A Guided Walk Through the CMSSM

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walk with R. Ruiz de Austri and R. Trotta

hep-ph/0602028 $\longrightarrow$ JHEP

DM after WMAP...

Post WMAP (Mar 06) ...+ACBAR+CBI+...

$\Omega = \rho/\rho_{crit}$

Hubble $H_0 = 100\, h\,$ km/ s/ Mpc

- $\Omega_{\rm m} h^2 = 0.131 \pm 0.009$
- $\Omega_{\rm b} h^2 = 0.0223 \pm 0.0008$
- $\ldots$

$\Rightarrow$ most matter non–baryonic

(DM problem)

- observations: haloes, etc.
- numerical simulations of LSS

$\Rightarrow$ cold DM

(non–relativistic)

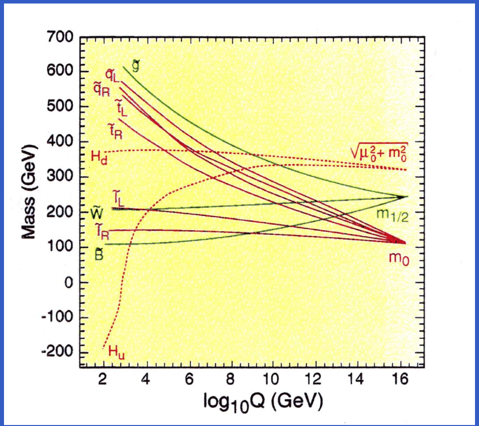
$$\Omega_{\rm CDM} h^2 = 0.109 \pm 0.009$$

(after WMAP–yr 1: $\Omega_{\rm CDM} h^2 = 0.129 \pm 0.009$)

Constrained MSSM

...aka mSUGRA

At M_{GUT}:



- gauginos $M_1 = M_2 = m_{\widetilde{g}} = m_{1/2}$ (c.f. MSSM)
- scalars $m^2_{\widetilde{q}_i} = m^2_{\widetilde{l}_i} = m^2_{H_b} = m^2_{H_t} = m^2_0$
- 3–linear soft terms $A_b = A_t = A_0$
- radiative EWSB

$$\mu^2 = \frac{\left(m^2_{H_b}+\Sigma^{(1)}_b\right)-\left(m^2_{H_t}+\Sigma^{(1)}_t\right)\tan^2\beta}{\tan^2\beta-1} - \frac{m^2_Z}{2}$$

- five independent parameters: $\tan\beta,\ m_{1/2},\ m_0,\ A_0,\ \mathrm{sgn}(\mu)$
- mass spectra at m_Z:
  run RGEs, 2–loop for g.c. and Y.c, 1-loop for masses
- some important quantities $(\mu, m_A, \ldots)$ very sensitive to procedure of computing EWSB & minimizing V_H

we use Suspect

Constrained MSSM

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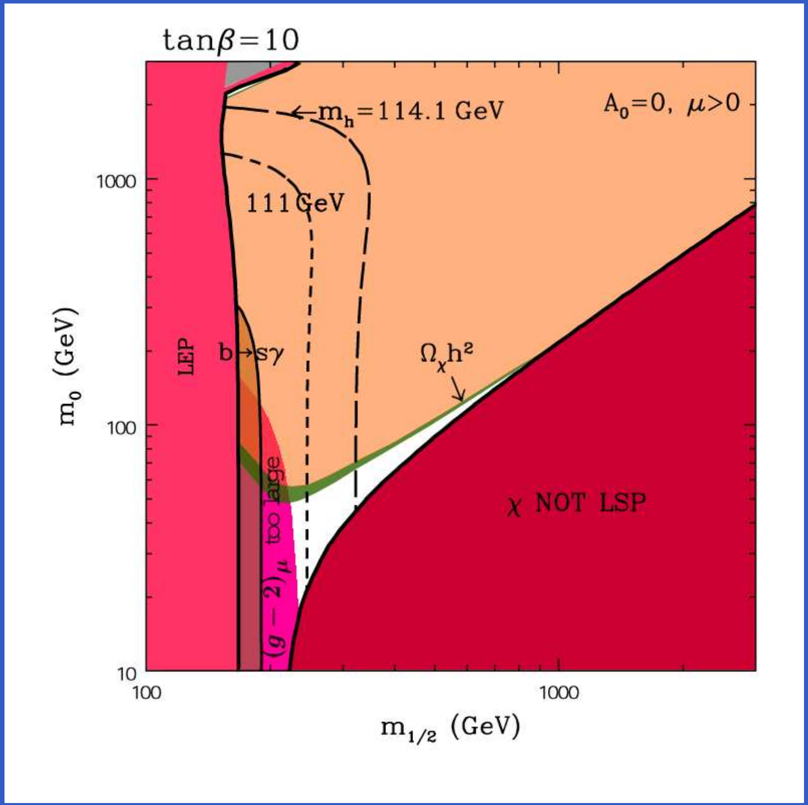
$\tan\beta \lesssim 45$



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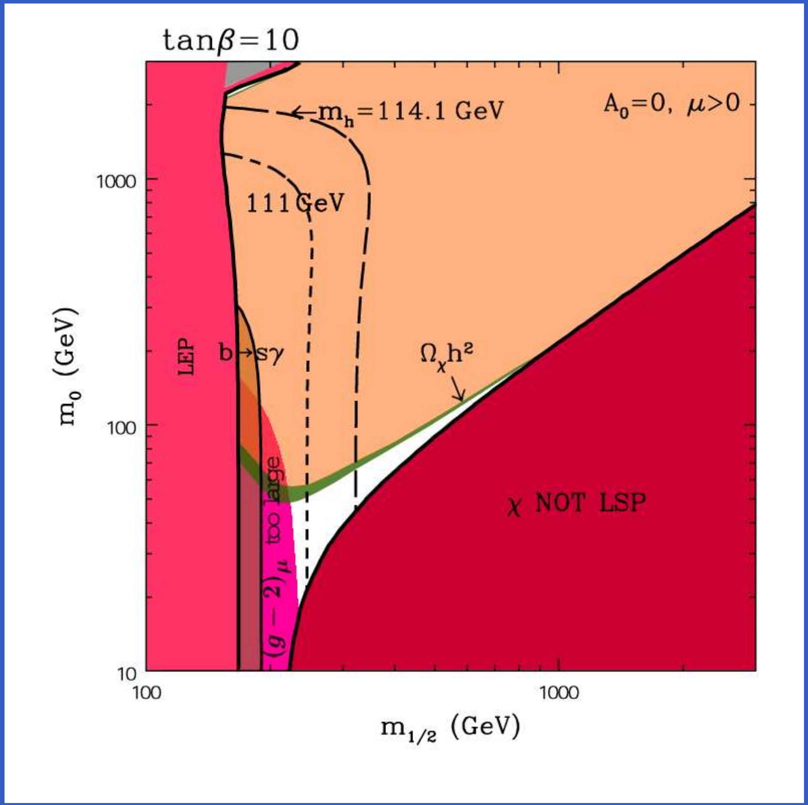

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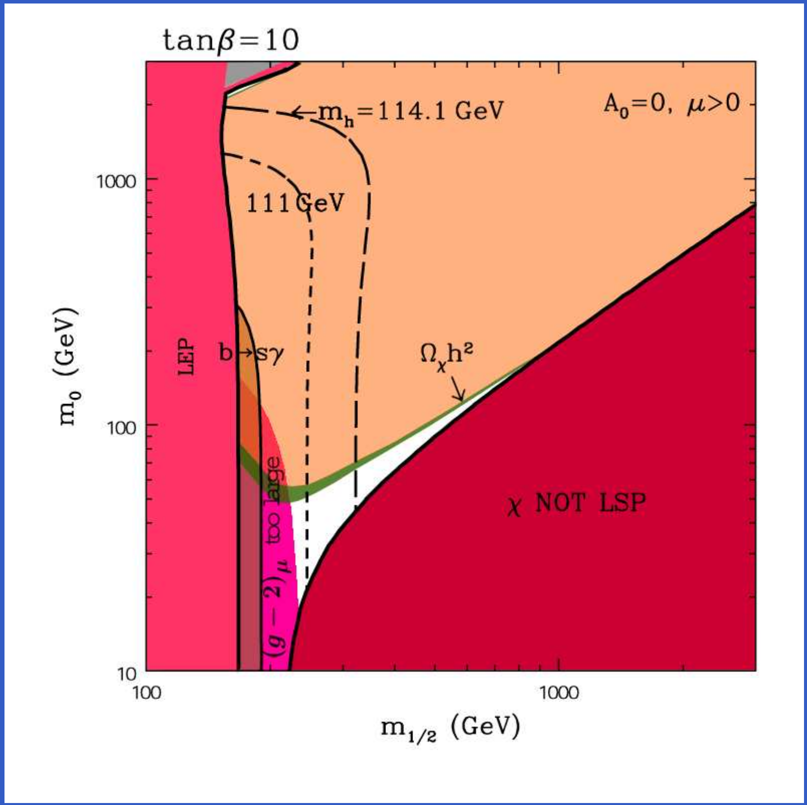

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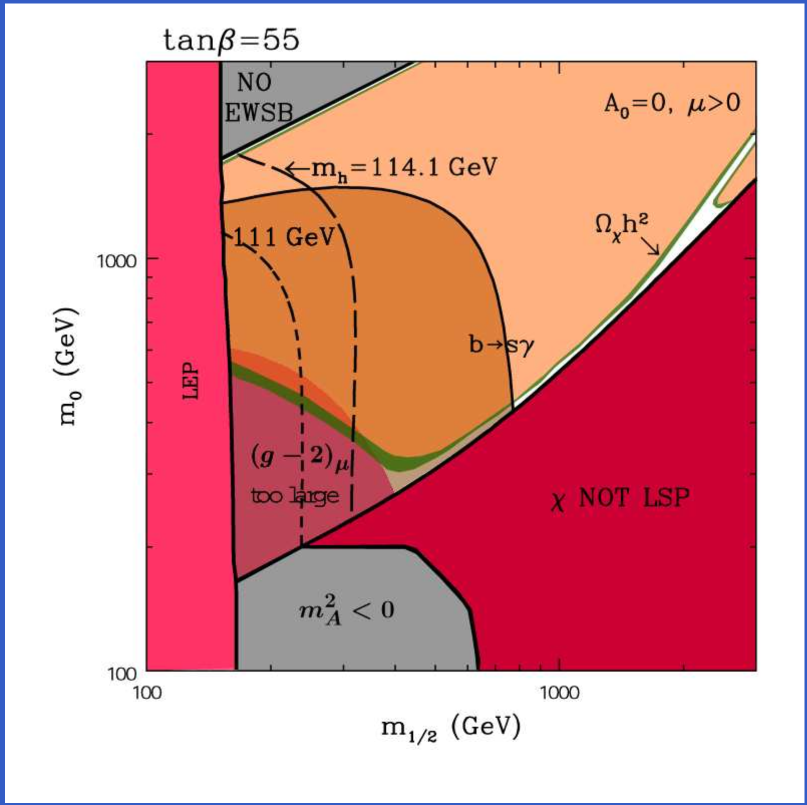

. . .left allowed

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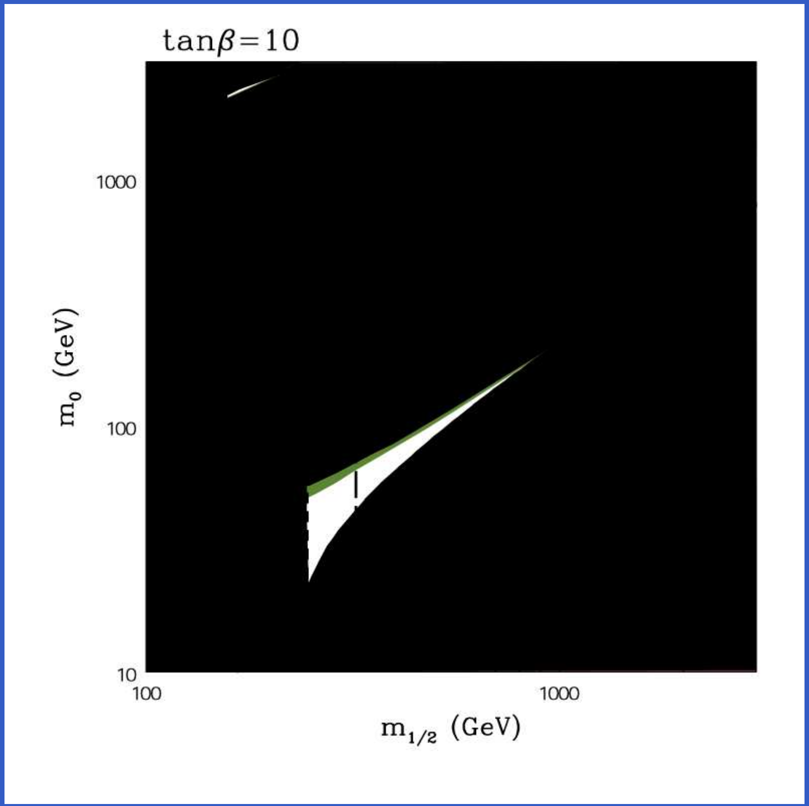

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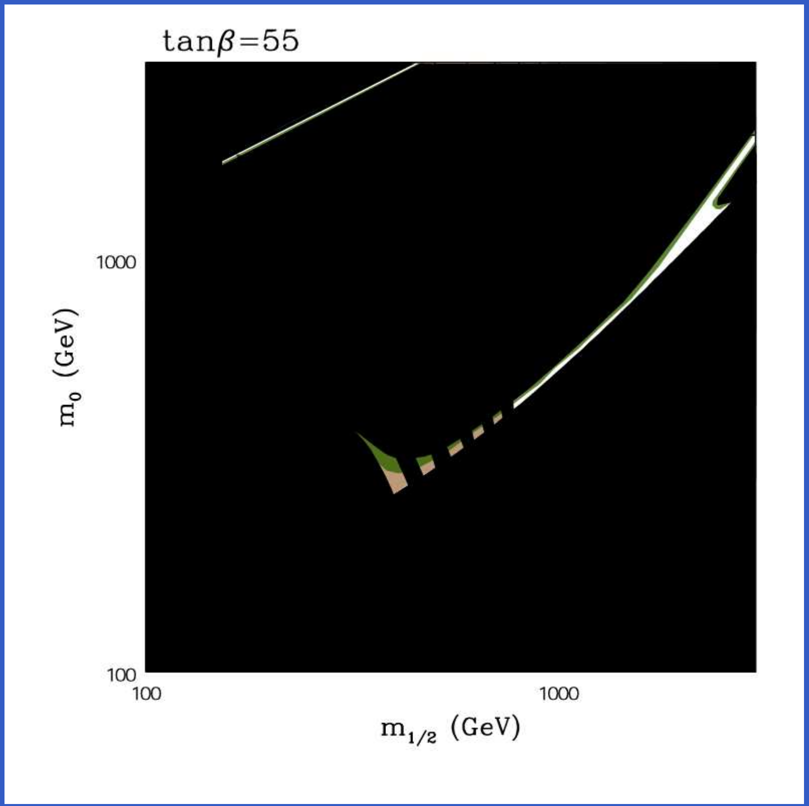
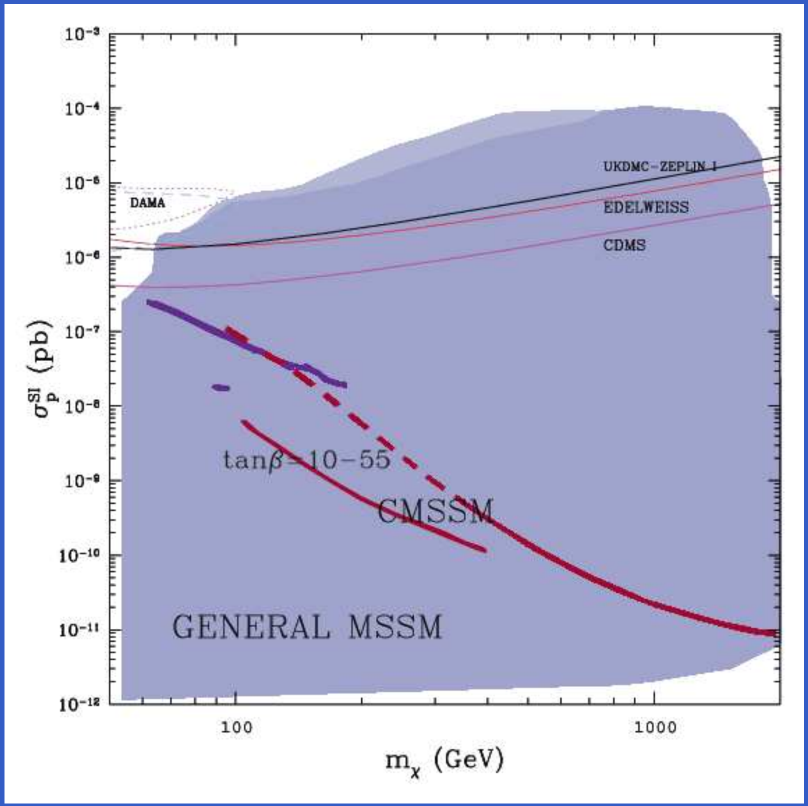


Expectations for σ_p^{SI} with unification



σ_p^{SI} – WIMP–proton SI elastic scatt. c.s.

blue: general MSSM

red: Constrained MSSM

$A_0 = 0, \tan\beta = 10, 55$

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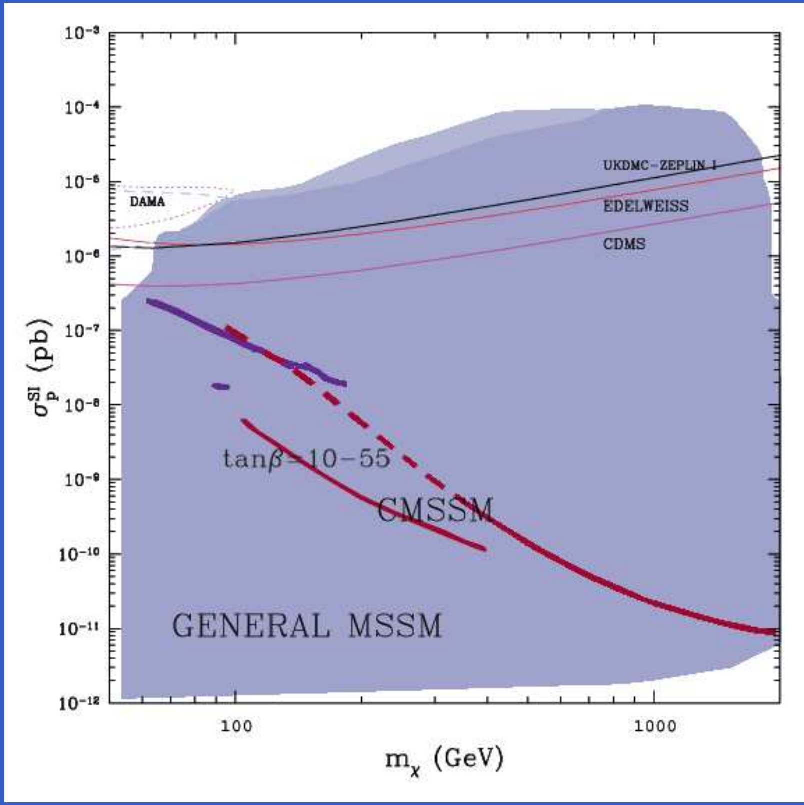

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- effect of varying CMSSM and SM inputs???

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fixed grid scans

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disadvantages

- nr of scan points $\propto k^N$ (where k: nr of steps, N: nr of parameters)
- $\Rightarrow$ highly inefficient for $N \gtrsim$ a few
- narrow wedges may be missed
- hard to inocoporate other uncertainties (*e.g.*, SM error bars)

MCMC + Bayesian Analysis

a probabilistic approach

(MCMC=Markov Chain Monte Carlo)

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- random scan of points (not strictly controlled)
- ‘allowed’ regions function of probability

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a powerful method of exploring multi–parameter models

Bayesian Analysis

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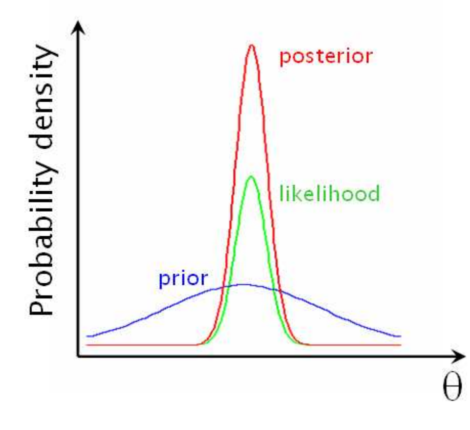

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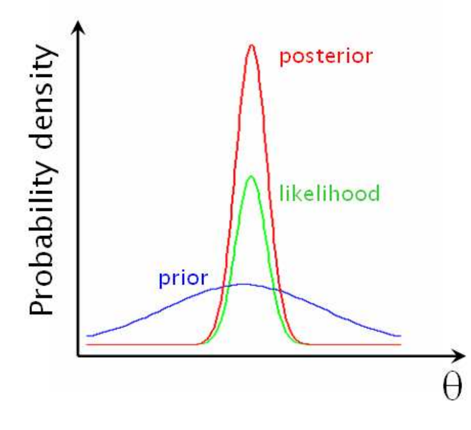
- d: data
- Bayes' theorem: posterior pdf

$$p(\theta, \psi | d) = \frac{p(d|\xi)\pi(\theta,\psi)}{p(d)}$$



- $p(d|\xi)$: likelihood
- $\pi(\theta, \psi)$: prior pdf
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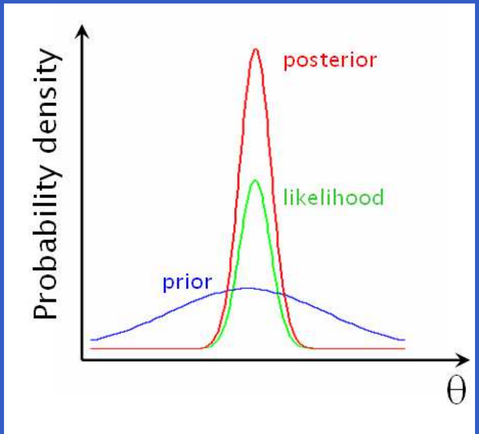


- $p(d|\xi)$: likelihood
- $\pi(\theta, \psi)$: prior pdf
- $p(d)$: evidence (normalization factor)
- usually marginalize (integrate) over nuissance parameters

$$p(\theta|d) = \int p(\theta, \psi | d) \, d\psi$$

Bayesian Analysis of the CMSSM

- $\theta = (m_0, m_{1/2}, A_0, \tan\beta)$: main CMSSM parameters
- $\psi = (m_t^{\rm pole}, m_b(m_b)^{\overline{MS}}, \alpha_{\rm em}(M_Z), \alpha_s)$: SM (nuissance) parameters
- priors – assume flat distributions and ranges as:

CMSSM parameters θ	
"2 TeV range"	"4 TeV range"
$50 < m_0 < 2$ TeV	$50 < m_0 < 4$ TeV
$50 < m_{1/2} < 2$ TeV	$50 < m_{1/2} < 4$ TeV
$\lvert A_0\rvert < 5$ TeV	$\lvert A_0\rvert < 7$ TeV
$2 < \tan\beta < 62$	
SM (nuisance) parameters ψ	
$160 < m_t^{\rm pole} < 190$ GeV	
$4 < m_b(m_b)^{\overline{MS}} < 5$ GeV	
$127.5 < 1/\alpha_{\rm em}(M_Z) < 128.5$	
$0.10 < \alpha_s < 0.13$	

Experimental Measurements

Derived observable	Mean value	Uncertainties	
	μ	σ (exper.)	τ (theor.)
M_W	80.425 GeV	34 MeV	13 MeV
$\sin^2\theta_{\rm eff}$	0.23150	16×10^{-5}	25×10^{-5}
$\delta a_\mu^{\rm SUSY}\times10^{10}$	25.2	9.2	1
$\mathrm{BR}(\bar{\mathrm{B}}\to \mathrm{X_s}\gamma)\times10^4$	3.39	0.30	0.30
$\Omega_\chi h^2$	0.119	0.009	$0.1\,\Omega_\chi h^2$

- other quantities more precisely known: *e.g.*

  $M_Z = 91.1876(21)$ GeV

  $G_F = 1.16637(1)\times10^{-5}$ GeV^{-2}

Experimental Limits

Derived observable	upper/lower limit	Constraints $\xi_{\rm lim}$	τ (theor.)
σ_p^{SI}	UL	WIMP mass dependent	$\sim 100\%$
$\mathrm{BR}(B_s \to \mu^+\mu^-)$	UL	1.5×10^{-7}	14%
m_h	LL	114.4 GeV (91.0 GeV)	3 GeV
$\zeta_h^2 \equiv g^2_{ZZh}/g^2_{ZZH_{\rm SM}}$	UL	$f(m_h)$	3%
m_χ	LL	50 GeV	5%
$m_{\chi_1^\pm}$	LL	103.5 GeV (92.4 GeV)	5%
$m_{\tilde{e}_R}$	LL	100 GeV (73 GeV)	5%
$m_{\tilde{\mu}_R}$	LL	95 GeV (73 GeV)	5%
$m_{\tilde{\tau}_1}$	LL	87 GeV (73 GeV)	5%
$m_{\tilde{\nu}}$	LL	94 GeV (43 GeV)	5%
$m_{\tilde{t}_1}$	LL	95 GeV (65 GeV)	5%
$m_{\tilde{b}_1}$	LL	95 GeV (59 GeV)	5%
$m_{\tilde{q}}$	LL	318 GeV	5%
$m_{\tilde{g}}$	LL	233 GeV	5%

The Likelihood

incorporates information about the observational data

- the mapping $\xi(\theta, \chi)$ comes with uncertainties
- experimental uncertainty σ_i
- theoretical uncertainty τ_i
- introduce "exact" mapping $\hat{\xi}(\theta, \chi)$
- the likelihood:

$$p(d|\xi) = \int p(d|\hat{\xi}) p(\hat{\xi}|\xi) d^m \hat{\xi}$$

where

$$p(\hat{\xi}|\xi) = \frac{1}{(2\pi)^{m/2} |C|^{1/2}} \exp\left(-\frac{1}{2}(\xi - \hat{\xi}) C^{-1} (\xi - \hat{\xi})^T\right)$$

C – $m \times m$ covariance matrix

if uncorrelated: $C = \mathrm{diag}\,(\tau_1, \ldots, \tau_m)$

$$p(d|\hat{\xi}) = \frac{1}{(2\pi)^{m/2} |D|^{1/2}} \exp\left(-\frac{1}{2}(d - \hat{\xi}) D^{-1} (d - \hat{\xi})^T\right)$$

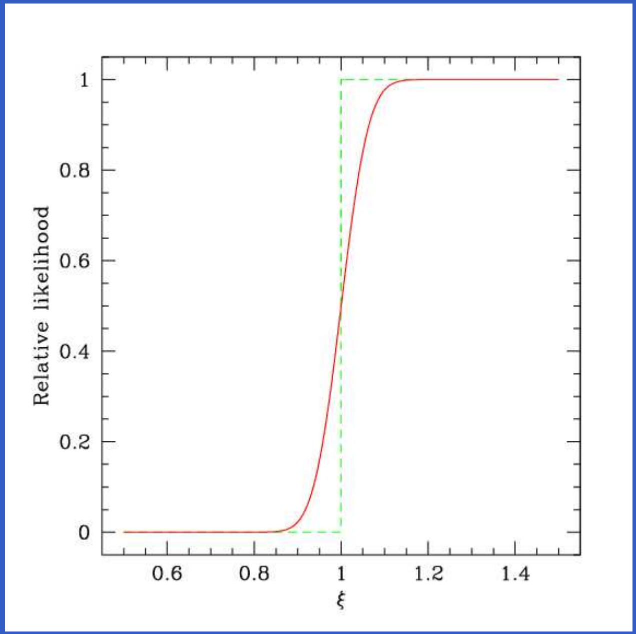
if uncorrelated: $D = \mathrm{diag}\,(\sigma_1, \ldots, \sigma_m)$

- total error for each observable: $s_i = \sqrt{\sigma_i^2 + \tau_i^2}$

The Likelihood: 1–dim example

1–dim example: limit with $\sigma = 0$ and $\tau = 0.05$

$$p(\sigma, \xi_{\rm lim}|\hat{\xi}) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma} & \text{for } \hat{\xi} \geq \xi_{\rm lim}, \\ \frac{1}{\sqrt{2\pi}\sigma} \exp -\frac{(\hat{\xi}-\xi_{\rm lim})^2}{2\sigma^2} & \text{for } \hat{\xi} < \xi_{\rm lim}, \end{cases}$$



$\xi_{\rm lim} = 1$

TH error “smears out” the EXPTAL limit

Full MCMC Scan of the CMSSM

Ruiz de Austri, Trotta, LR, hep-ph/0602028 –> JHEP

Bayesian analysis, relative probability density fn (pdf), flat priors

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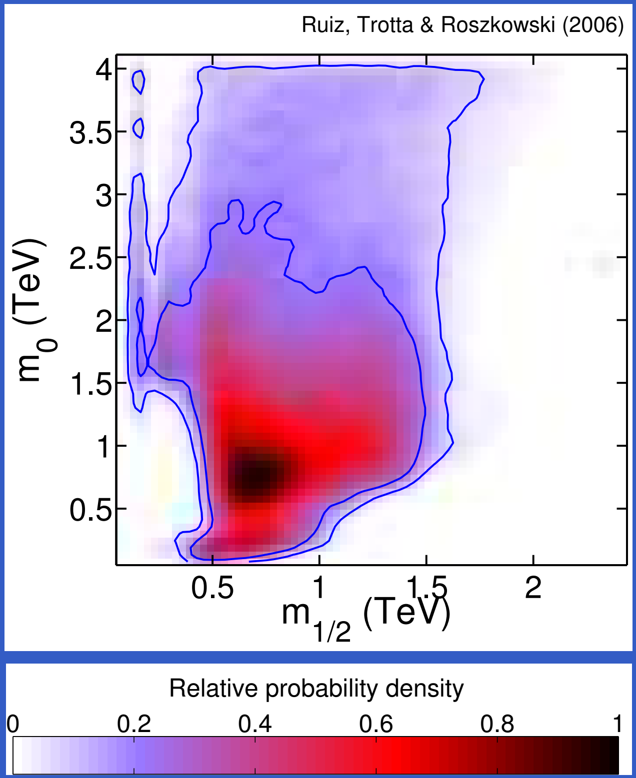

- project on $(m_0, m_{1/2})$

$p(m_0, m_{1/2}|d) = \int p(\theta|d)\, d\tan\beta\, dA_0$

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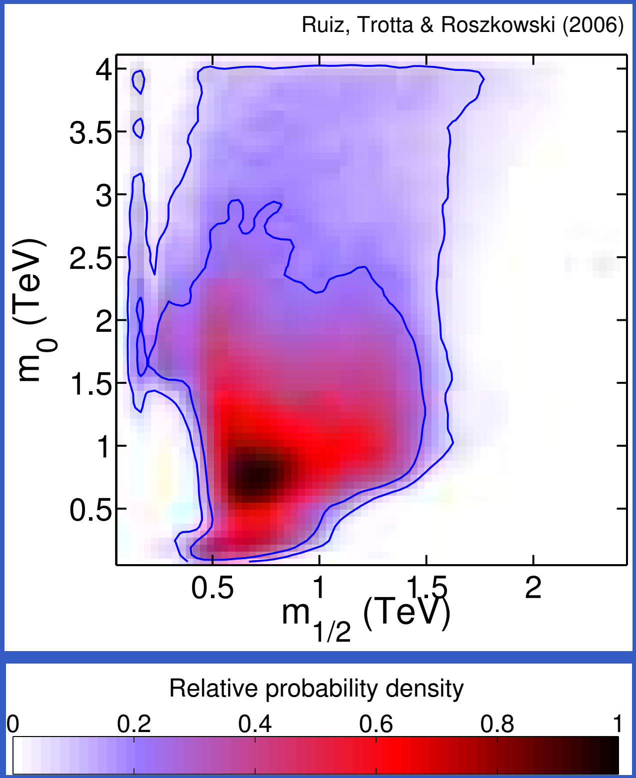
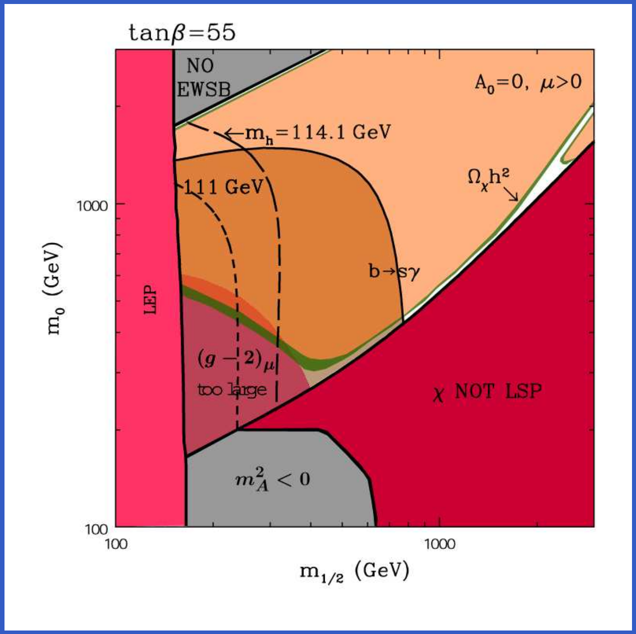


compare fixed grid:



$(\tan\beta \gtrsim 45)$

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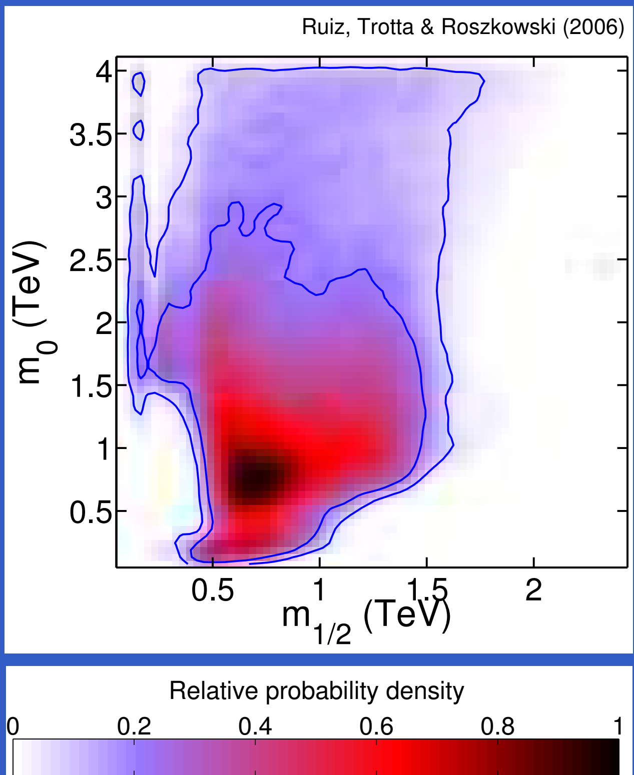

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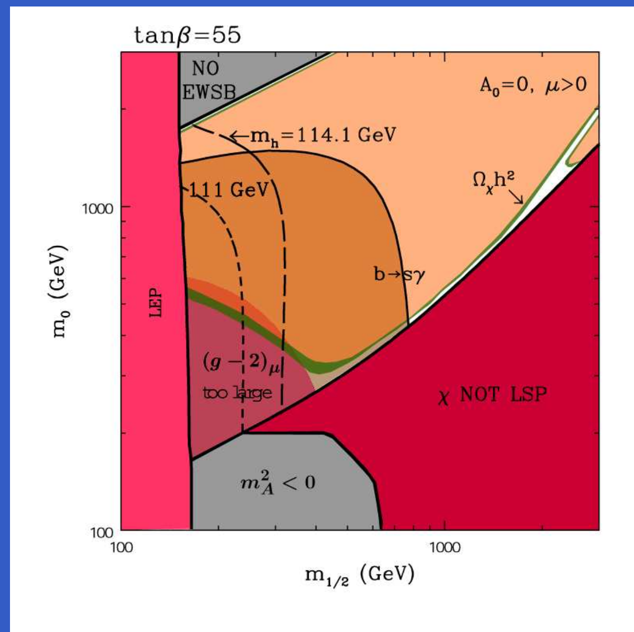

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related study by Allanach+Lester (no direct detection), Ellis, et al (EHOW, χ^2 approach)

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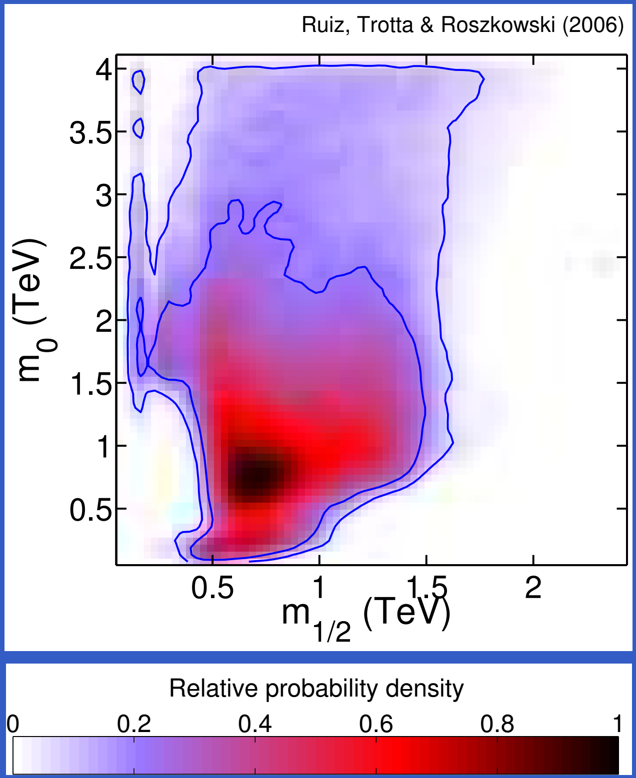

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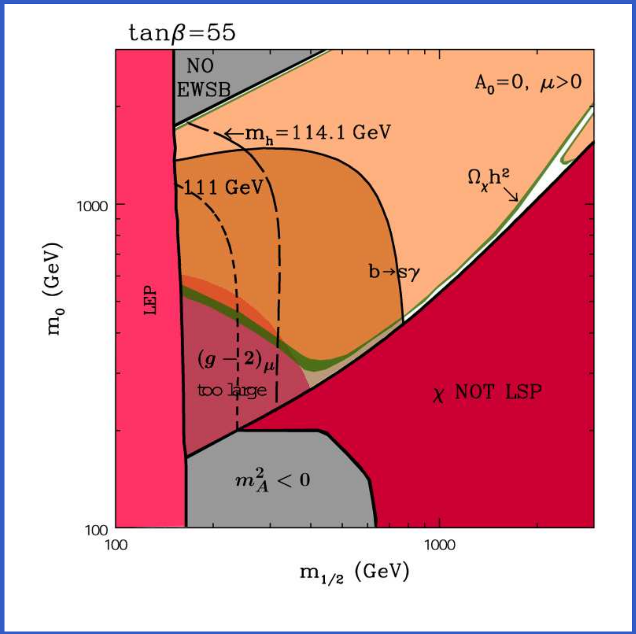

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unlike others (except for A+L), we vary also SM parameters

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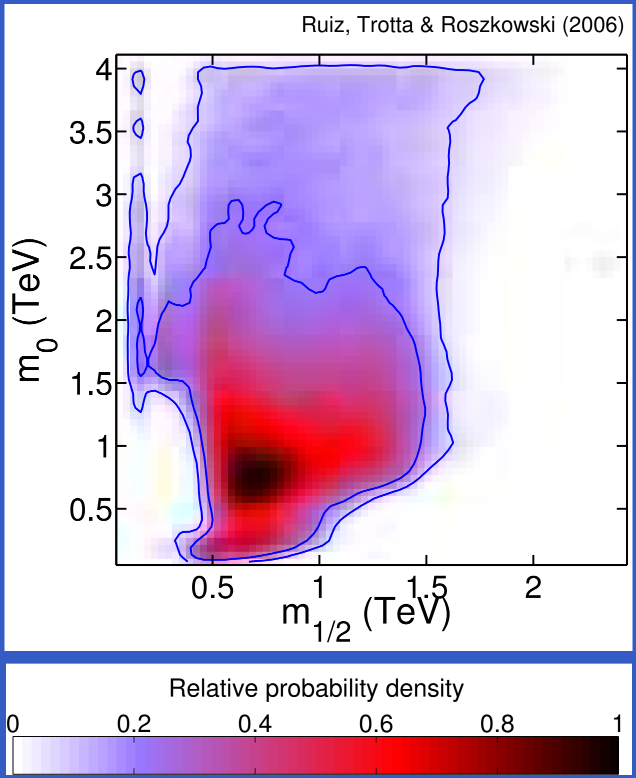

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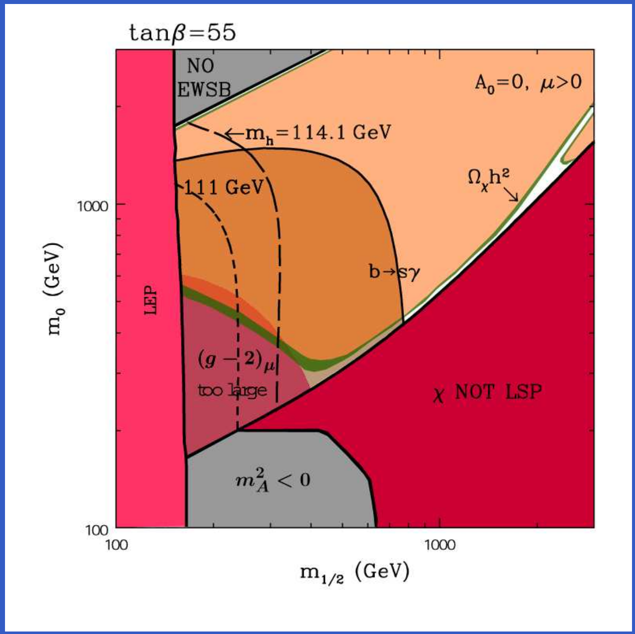
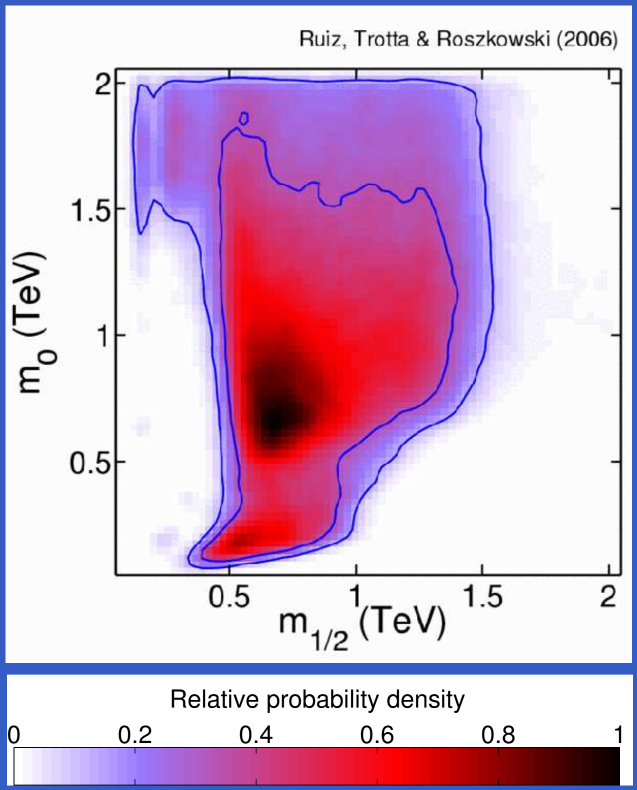


$(\tan\beta \gtrsim 45)$

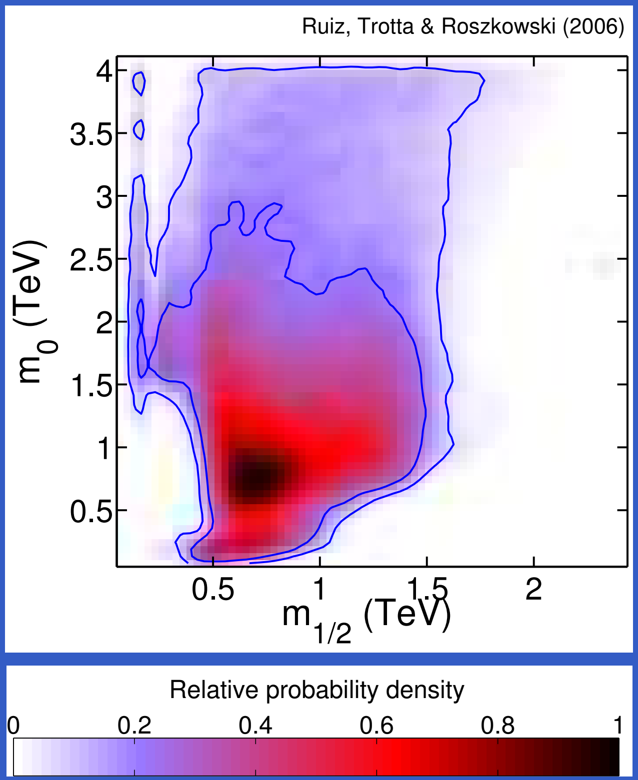
⇒ low energy SUSY!!! $(m_{1/2}, m_0 \lesssim \mathcal{O}(1\,\text{TeV}))$

Impact of Priors

$m_{1/2}, m_0 < 2$ TeV



$m_{1/2}, m_0 < 4$ TeV



m_0: strong, $m_{1/2}$: weak

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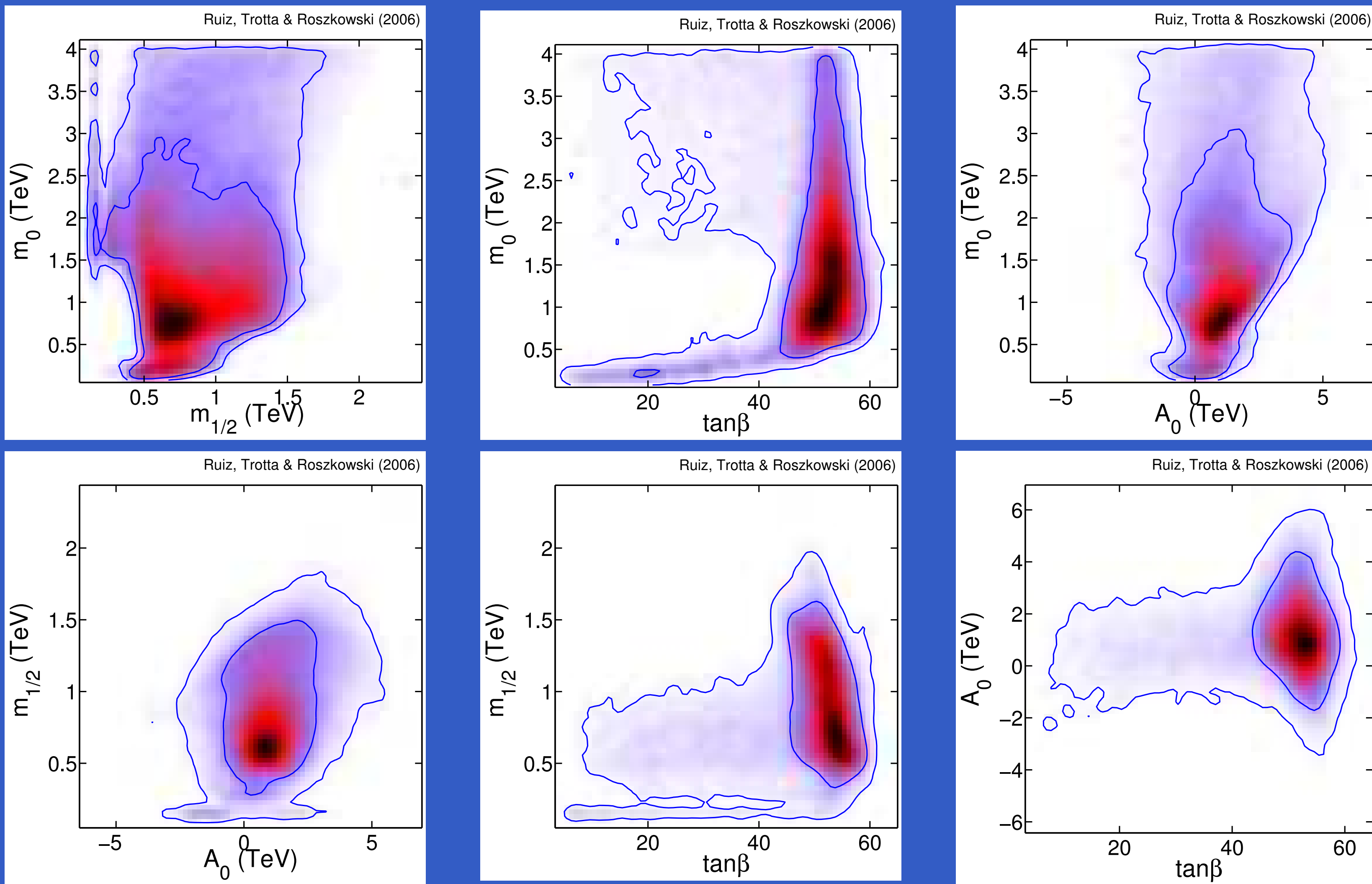

$A_0 \simeq 700$ GeV most probable

(usually assumed: $A_0 = 0$)

Implications for σ_p^{SI}

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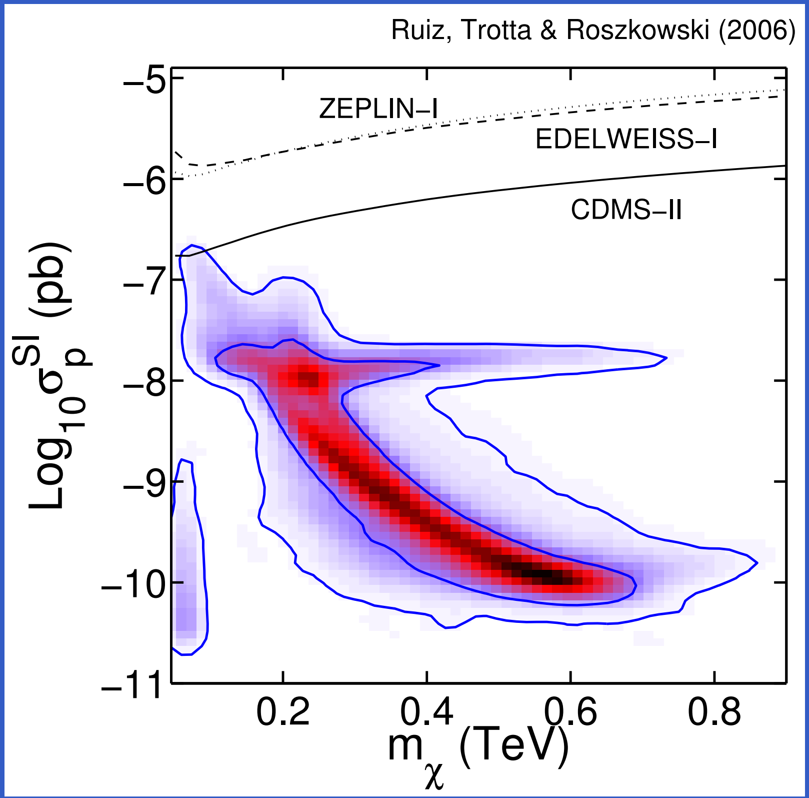
MCMC+Bayesian analysis



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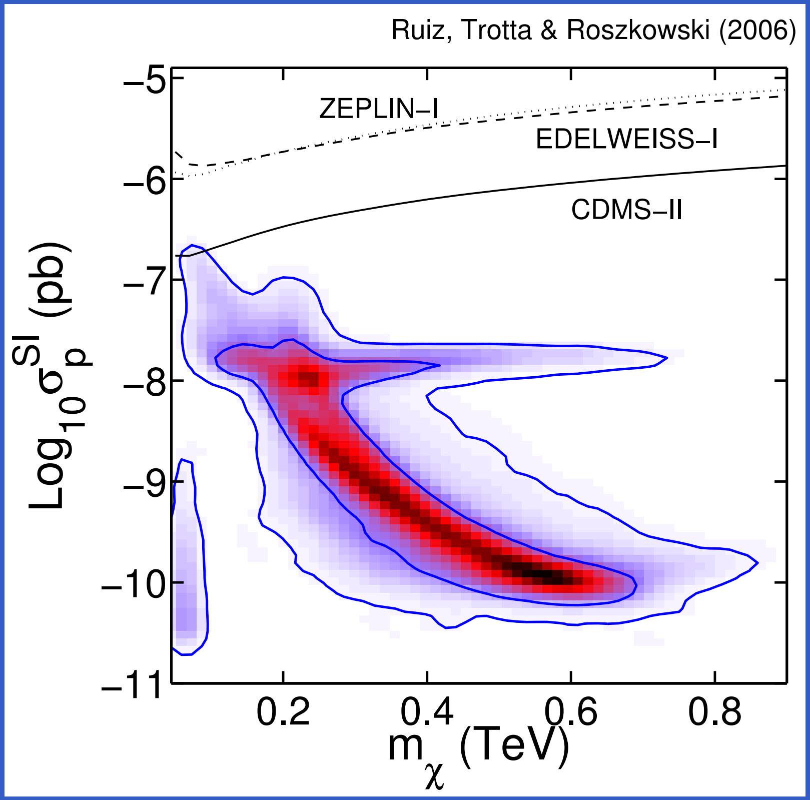
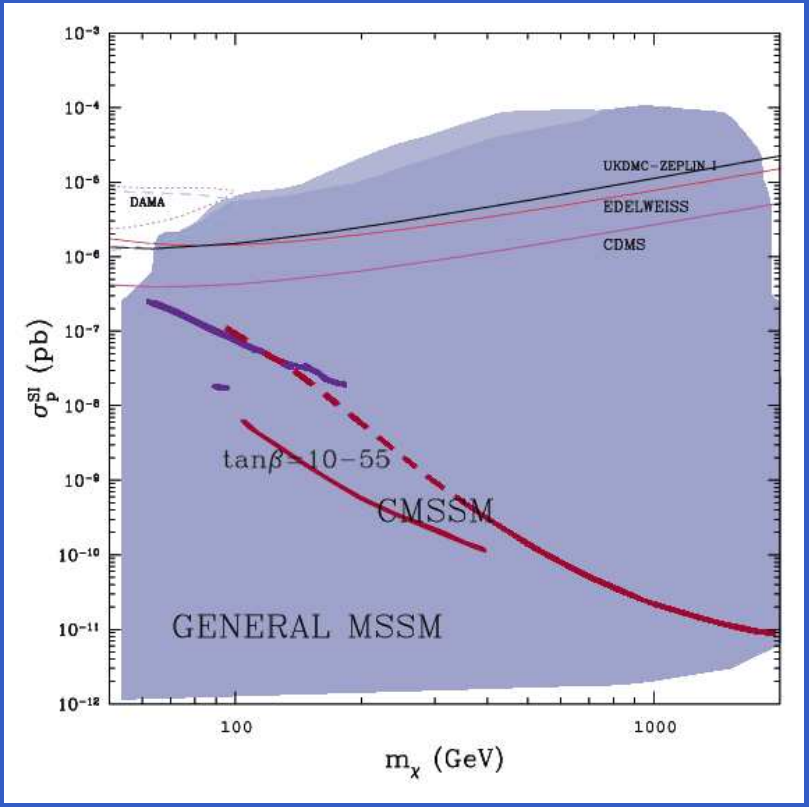
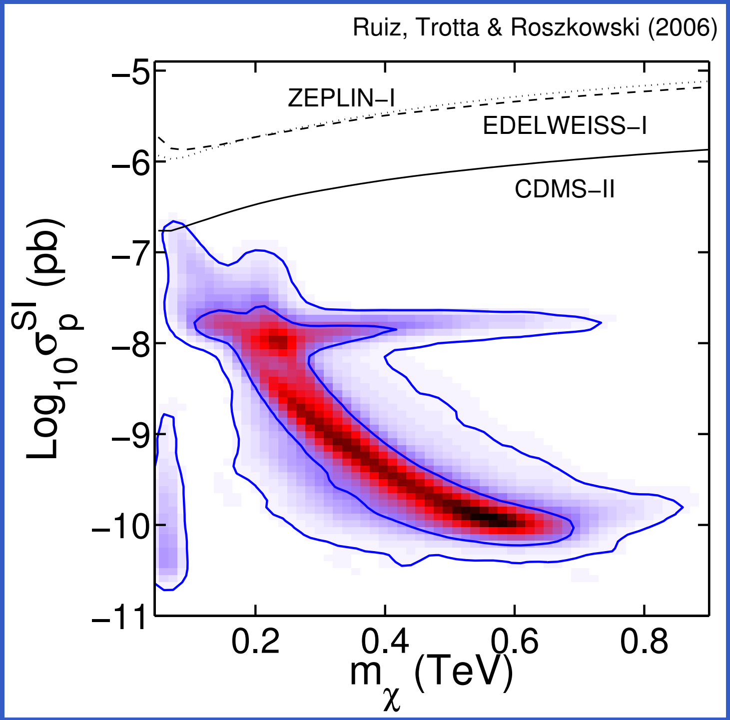


compare: fixed grid scan



Full MCMC Scan of the CMSSM

implications for σ_p^{SI}



internal (external): 68% (95%) region

Full MCMC Scan of the CMSSM

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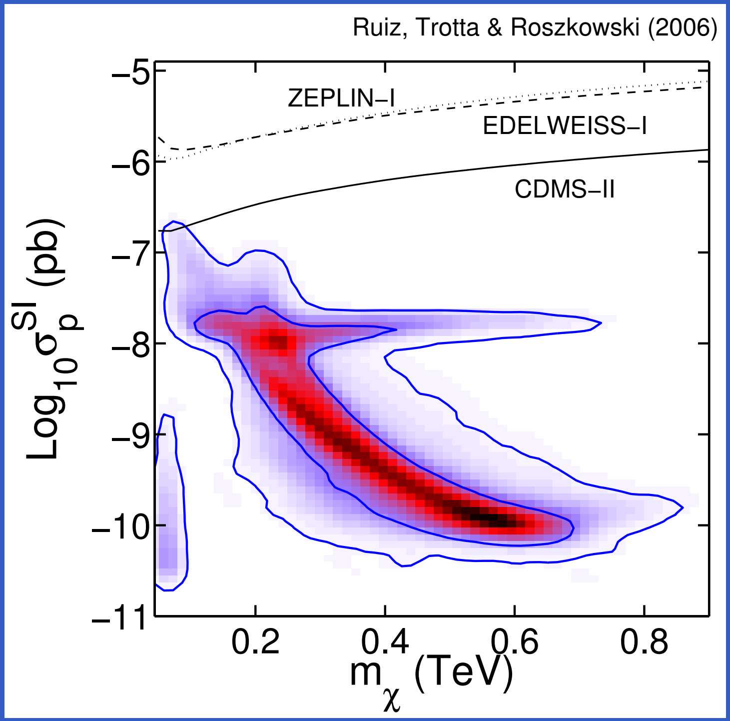

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CDMS (this year?):

will reach down to $\sigma_p^{SI} \sim 10^{-8}$ pb:

also Edelweiss–II and/or ZEPLIN–II (?)

$\Rightarrow$ explore the FP region

(large $m_0 \gg m_{1/2}$), outside of the LHC reach

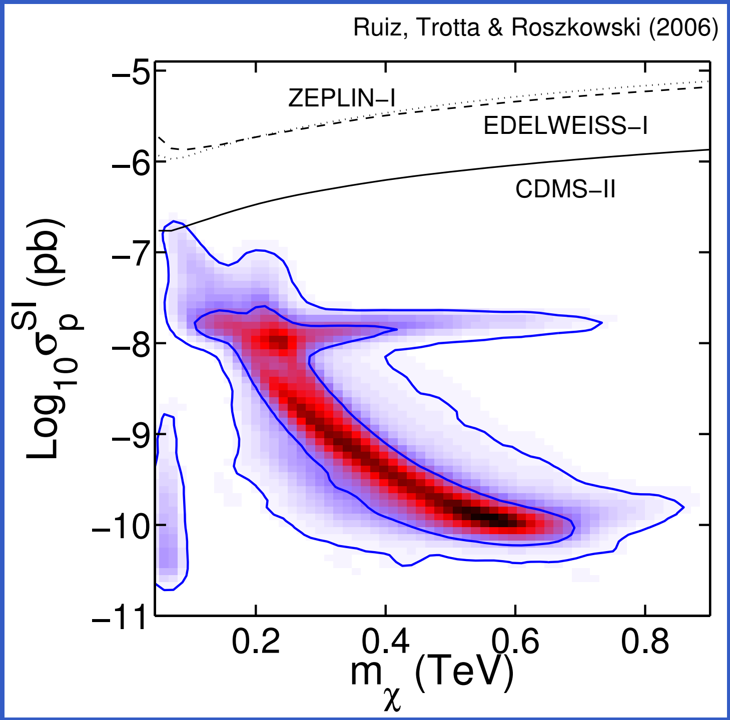
ultimately: "1 tonne" detectors:

$\sigma_p^{SI} \lesssim 10^{-10}$ pb

will cover all 68% region

Full MCMC Scan of the CMSSM

implications for σ_p^{SI}



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will cover all 68% region

most probable range: 10^{-8} pb $\lesssim \sigma_p^{SI} \lesssim 10^{-10}$ pb

partly outside of the LHC reach ($m_\chi \lesssim 400$ GeV)

σ_p^{SI}: 1–dim Probability Distribution

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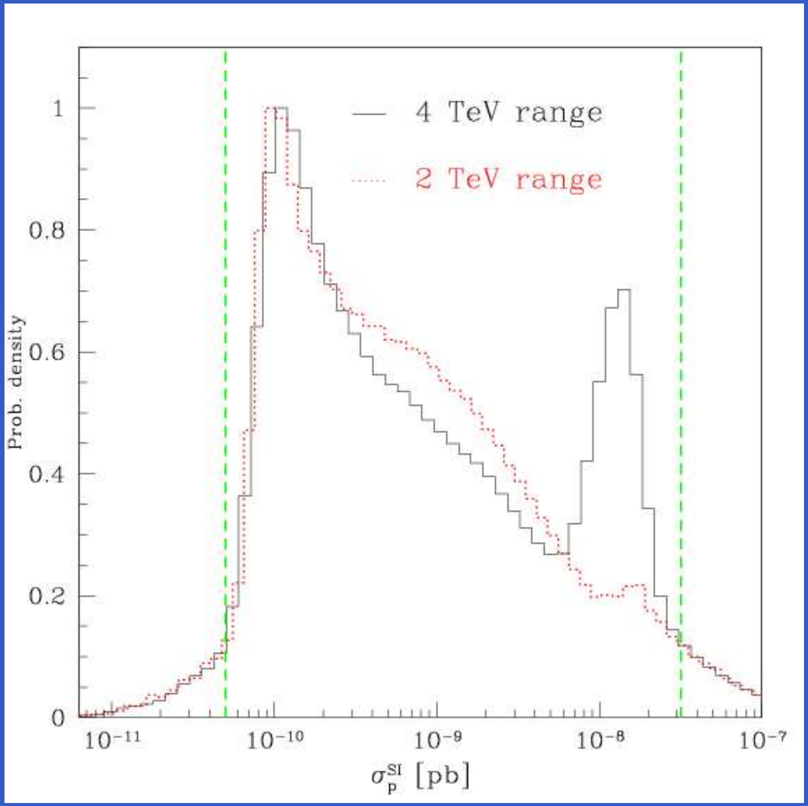
impact of priors



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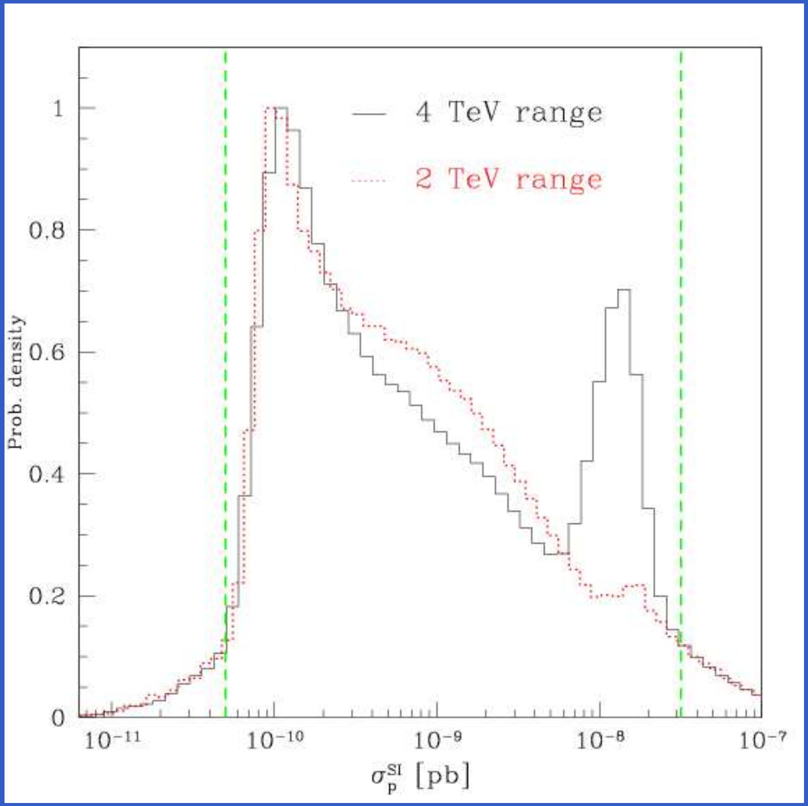
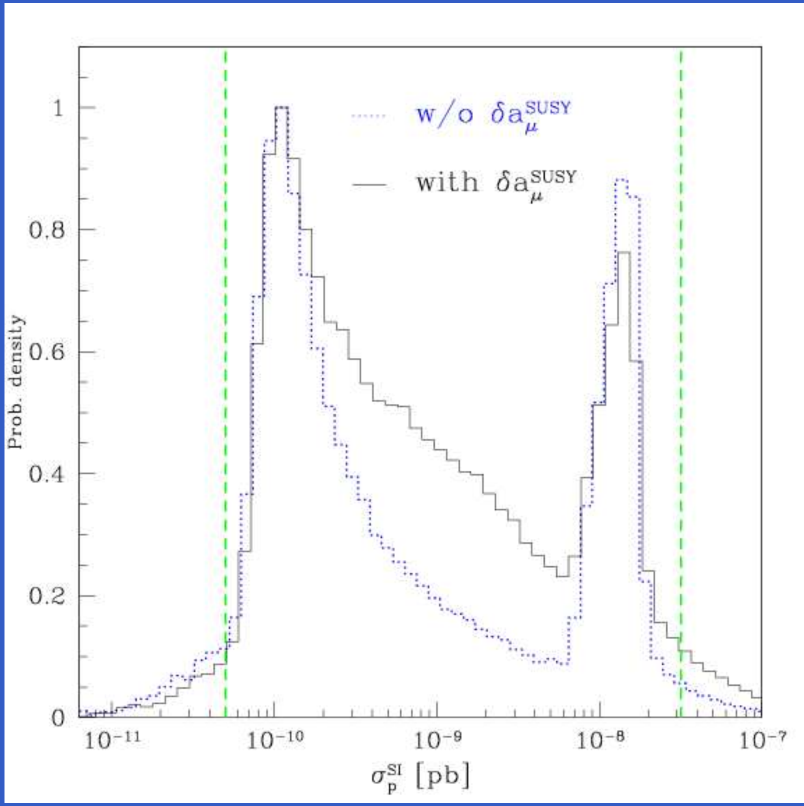


impact of $(g-2)_\mu$

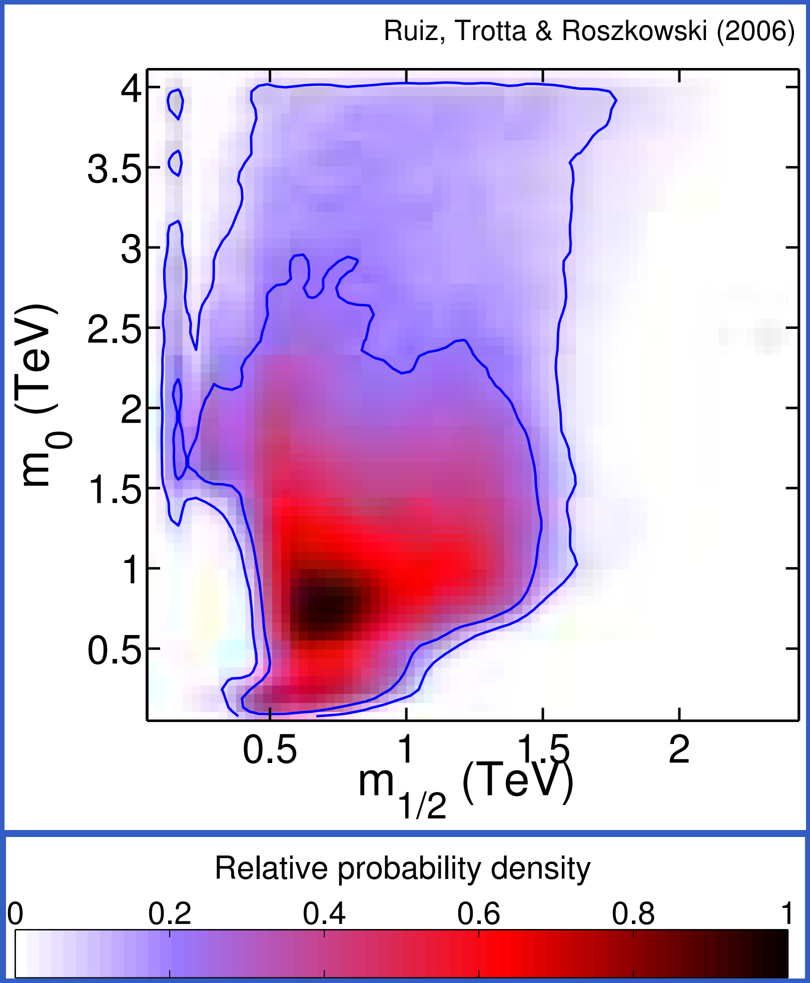


between vertical lines: 95% range

Bayesian vs Mean Like

Bayesian vs Mean Like

posterior pdf



volume effect of PS

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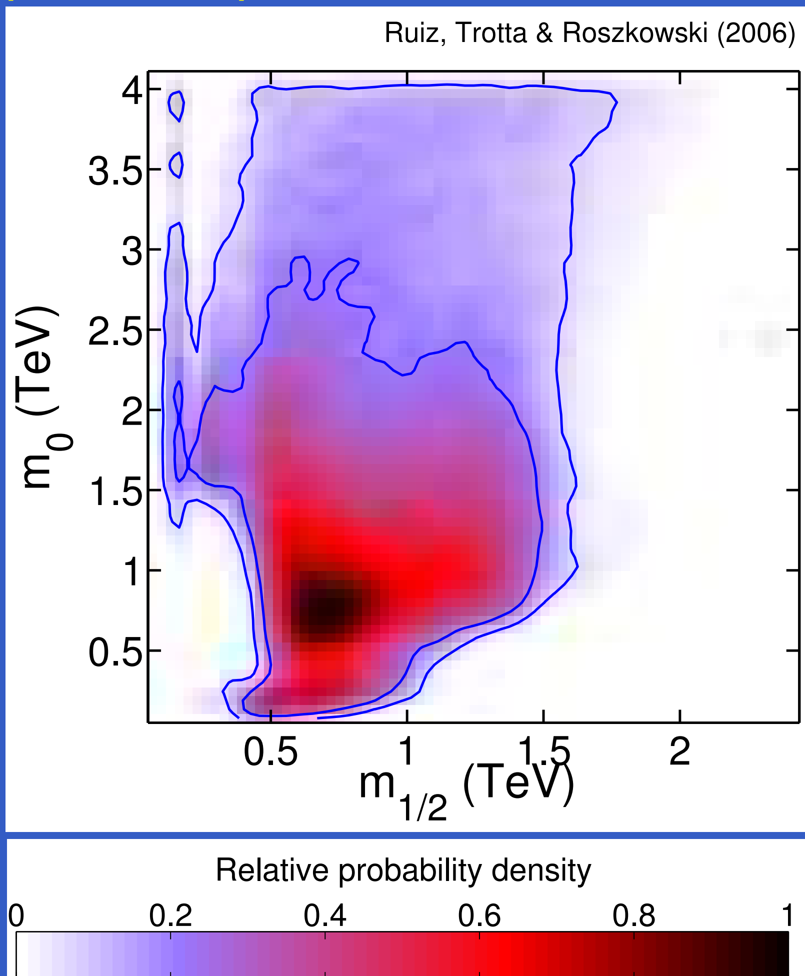

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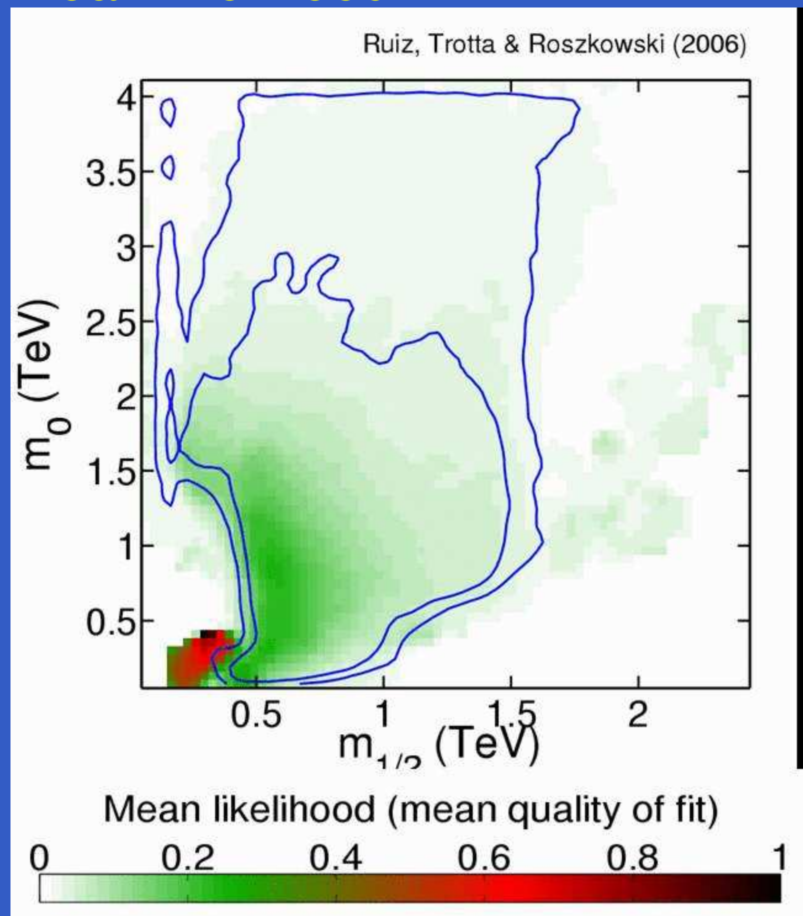

akin to χ^2 (goodness of fit) statistics

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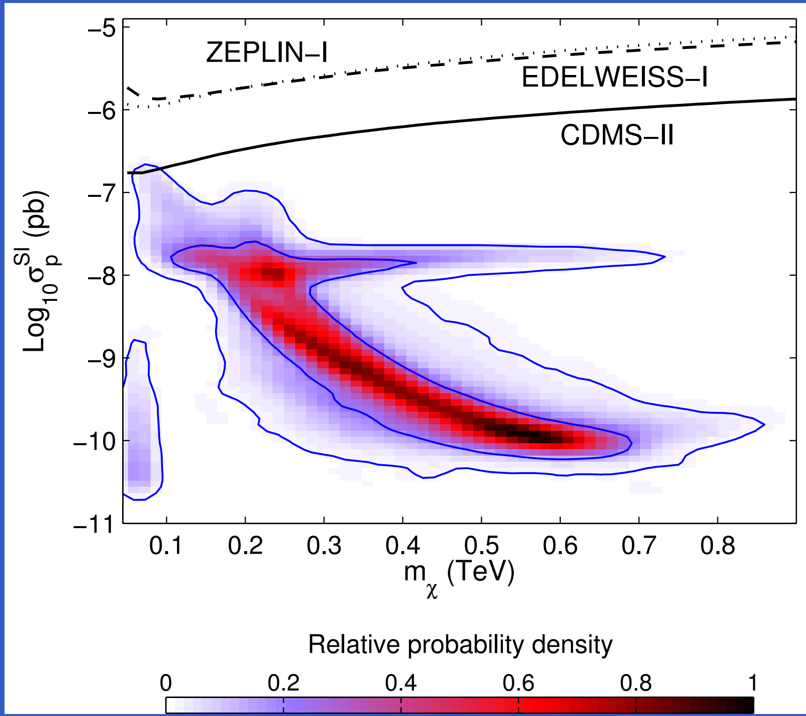

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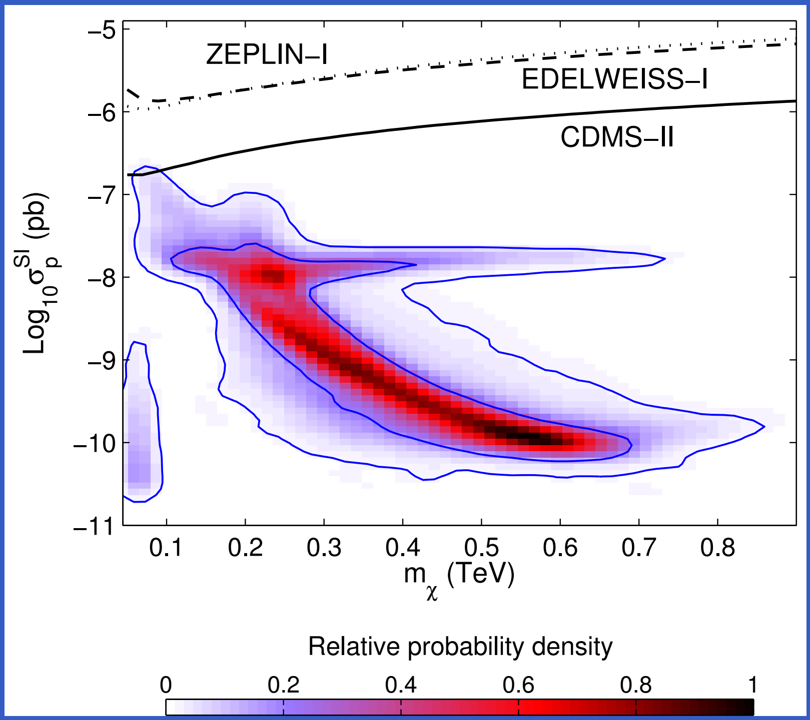

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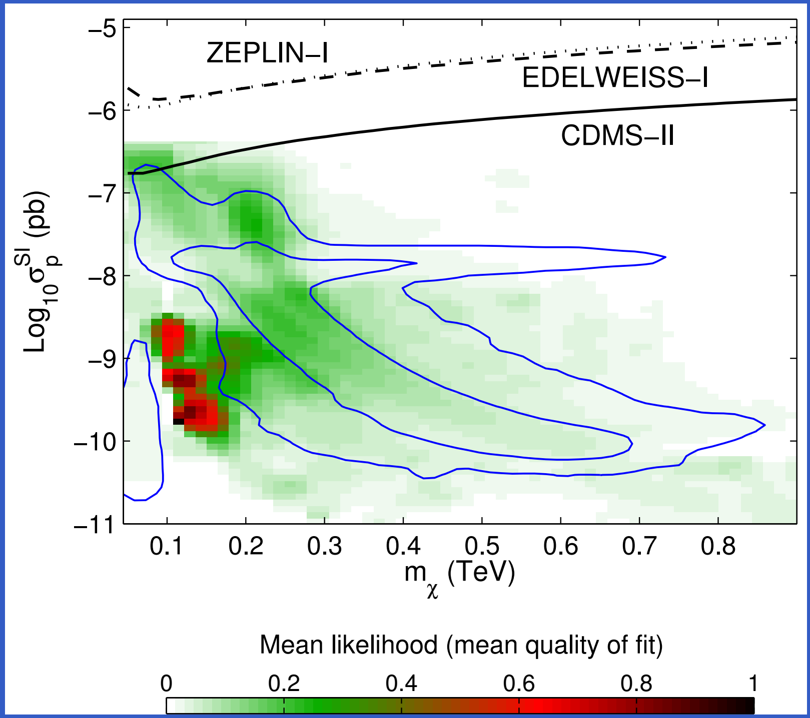

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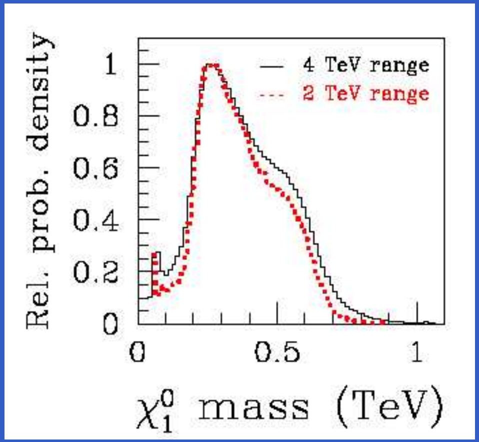
Neutralino mass range

priors



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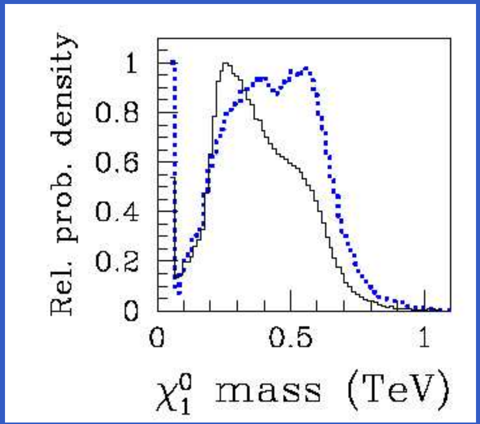
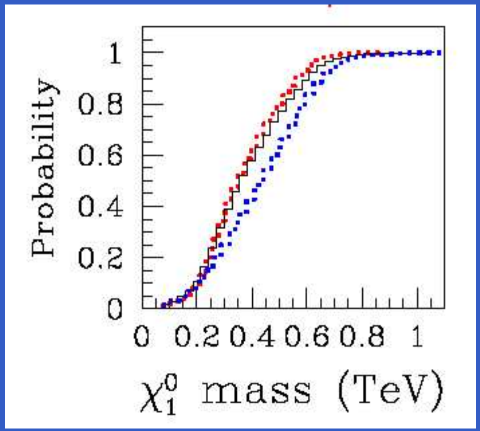
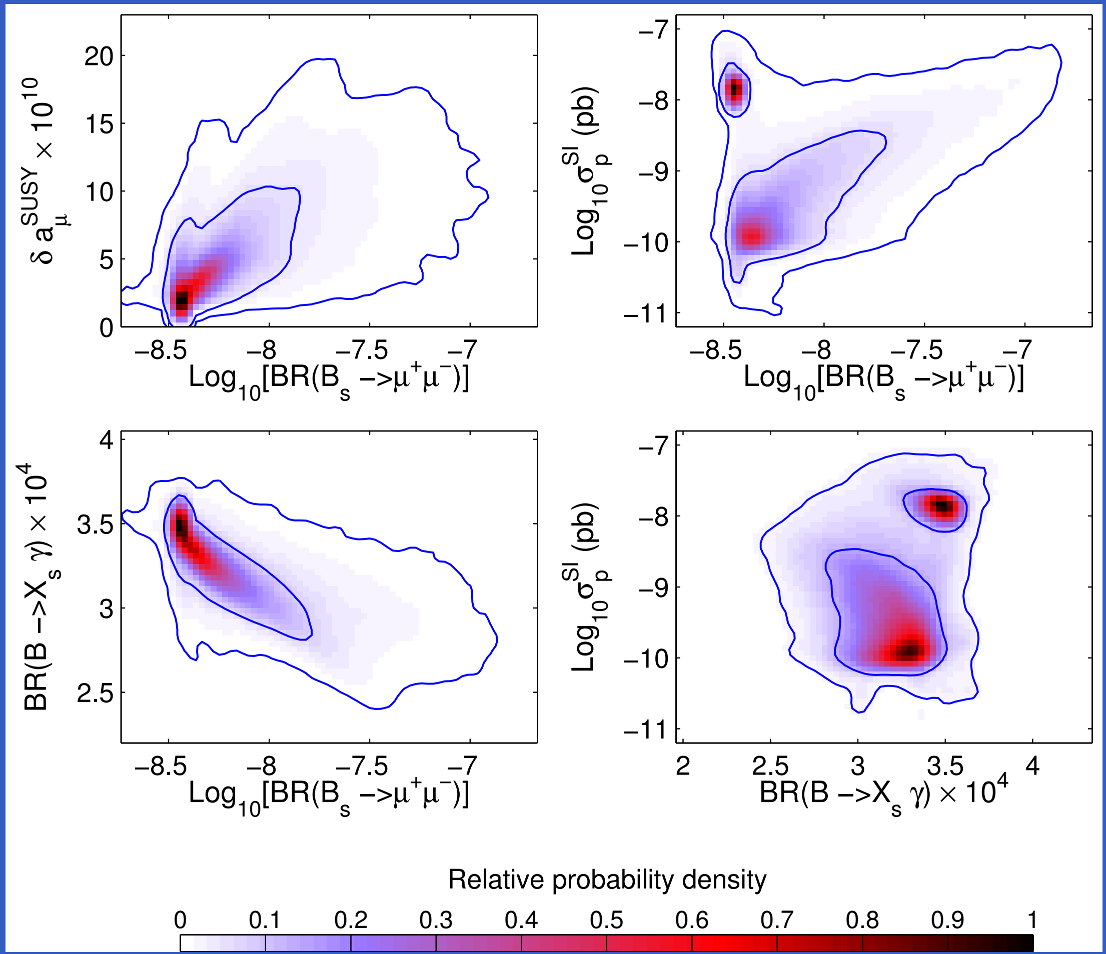


integrated prob.



Some Correlations



new features due to FP

some testable

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...not a time–independent statement