2}$$

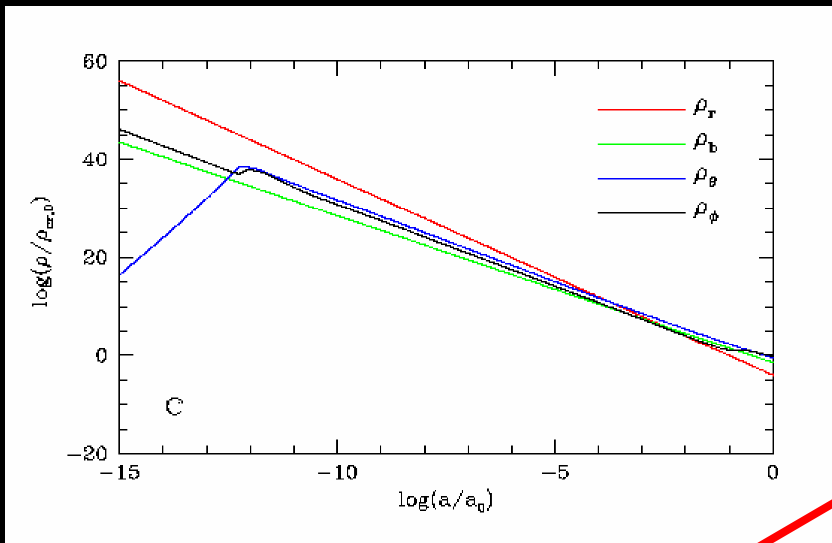
DM an DE in fair proportion from one parameter

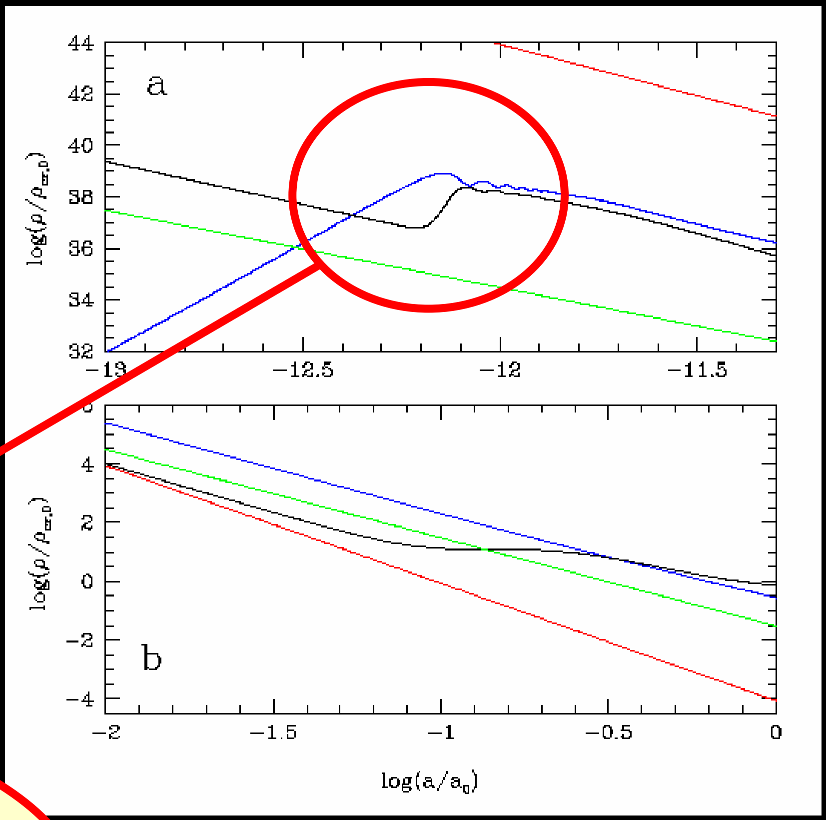


$$\Omega_{DM} = 0.3 \Rightarrow \Lambda \approx 10^{10}\,GeV$$

Note: in a model with dynamical DE (coupled or uncoupled) once Ω_{DM} is assigned Λ can be arbitrarily fixed. Here **Λ is univocally determined**

Dual-axion model: *background evolution*





$$\ddot{\phi} + 2\frac{\dot{a}}{a}\dot{\phi} + a^2 V'(\phi) = \phi\dot{\theta}^2$$

once coupling term $\phi\dot{\theta}^2 >> a^2 V'$

ϕ settles on different tracker solution

Density fluctuations: linear evolution

From linear theory:
DM and baryons density fluctuations described by 2 coupled Jeans' equations

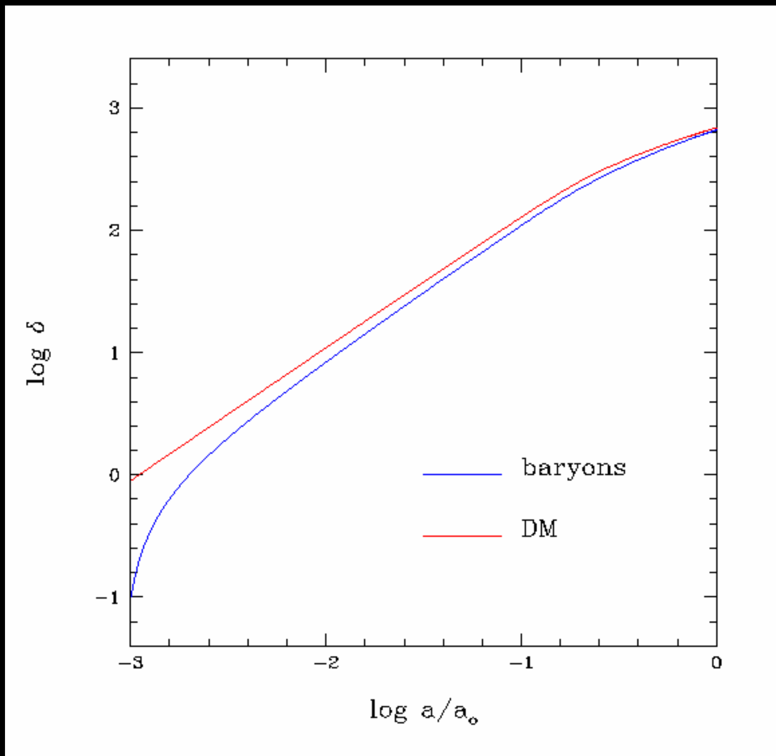
$$\ddot{\delta}_b + \frac{\dot{a}}{a}\dot{\delta}_b = 4\pi\left(G\rho_c\delta_c + G\rho_b\delta_b\right)a^2$$

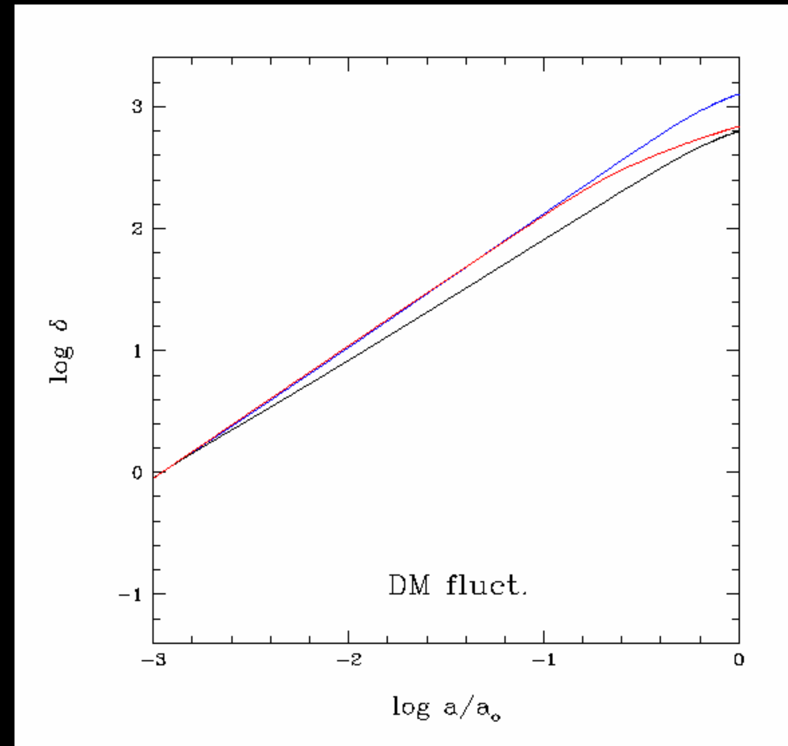
$$\ddot{\delta}_c + \left(\frac{\dot{a}}{a} - C\dot{\phi}\right)\dot{\delta}_c = 4\pi\left(G^*\rho_c\delta_c + G\rho_b\delta_b\right)a^2$$

modified friction term   modified source term

As a consequence….. Linear bias $\delta_b = b\delta_c$

Density fluctuations: linear evolution





Red: dual axion C(ϕ)=1/ϕ
Blue: Coupled DE C=cost=<C(ϕ)>
Black: ΛCDM

Differences from ΛCDM:

- objects should form earlier
- baryons fluct. keep below DM fluct. until very recently

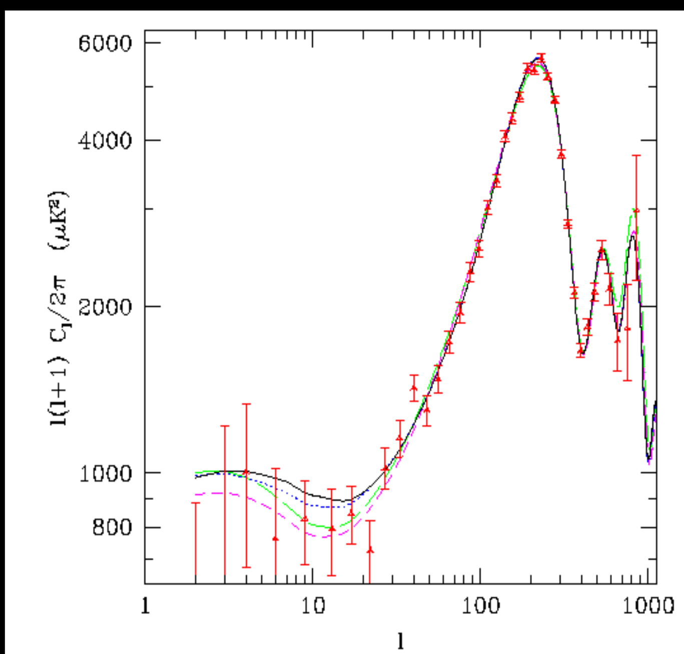
Dual-axion model: *fit to WMAP data*

Best fit parameters:

Uncoupled SUGRA		
x	$\langle x \rangle$	σ_x
$\Omega_b h^2$	0.025	0.001
$\Omega_{dm} h^2$	0.12	0.02
h	0.63	0.06
τ	0.21	0.07
n_s	1.04	0.04
A	0.97	0.13
Log Λ	3.0	7.7

SUGRA with C=cost		
x	$\langle x \rangle$	σ_x
$\Omega_b h^2$	0.024	0.001
$\Omega_{dm} h^2$	0.11	0.02
h	0.74	0.11
τ	0.18	0.07
n_s	1.03	0.04
A	0.92	0.14
Log Λ	-0.5	7.6
β	0.10	0.07

SUGRA with C= 1/ϕ		
x	$\langle x \rangle$	σ_x
$\Omega_b h^2$	0.025	0.001
$\Omega_{dm} h^2$	0.11	0.02
h	0.93	0.05
τ	0.26	0.04
n_s	1.23	0.04
A	1.17	0.10
Log Λ	4.8	2.4



- **Main differences at low l**
- **χ^2_{eff} from 1.064 (uncoupled SUGRA) to 1.071 (C=1/ϕ)**
- **SUGRA with C=1/ϕ → h=0.93±0.05**

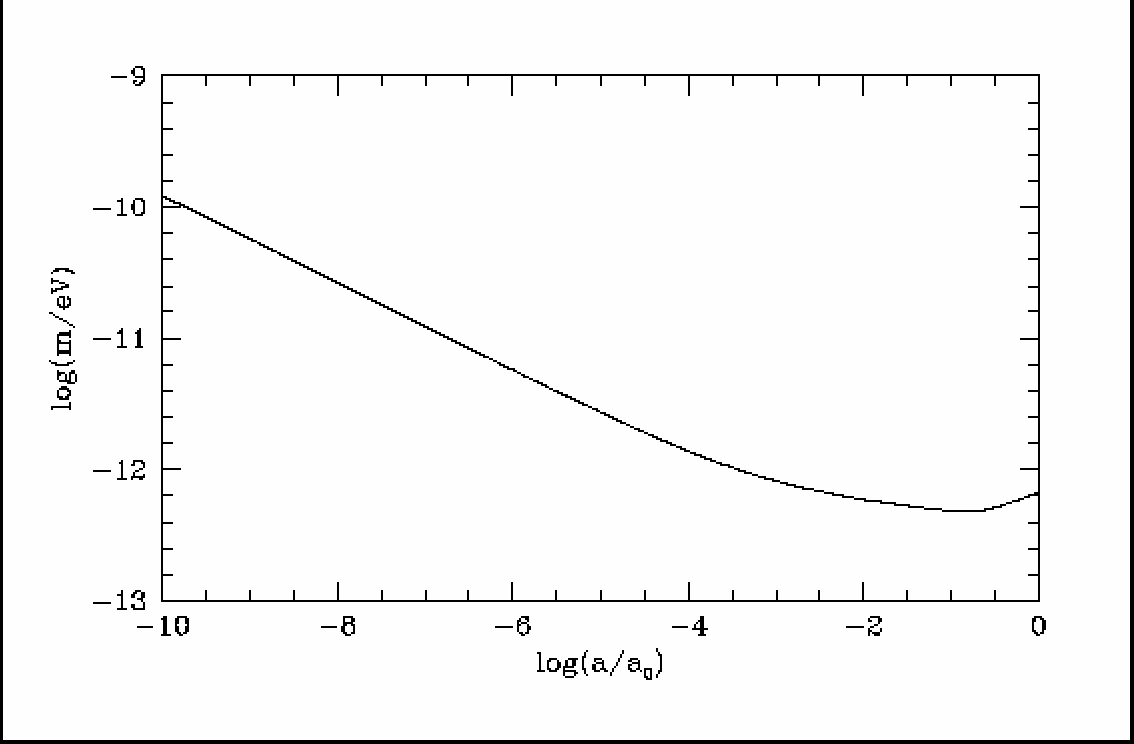
Uncoupled SUGRA
SUGRA with C = cost
SUGRA with C= 1/ϕ
SUGRA Dual-Axion

Conclusions

Dual-axion model

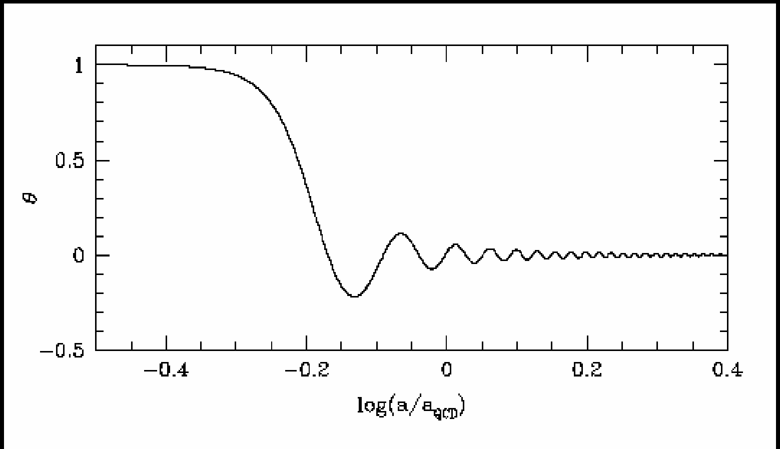
- One complex scalar field yields both DM & DE
- Fair DM-DE proportions from one parameter : Λ/GeV~10^{10}
- Strong CP problem solved
- DM-DE coupling fixed (C=1/ϕ)
- Fair fit to WMAP data, constraining Λ in the expected range
- SUGRA potential: large H_0 (~90 $\pm$ 10 km/s/Mpc)
- Fluctuation growth is fair
- Structure formation earlier than in ΛCDM

Axion mass



Φ variation cause a dependence of the axion mass on scale factor a
Rebounce at z=10 could be critical for structure formation

Maccio' et al 2004, Phys. Rev. D69, 123516



1
0.5
0
−0.5
θ
−0.4
−0.2
0
0.2
0.4
log(a/a_QCD)