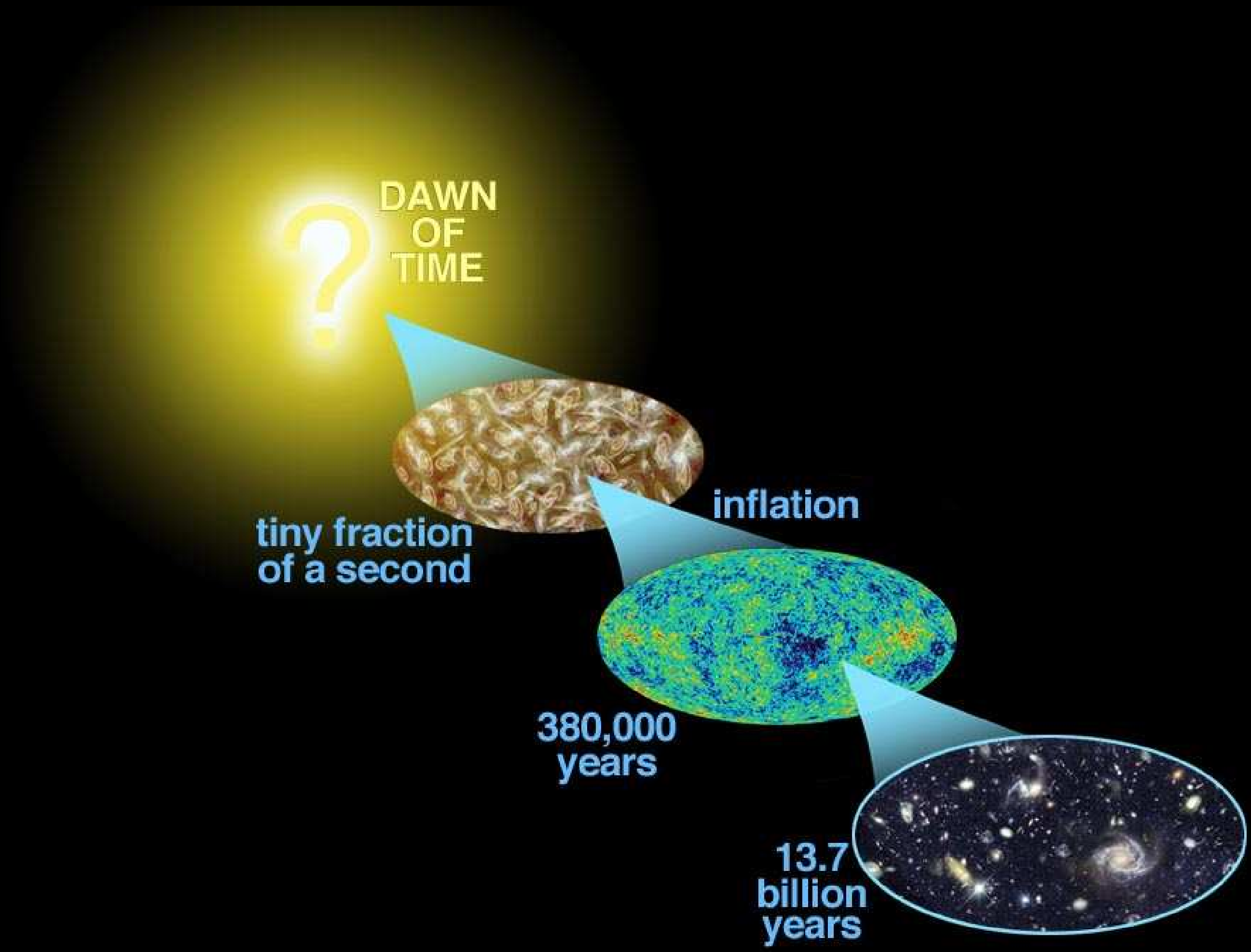
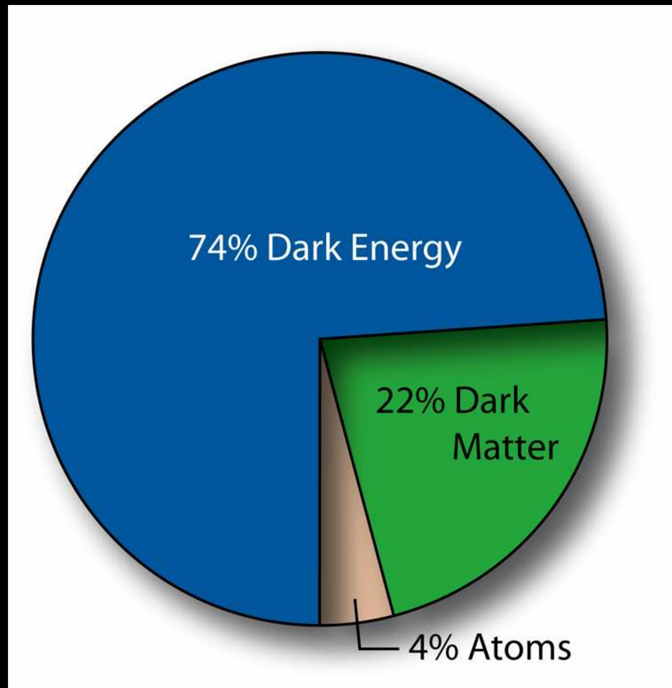
What have we learnt from *WMAP* ?
dark matter versus dark energy





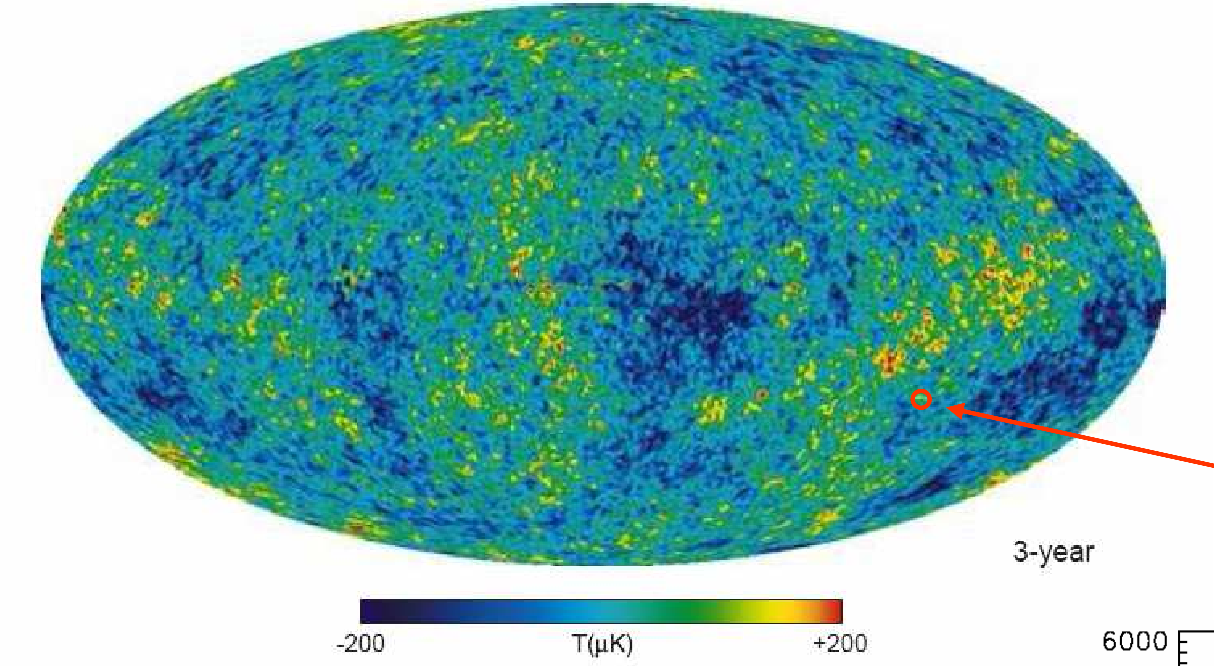
Courtesey: NASA/WMAP Science Team

Subir Sarkar

University of Oxford

The Dark Side of the Universe, Madrid, 20 June 2006

‘Internal Linear Combination’ map (*circa* March 2006)



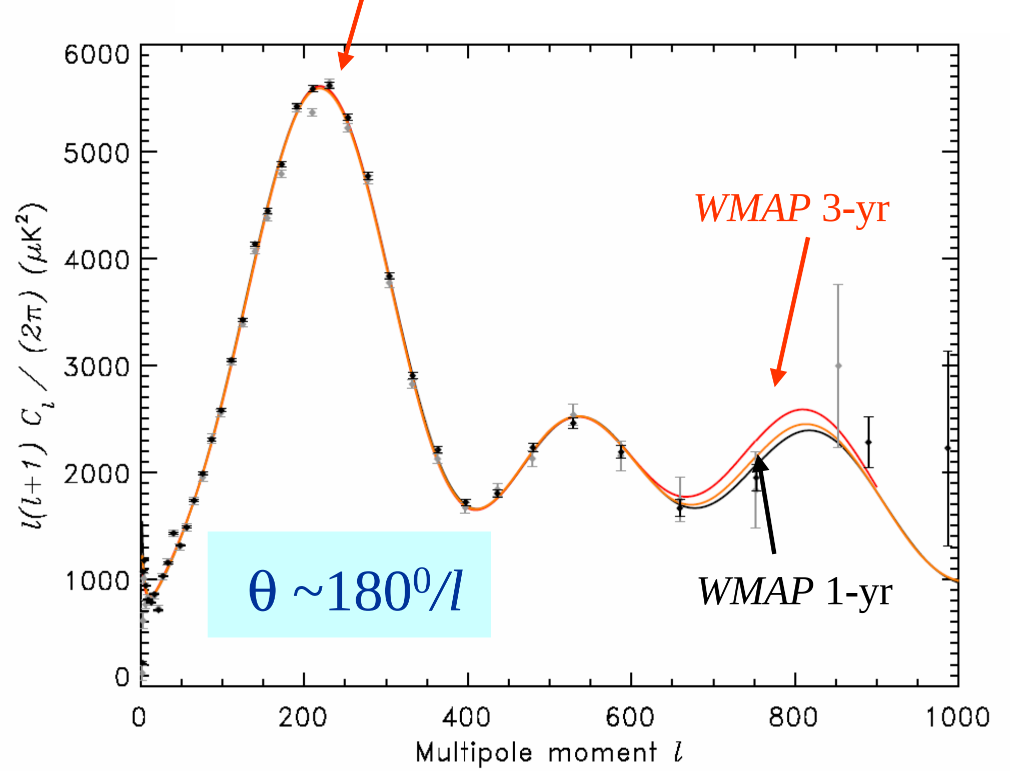
Coherent oscillations in photon-baryon plasma, excited by primordial density perturbations on *super*-horizon scales …

(Hubble radius at $t_{\rm rec}$)

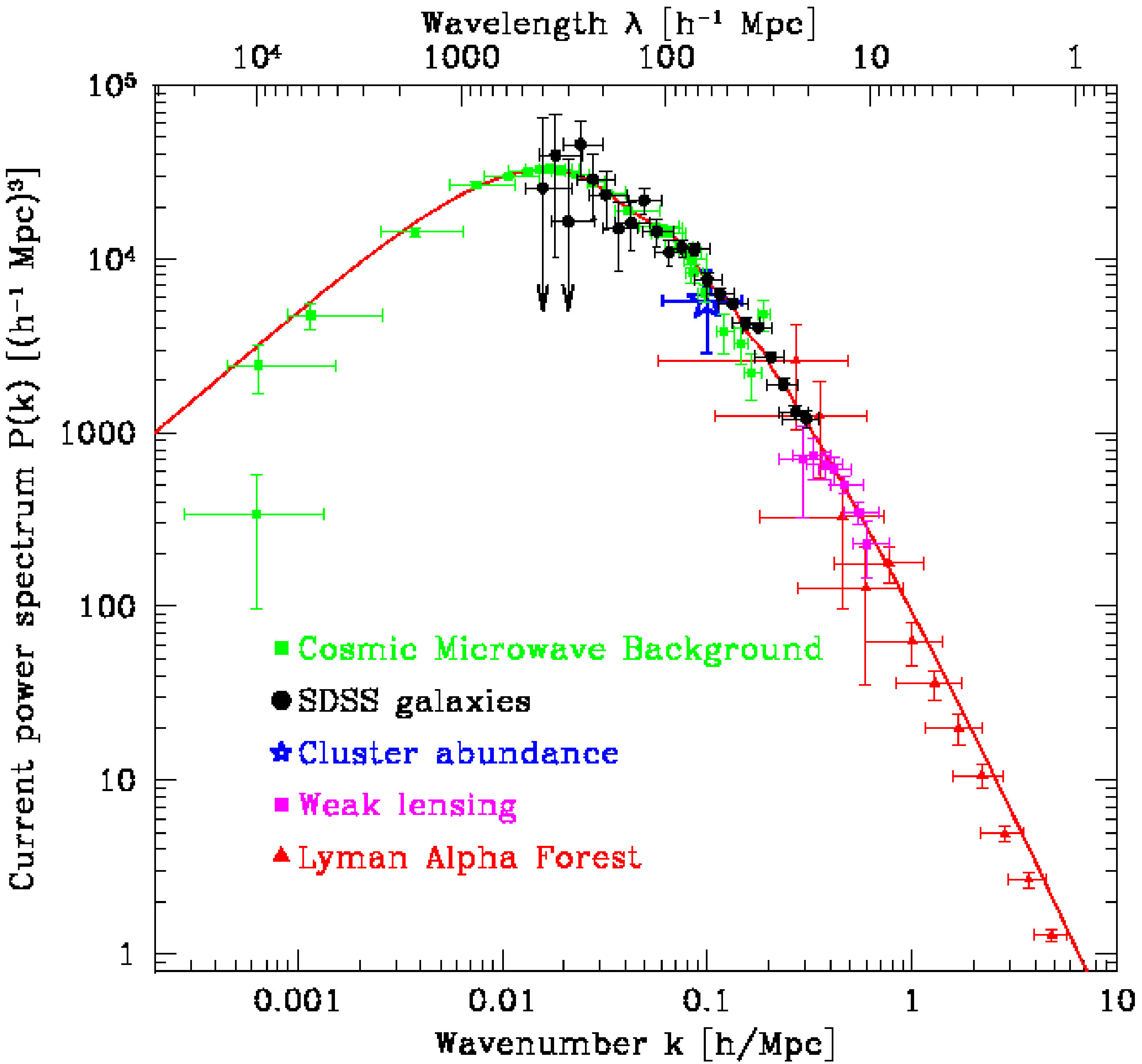
$$\Delta T(\mathbf{n}) = \sum a_{lm} Y_{lm}(\mathbf{n})$$

$$C_l \equiv \frac{1}{2l+1} \sum |a_{lm}|^2$$

***C_l*’s mildly correlated since (due to Galactic foreground) only ~85% of sky can be used**

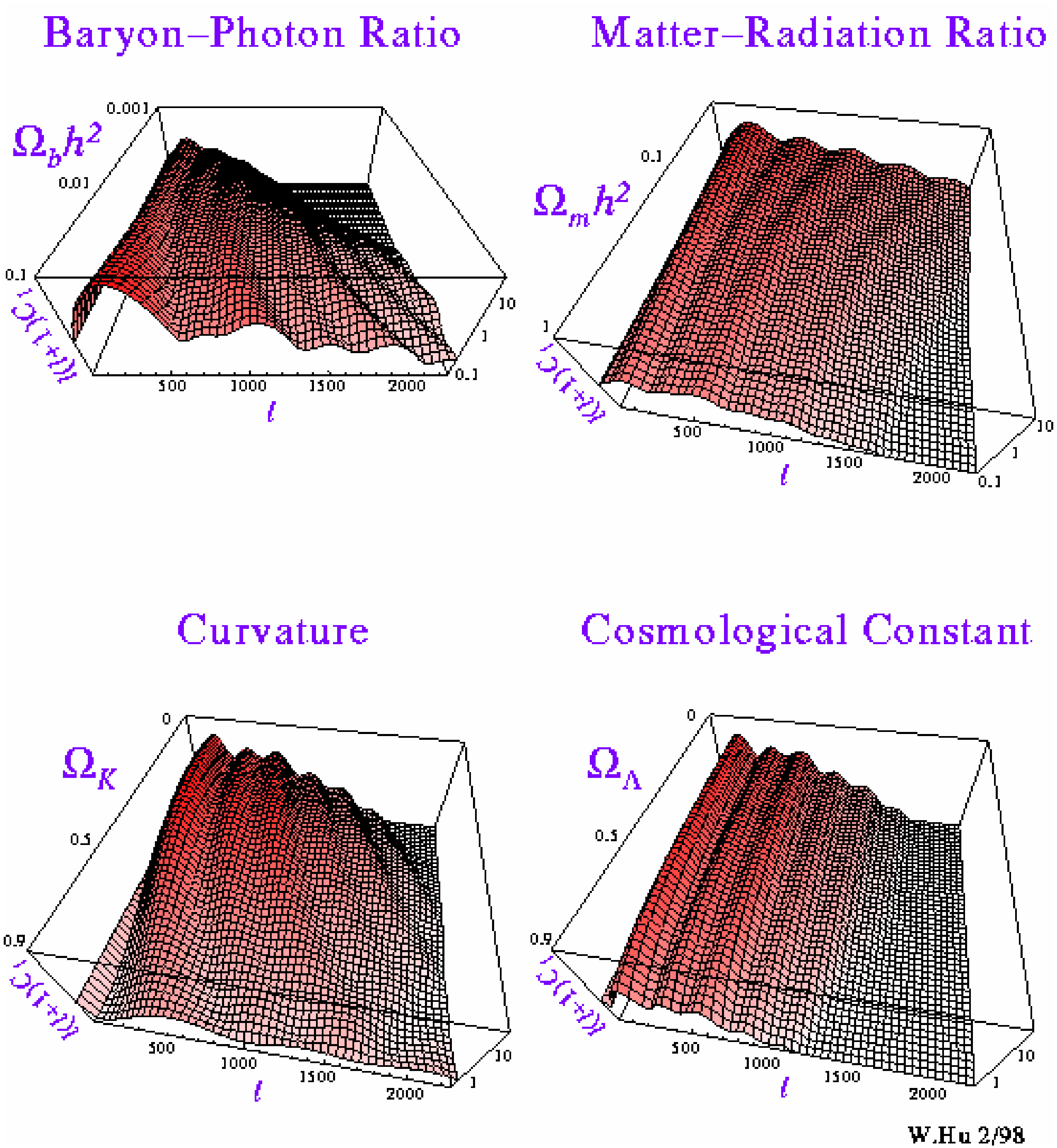


The CMB and large-scale structure data have supposedly established a 'standard model' of cosmology – a spatially flat ΛCDM universe with a ~scale-invariant spectrum of adiabatic primordial density fluctuations



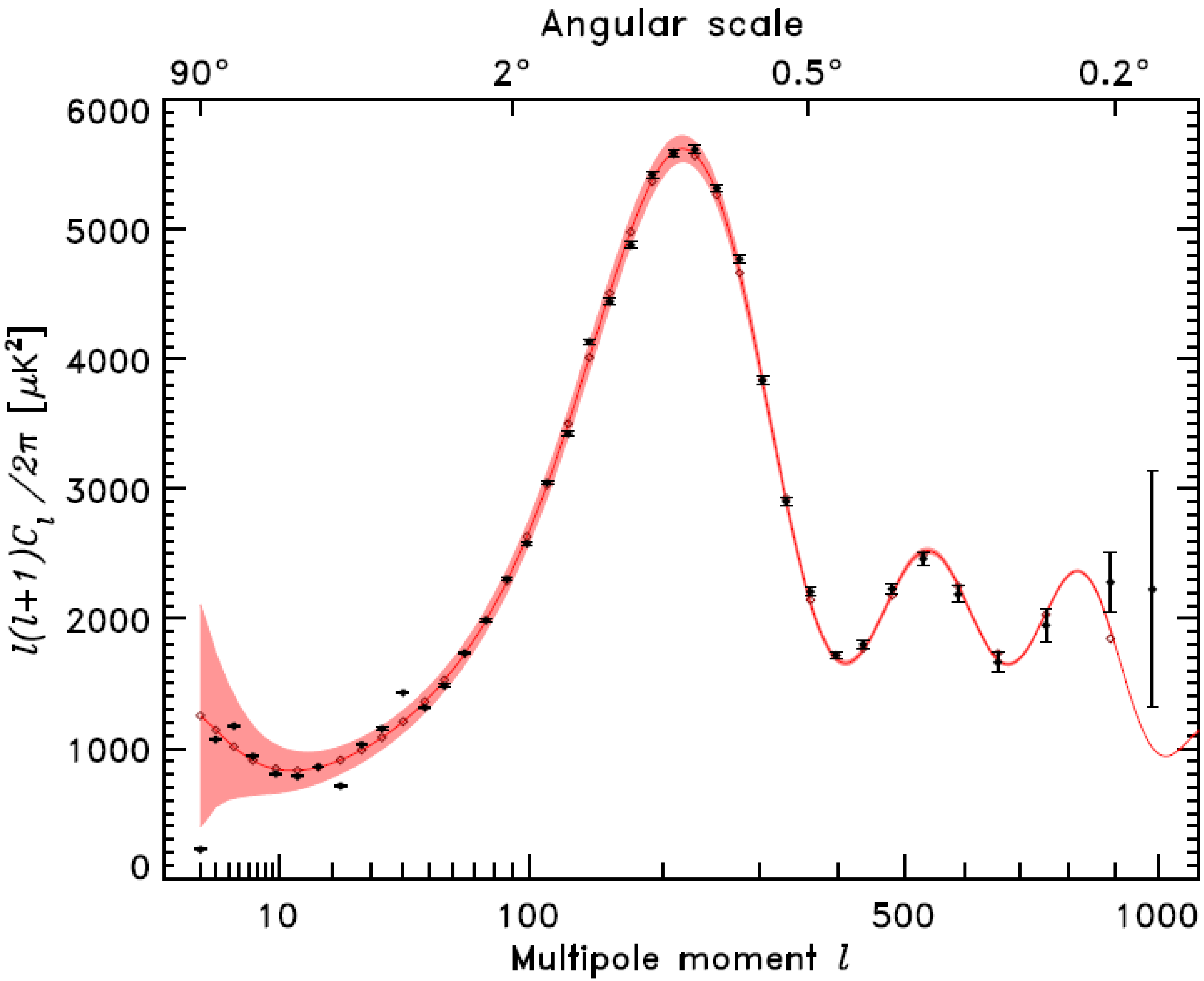
Tegmark. (2004)

Cosmological parameters in the CMB



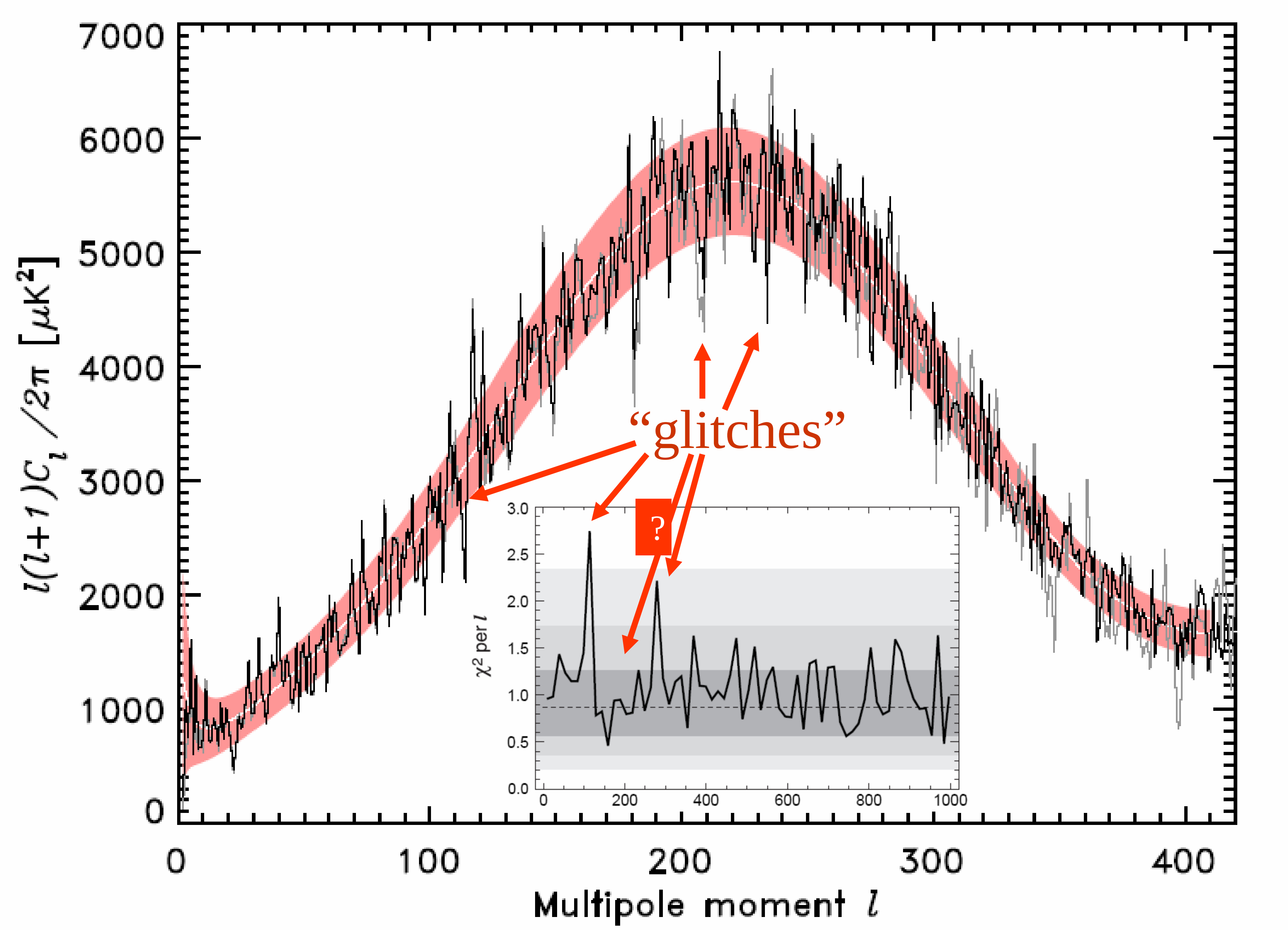
WMAP-3 is supposed to have confirmed the power-law *Λ*CDM model

Best-fit: $\Omega_m h^2 = 0.13 \pm 0.01$, $\Omega_b h^2 = 0.022 \pm 0.001$, $h = 0.73 \pm 0.05$, $n = 0.95 \pm 0.02$

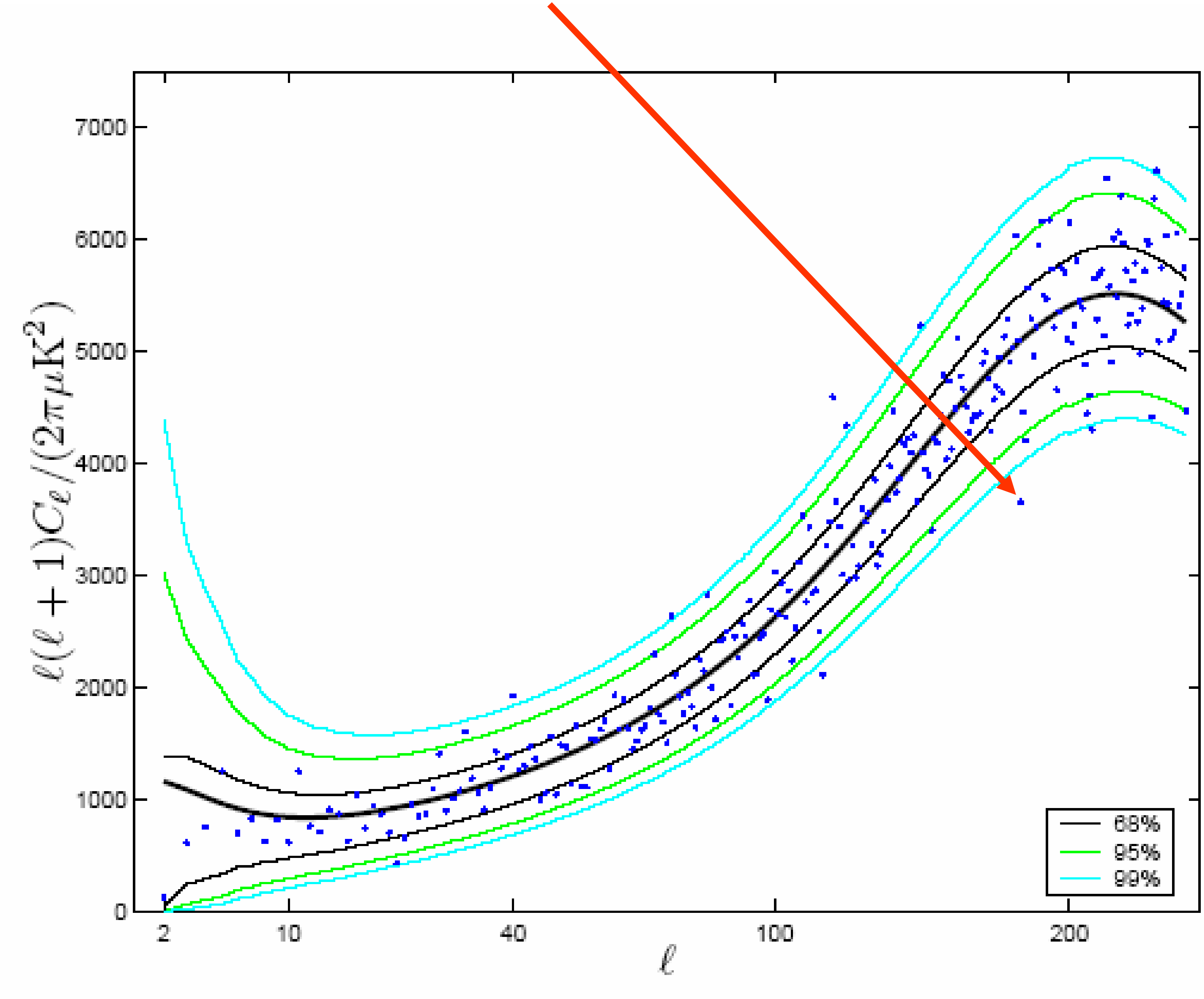


But the χ^2/dof = 1049/982 ⇒ probability of only ~7% that this model is correct!

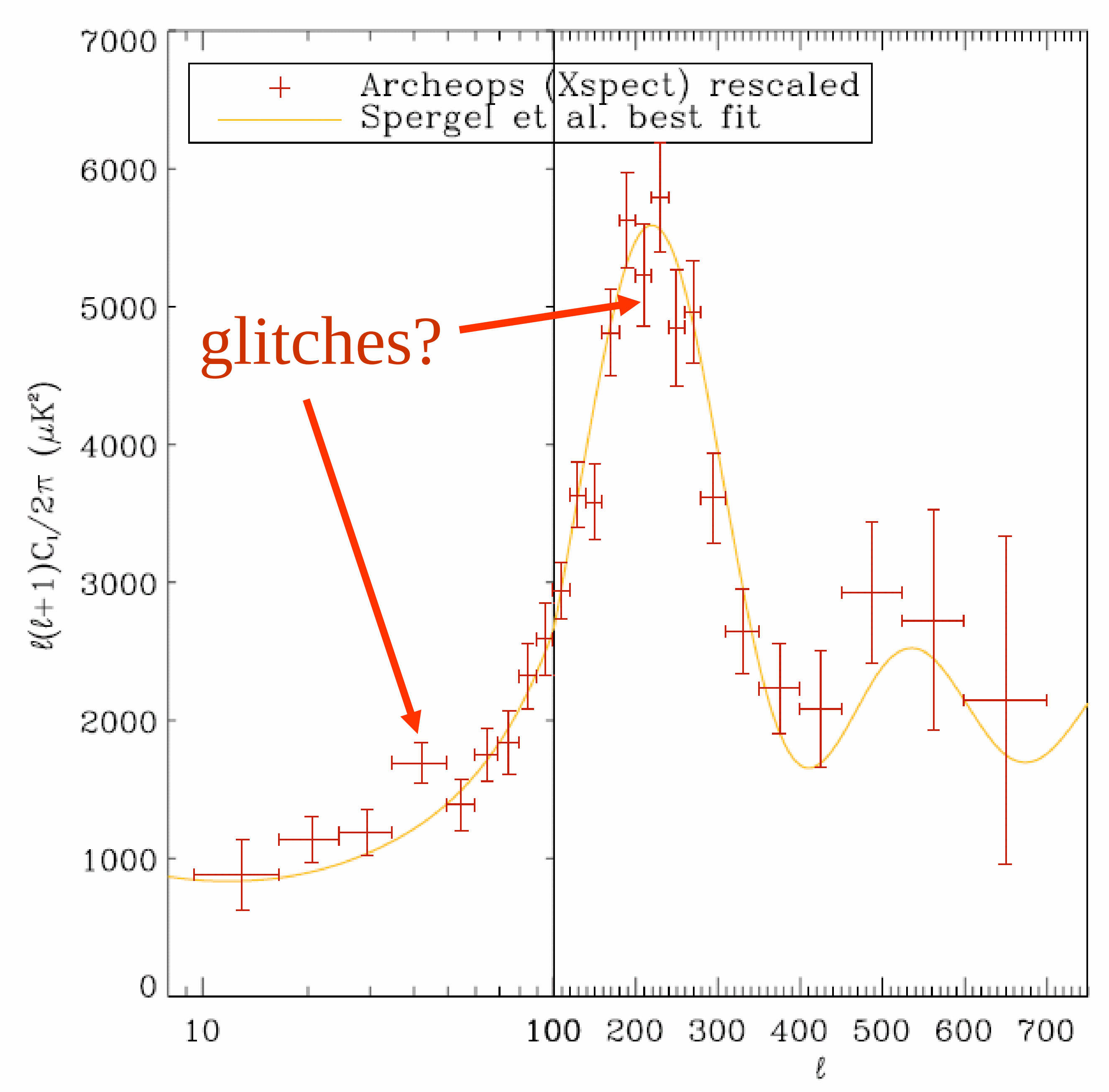
The excess χ^2 comes mostly from the *outliers* in the TT spectrum



WMAP-1: Only 3 out of 16000 simulations would have a lower value of C_{181} than that observed (Lewis 2004)



Similar outliers have been seen by *Archeops* (although less significant)



suggests that the primordial density perturbation is *not* scale-free, as is usually assumed

“In the absence of an established theoretical framework in which to interpret these glitches … they will likely remain curiosities”

Spergel *et al* (2006)

Then why not also say:

“In the absence of an established theoretical framework in which to interpret *dark energy* … the *apparent acceleration of the universe* will likely remain a curiosity”

The formation of large-scale structure is akin to a scattering experiment

The **Beam:** **inflationary density perturbations**

No 'standard model' – usually *assumed* to be adiabatic and ~scale-invariant

The **Target:** **dark matter (+ baryonic matter)**

Identity unknown - usually taken to be cold (sub-dominant 'hot' component?)

The **Detector:** **the universe**

Modelled by a 'simple' FRW cosmology with parameters h, Ω_{CDM} , Ω_b , Ω_Λ , Ω_k ...

The **Signal:** **CMB anisotropy, galaxy clustering …**

measured over scales ranging from ~ 1 – 10000 Mpc ($\Rightarrow$ ~8 e-folds of inflation)

We cannot simultaneously determine the properties of *both* the **beam** *and* the **target** with an unknown **detector**

… hence need to adopt suitable 'priors' on h, Ω_{CDM}, etc
in order to break inevitable parameter *degeneracies*

Astronomers have traditionally *assumed* a Harrison-Zeldovich spectrum:

$$P(k) \propto k^n, \ \ n = 1$$

But models of inflation generally predict departures from scale-invariance

… even in **single-field slow-roll models**: $n = 1 + 2V''/V - 3(V'/V)^2$

Since the potential $V(\phi)$ steepens towards the end of inflation, there will be a *scale-dependent spectral tilt* on cosmologically observable scales:

e.g. in simplest 'new inflation' model: $V(\phi) \simeq V_o - \eta\phi^2 + \ldots \Rightarrow n \simeq 1 - 2/N_* \sim 0.96$

where $N_* \approx 50 + \ln(k^{-1}/3000h^{-1}\ \mathrm{Mpc})$ is the # of e-folds from the *end* of inflation

This agrees with the best-fit value power-law index inferred from the *WMAP* data

In **hybrid models**, inflation is ended by the 'waterfall' field, *not* due to the steepening of $V(\phi)$, so spectrum can be close to scale-invariant …

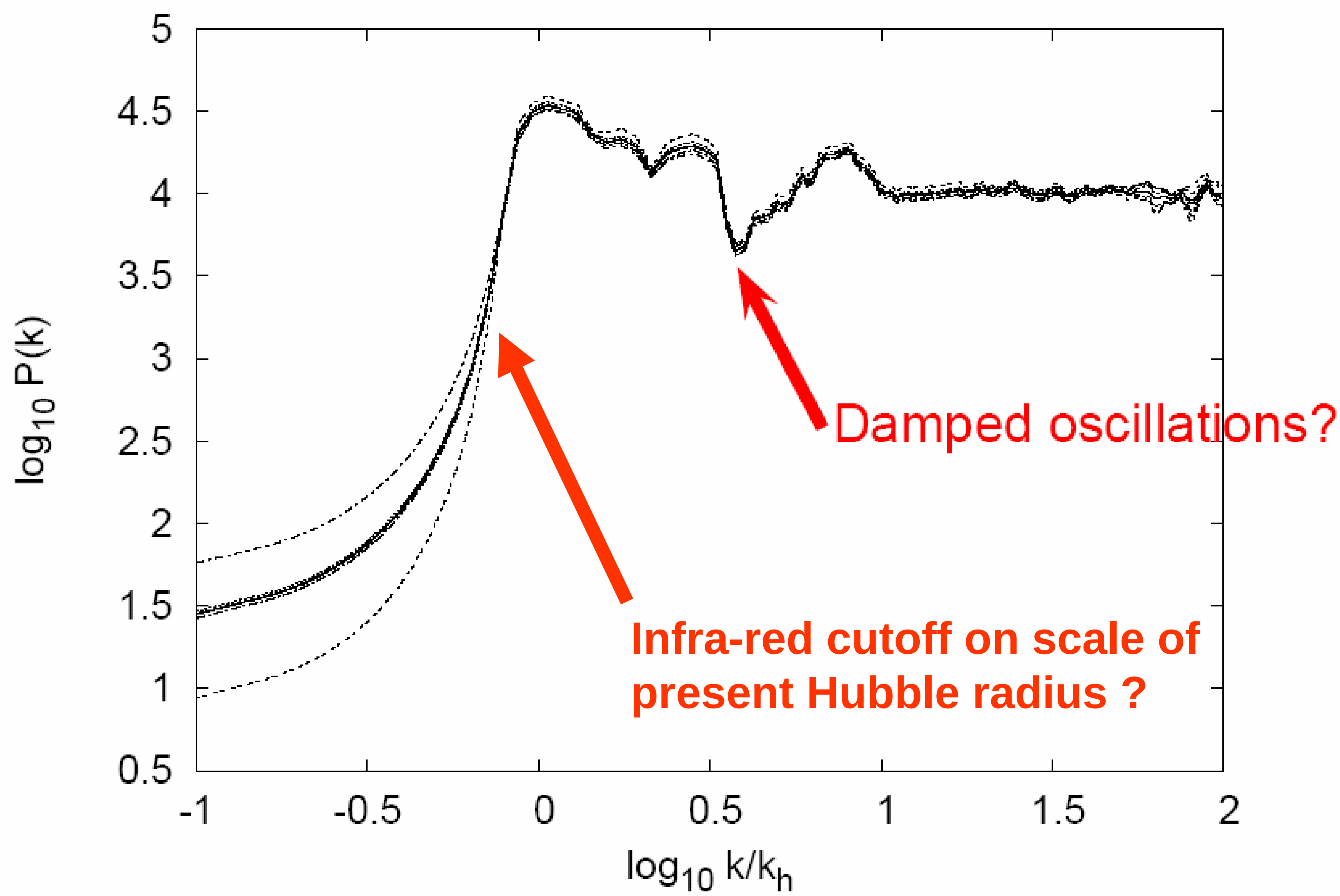
In general there would be *many* other fields present, whose own dynamics may *interrupt* the inflaton's slow-roll evolution (rather than terminate it altogether)

→ can generate features in the spectrum ('steps', 'oscillations', 'bumps' …)

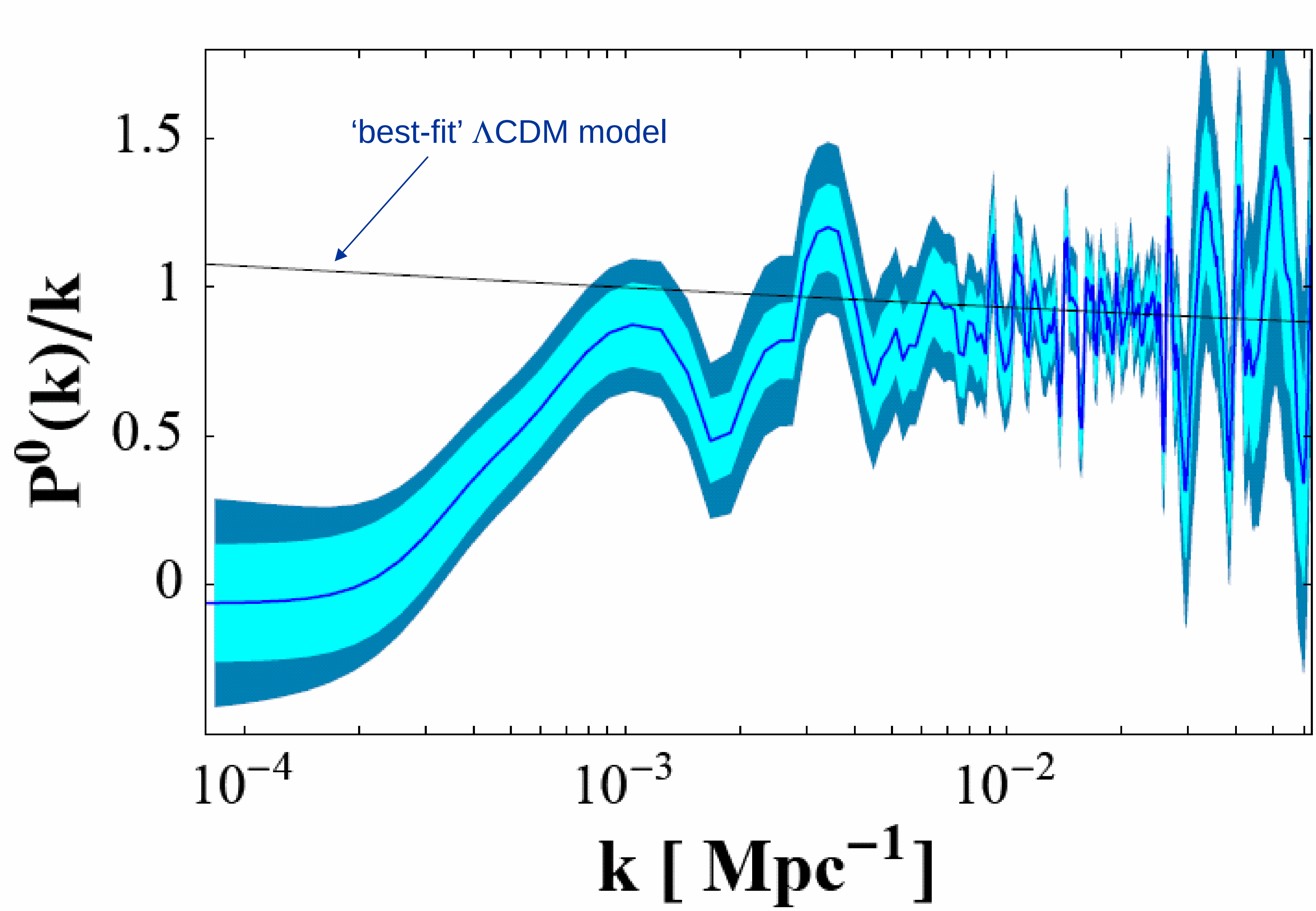
Many attempts made to reconstruct the primordial spectrum (*assuming* ΛCDM)

Bridle, Lewis, Weller & Efstathiou 2003; Cline, Crotty & Lesgourgues 2003, Mukherjee & Wang 2003; Hannestad 2004; Kogo, Sasaki & Yokoyama 2004; Tocchini-Valentini, Douspis & Silk 2004, …

… Richardson-Lucy inversion on *WMAP-1* data yields (Shafieloo & Souradeep 2004):



Similar results obtained by other (*non*-parametric) reconstruction methods

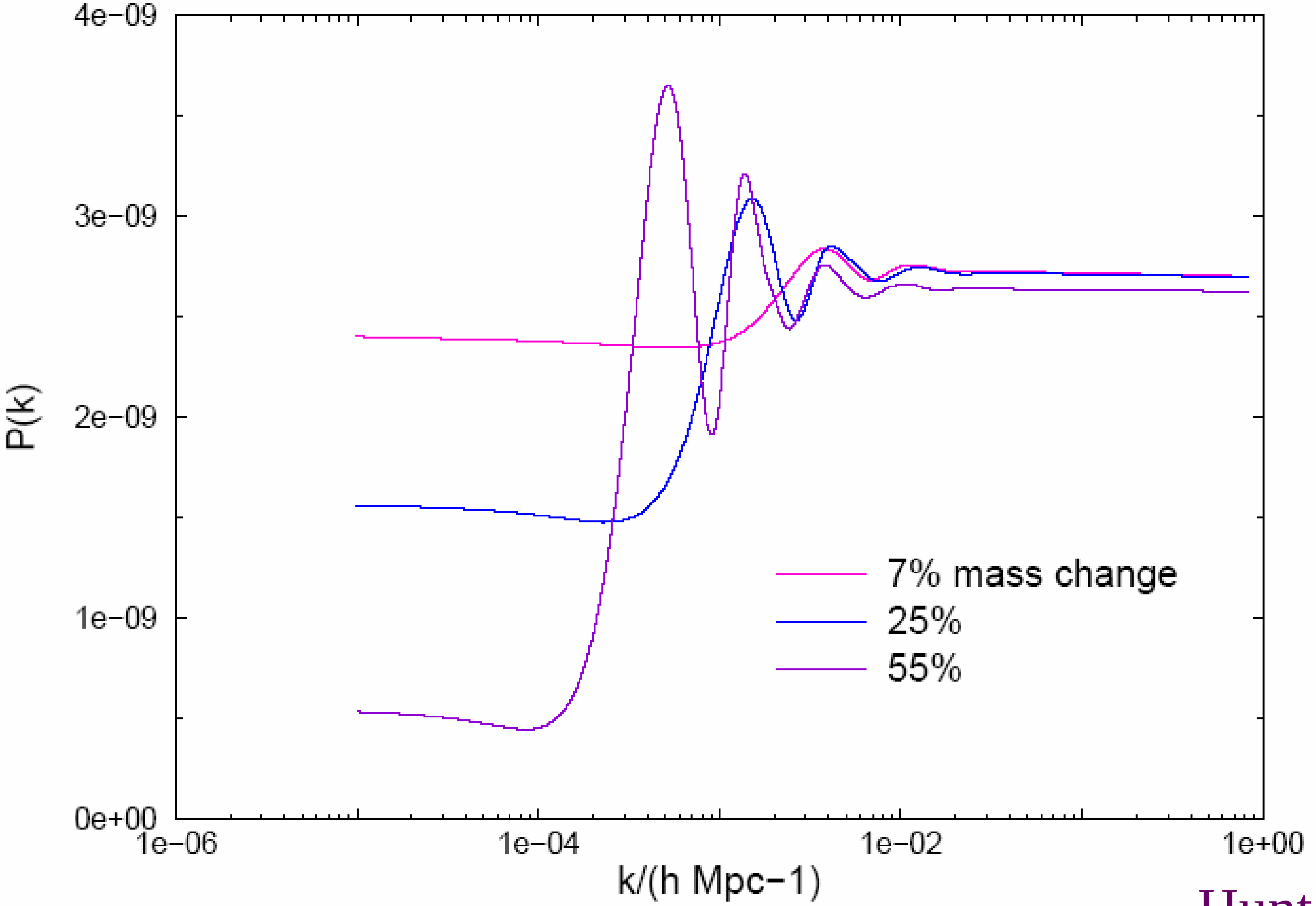


Tochhini-Valentini, Hoffman & Silk (2005)

Such spectra arise *naturally* if the inflaton mass changes suddenly, e.g. due to its coupling (through gravity) to a field which undergoes a fast symmetry-breaking phase transition in the rapidly cooling universe

(Adams, Ross & Sarkar 1997)

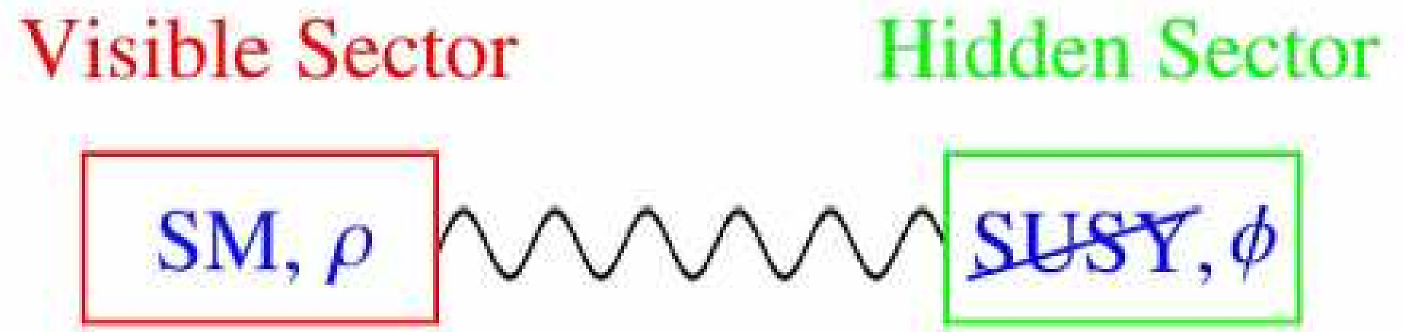
This must happen as cosmologically interesting scales 'exit the horizon' ... likely if (last phase of) inflation did not last longer than ~50-60 e-folds



Hunt & Sarkar (2005)

Consider inflation in context of *effective* field theory: $\mathcal{N}=1$ SUGRA

(successful description of gauge coupling unification, EW symmetry breaking, …)



The visible sector could be important during inflation if gauge symmetry breaking occurs

Supersymmetric theories contain ‘flat directions’ in field space where the potential vanishes in the limit of unbroken SUSY

This is due to various symmetries and non-renormalisation theorems

Flat directions are lifted by

- ~~SUSY~~ .
- Higher dimensional operators $\rho^n/M_\mathrm{P}^{n-4}$ which appear after integrating out heavy degrees of freedom

These fields acquire a large mass during inflation, thus perturbing the inflaton …

For canonically normalised fields with

$$K = \sum_{\alpha} |\phi_\alpha|^2$$

the SUGRA scalar potential is

$$\begin{aligned}
V_{\mathrm{SUGRA}} &= \exp\left(\frac{K}{M_{\mathrm{P}}^2}\right)\left[\sum_{\alpha,\beta}\left(\frac{\partial^2 K}{\partial\overline{\phi}_\alpha \partial\phi_\beta}\right)^{-1}\left(\frac{\partial W}{\partial\phi_\alpha} + \frac{W}{M_{\mathrm{P}}^2}\frac{\partial K}{\partial\phi_\alpha}\right)\right. \\
&\quad \left. \times\left(\frac{\partial\overline{W}}{\partial\overline{\phi}_\beta} + \frac{\overline{W}}{M_{\mathrm{P}}^2}\frac{\partial K}{\partial\overline{\phi}_\beta}\right) - \frac{3}{M_{\mathrm{P}}^2}|W|^2\right] + \text{D-terms} \\
&= \exp\left(\frac{1}{M_{\mathrm{P}}^2}\sum_{\gamma}|\phi_\gamma|^2 + \dots\right)\left\{\sum_{\alpha,\beta}(\delta_{\alpha\beta} + \dots)\right. \\
&\quad \times\left[\frac{\partial W}{\partial\phi_\alpha} + \frac{W}{M_{\mathrm{P}}^2}(\overline{\phi}_\alpha + \dots)\right]\left[\frac{\partial\overline{W}}{\partial\overline{\phi}_\beta} + \frac{\overline{W}}{M_{\mathrm{P}}^2}(\phi_\beta + \dots)\right] \\
&\quad \left. - \frac{3}{M_{\mathrm{P}}^2}|W|^2\right\} \\
&\simeq V_{\mathrm{global}} \pm \frac{V_{\mathrm{global}}}{M_{\mathrm{P}}^2}\sum_{\alpha}|\phi_\alpha|^2, \quad V_{\mathrm{global}} = \sum_{\alpha}\left|\frac{\partial W}{\partial\phi_\alpha}\right|^2
\end{aligned}$$

i.e. there is a contribution of $\pm H^2$ to the mass-squared of all scalar fields

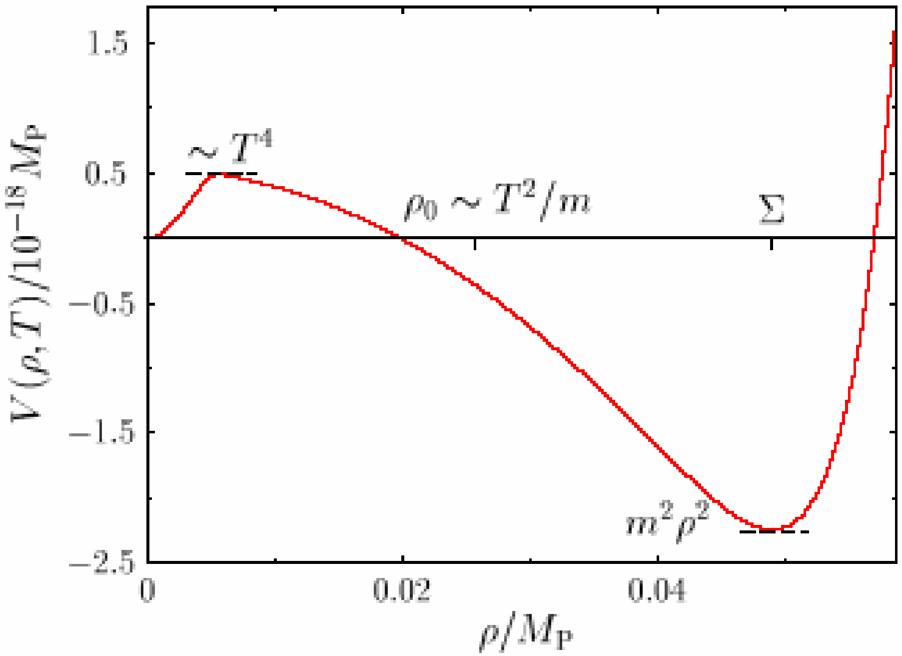
If m^2 is negative, ρ is stabilised at $\Sigma \sim \mathcal{O}\left(M_{\rm P}^2 \left|m^2\right|\right)^{1/(n-4)}$, by $\rho^n/M_{\rm P}^{n-4}$ terms

Assume that in the era preceding observable inflation, all fields (with gauge and/or Yukawa couplings) are in thermal equilibrium

Including the one-loop finite temperature correction

$$V(\rho,T) \quad \simeq \quad \begin{cases} C_1 T^2 \rho^2, & \text{for} \quad \rho \ll T \\ -m^2\rho^2 + \frac{1}{90}\pi^2 N_{\rm h}(T) T^4 + \frac{\gamma \rho^n}{M_{\rm P}^{n-4}}, & \text{for} \quad T \ll \rho < \Sigma \end{cases}$$

Here $N_{\rm h}(T)$ is the number of helicity states with mass much less than the temperature

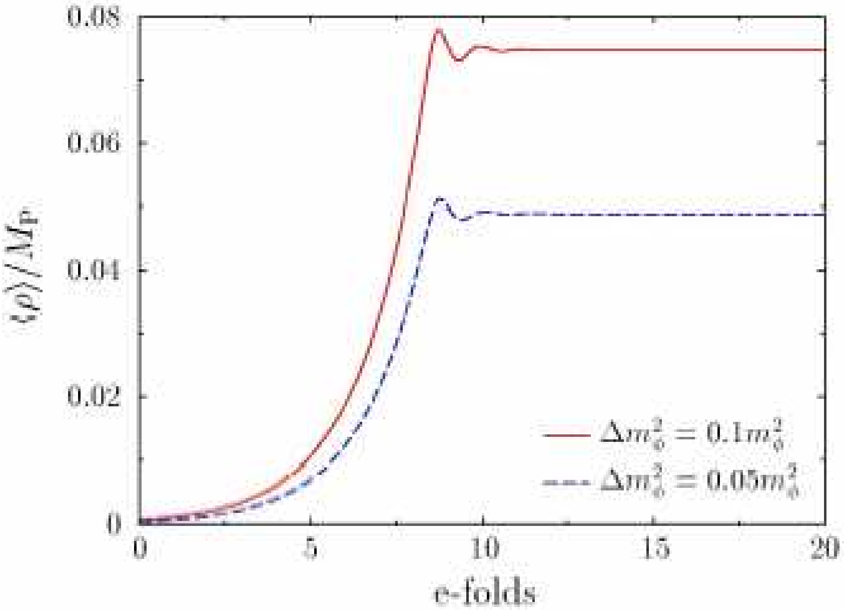


The tunneling rate through the thermal barrier between $\rho = 0$ and $\rho \sim T^2/m$ is negligible, so $\rho = 0$ until $T \sim m$ when the barrier disappears (Yamamoto 1985)

ρ evolves to the global minimum at Σ as

$$\ddot{\rho} + 3H\dot{\rho} = -\frac{dV}{d\rho}$$

$$\Rightarrow \rho \quad \simeq \quad \begin{cases} \rho_0 \exp\left[\frac{3Ht}{2}\left(\sqrt{1+\frac{8m^2}{9H^2}}\right) - 1\right], & \langle\rho\rangle \ll \Sigma \\ \Sigma + K_1 \exp\left(-\frac{3Ht}{2}\right) \sin\left[\frac{3Ht}{2}\sqrt{(n-2)\frac{8m^2}{9H^2} - 1} + K_2\right], & \langle\rho\rangle \sim \Sigma \end{cases}$$



After the phase transition slow-roll inflation continues but at a reduced scale

$$V(\phi) \quad \rightarrow \quad \left[1 - (\Sigma/M_{\mathrm{P}})^2\right] V(\phi)$$

For $\Sigma \ll M_{\mathrm{P}}$ the change is negligible and so H can be taken to be sensibly constant

However ρ and ϕ are coupled by gravity.

Then with $K \subset \kappa\phi\phi^\dagger\rho^2/M_{\rm P}^2$ for example

$$V(\phi,\rho) = V_0 - \frac{1}{2}m^2\phi^2 - \frac{1}{2}\mu^2\rho^2 + \frac{1}{2}\lambda\phi^2\rho^2 + \frac{\gamma}{M_{\rm P}^{n-4}}\rho^n + \ldots, \qquad \lambda = \frac{\kappa H^2}{M_{\rm P}^2},$$

$\Rightarrow$ change in inflaton effective mass-squared $m_\phi^2 \equiv {\rm d}^2V/{\rm d}\phi^2$

$$m_\phi^2 = -m^2 \quad \rightarrow \quad m_\phi^2 = -m^2 + \lambda\Sigma^2, \quad \Sigma \simeq \left(\frac{2m^2 M_{\rm P}^{n-4}}{n\gamma}\right)^{1/(n-2)}.$$

Phase transition must occur as cosmological scales are leaving the horizon for its effects to be observable (eg in LSS or CMB).

But we expect many flat directions which each cause a phase transition at a different temperature

$\Rightarrow$ increased likelihood that one will be observed.

This assumes that the **initial conditions are thermal** (so ρ starts at the origin) and (this *last* phase of) **inflation lasts just ~50 e-folds** so as to create our present Hubble volume

may seem fine-tuned but the data *does* indicate an **IR cutoff** at the present Hubble radius!

The Spectrum

Metric describing scalar perturbations in a flat universe can be written as

$$\mathrm{d}s^2 = a^2\left[(1+2A_\mathrm{s})\,\mathrm{d}\eta^2 - 2\partial_i B_\mathrm{s}\mathrm{d}\eta\mathrm{d}x^i - \{(1-2D_\mathrm{s})\,\delta_{ij} + 2\partial_i\partial_j E_\mathrm{s}\}\,\mathrm{d}x^i\mathrm{d}x^j\right].$$

Use Sasaki-Mukhanov variable

$$u = a\left(\delta\phi + H\frac{D_\mathrm{s}}{\dot\phi}\right) = -z\mathcal{R}, \quad z = \frac{a\dot\phi}{H}, \quad \mathcal{R} = D_\mathrm{s} + H\frac{\delta\phi}{\dot\phi}.$$

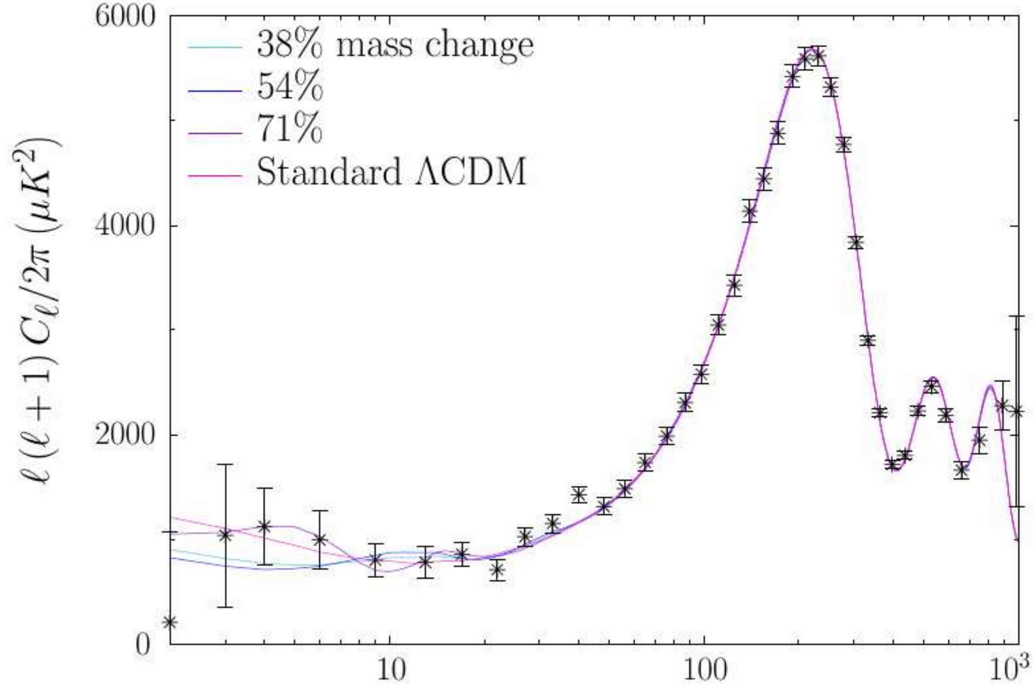
Fourier components of u satisfy

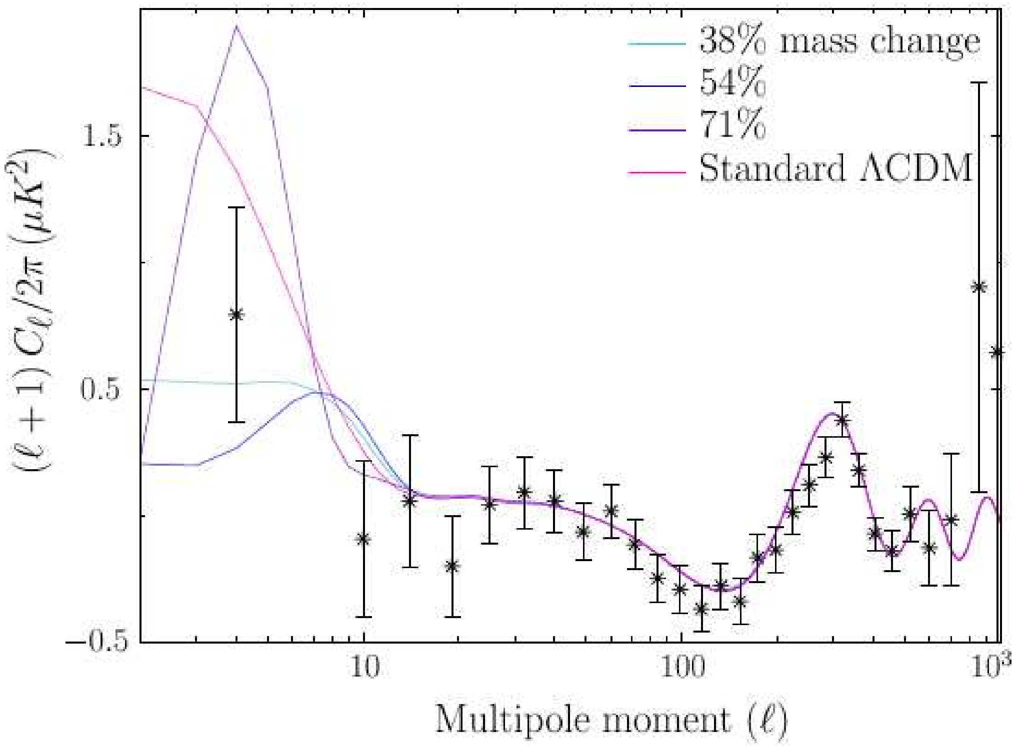
$$u_k'' + \left(k^2 - \frac{z''}{z}\right)u_k = 0, \quad \frac{z''}{z} = a^2\left(2H^2 + m^2 - \lambda\rho^2 - \frac{2\lambda\rho\dot\rho\phi}{\dot\phi}\right).$$

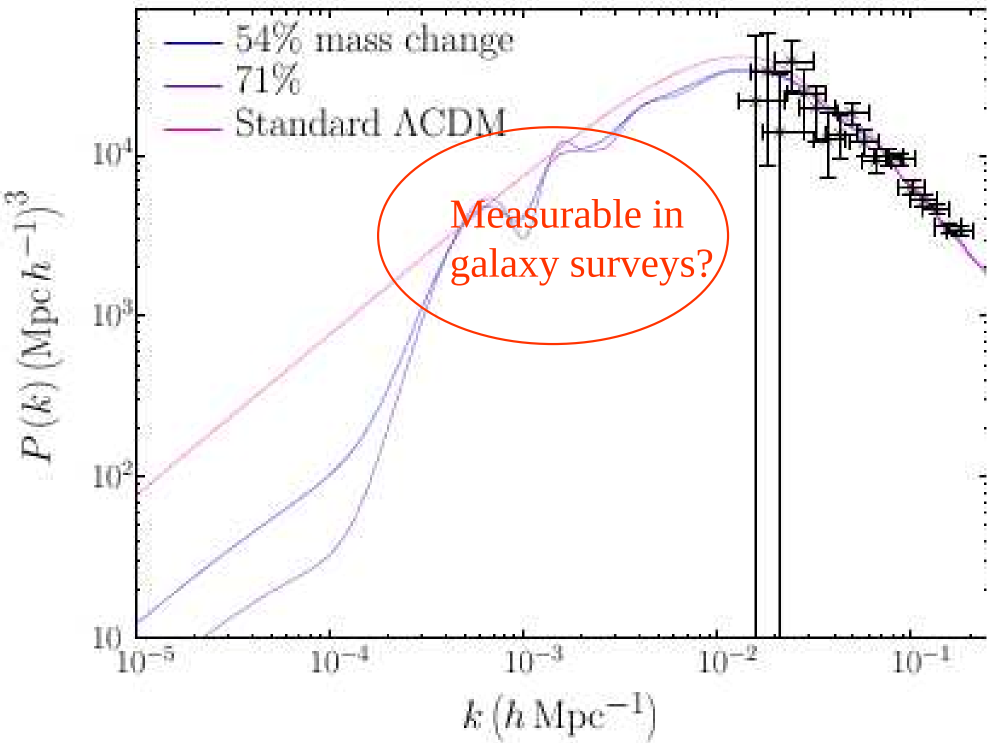
Spectrum is given by

$$\mathcal{P_R}^{1/2} = \sqrt{\frac{k^3}{2\pi^2}}\,|\mathcal{R}_k| = \sqrt{\frac{k^3}{2\pi^2}}\left|\frac{u_k}{z}\right|.$$

Use WKB method (Martin & Schwarz 2003) to obtain P_R when slow-roll is violated …







Fits are all acceptable but parameters change little except for large-scale amplitude

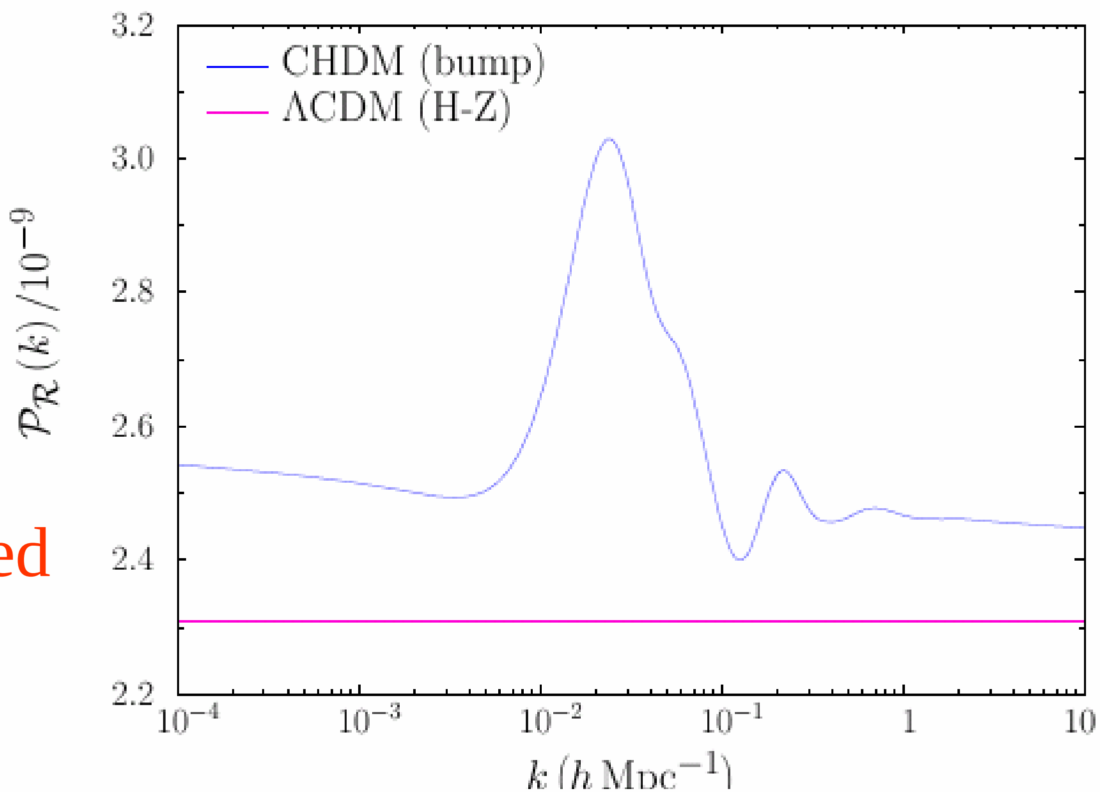
n	χ^2	$\frac{\Delta m_\phi^2}{m^2}$	$\Omega_b h^2$	$\Omega_c h^2$	H_0	τ	$10^4 k_0$	$10^{10} A_s$
15	5628.9	0.38	0.0237 $\pm$0.0010	0.0982 $\pm$0.0204	78.9 $\pm$8.1	0.150 $\pm$0.076	8.04 $\pm$5.84	9.54 $\pm$1.09
16	5629.4	0.54	0.0236 $\pm$0.0011	0.0992 $\pm$0.0217	78.8 $\pm$8.5	0.150 $\pm$0.075	7.89 $\pm$5.16	5.23 $\pm$0.49
17	5629.6	0.71	0.0238 $\pm$0.0011	0.1010 $\pm$0.0233	78.0 $\pm$9.2	0.131 $\pm$0.075	3.62 $\pm$4.74	2.21 0.20

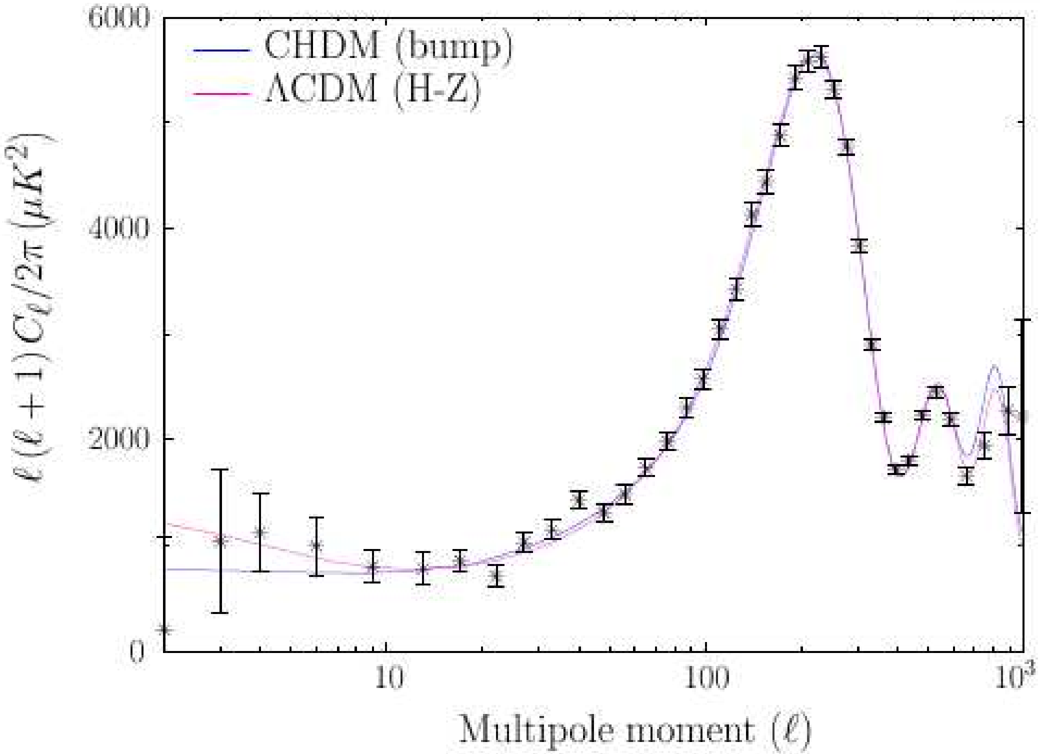
Hunt & Sarkar (2006)

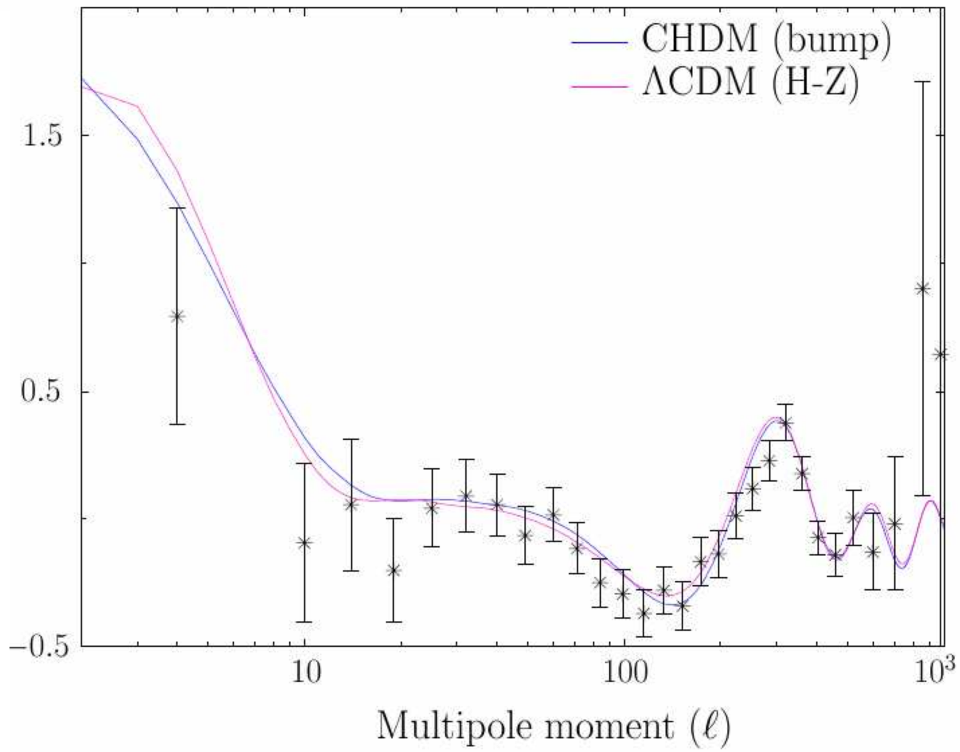
But if there are *many* flat direction fields, then two phase transitions may occur in quick succession, creating a ‘bump’ in the primordial spectrum on cosmologically relevant scales

The *WMAP* data can then be fitted with ***no need for dark energy***

($\Omega_m = 1$, $\Omega_\Lambda = 0$, $h = 0.46$)



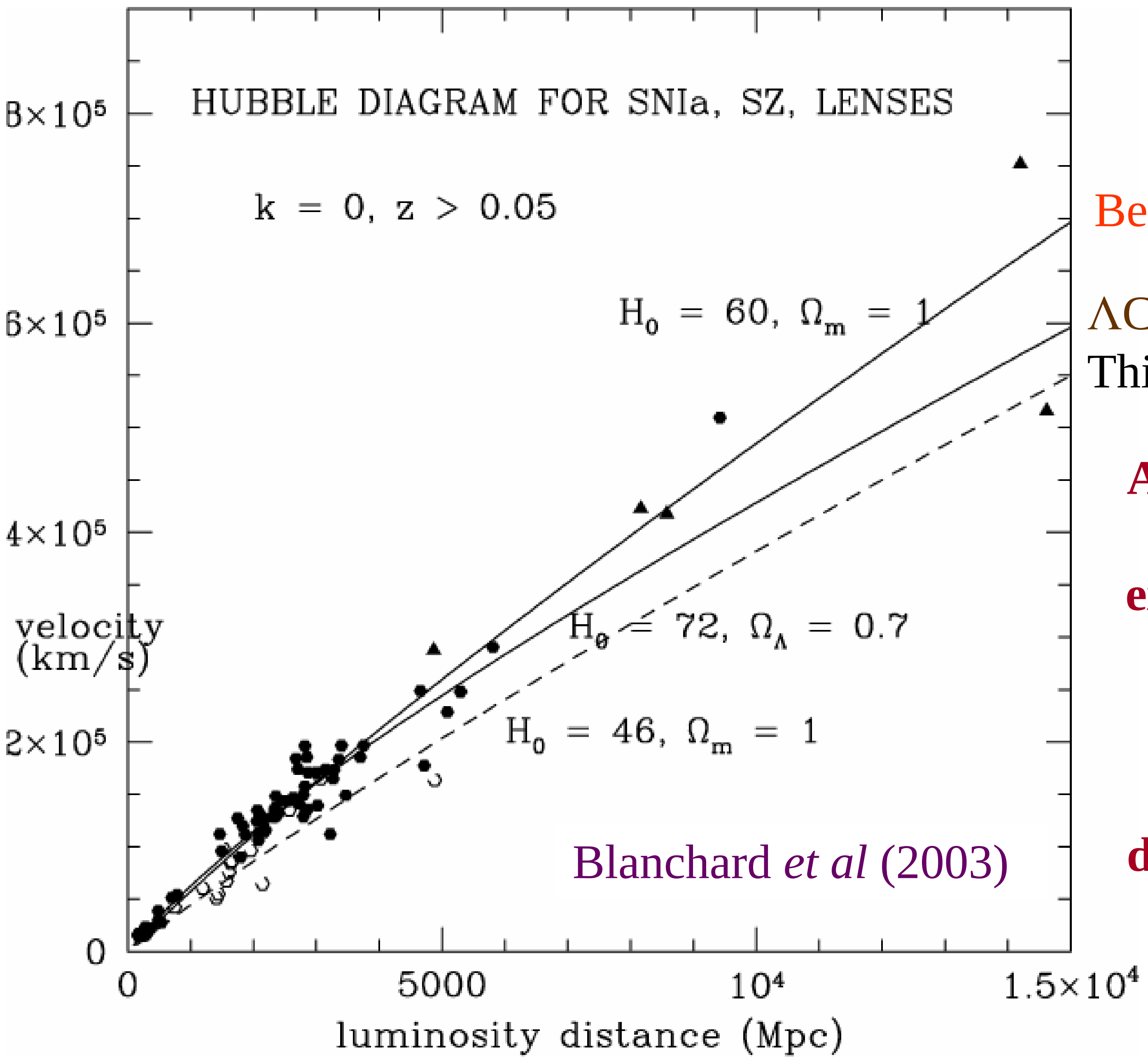




h = 0.46 is inconsistent with Hubble Key Project value (h = 0.72 ± 0.08)

but is in fact *indicated* by direct (and much deeper) determinations

e.g. gravitational lens time delays (h = 0.48 ± 0.03 ± ?)



Best fit

ΛCDM model

This model

Are we in a local void which is expanding faster than average?

… may explain SNIa Hubble diagram *without* acceleration!

… uncertainty in Hubble parameter determination comes from lens model

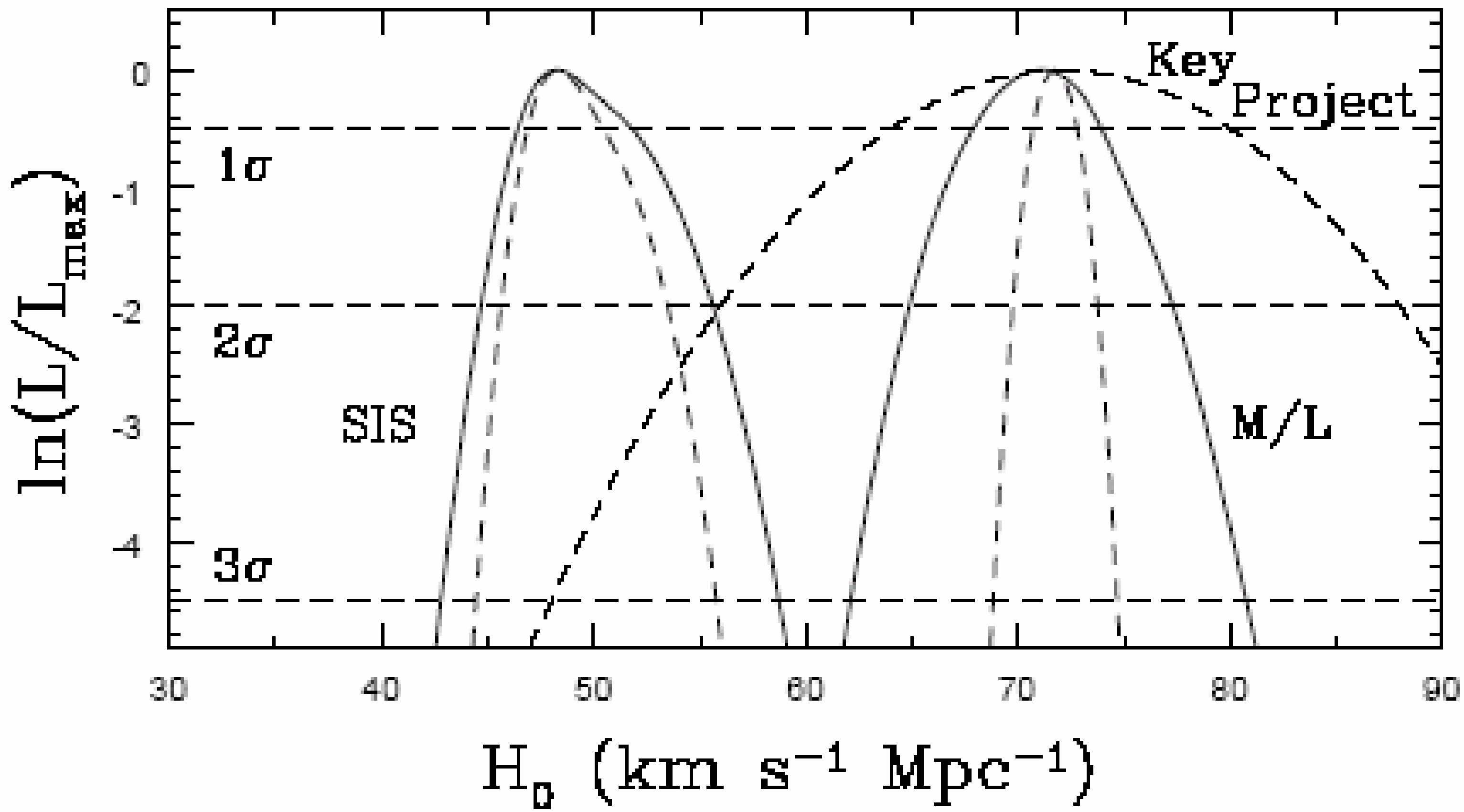
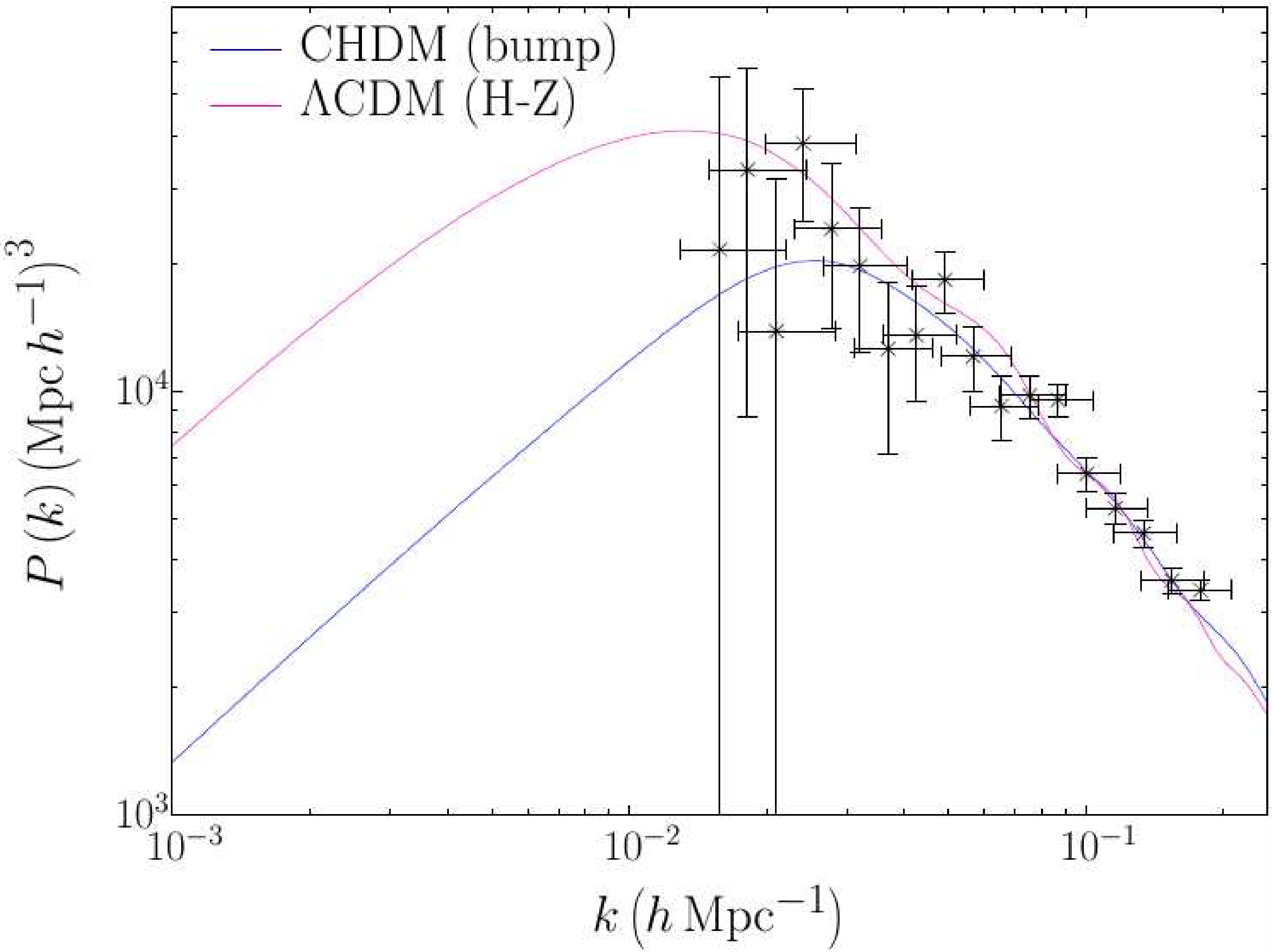


Fig. 1.4. H_0 likelihood distributions. The curves show the joint likelihood functions for H_0 using the four simple lenses PG1115+080, SBS1520+530, B1600+434, and HE2149–2745 and assuming either an SIS model (high $\langle\kappa\rangle$, flat rotation curve) or a constant M/L model (low $\langle\kappa\rangle$, declining rotation curve). The heavy dashed curves show the consequence of including the X-ray time delay for PG1115+080 from Chartas (2003) in the models. The light dashed curve shows a Gaussian model for the Key Project result that $H_0 = 72 \pm 8$ km s^{-1} Mpc^{-1}.

If lensing galaxies have *dark matter halos* then *h* ≈ 0.5 (Kochanek & Schechter 2004)

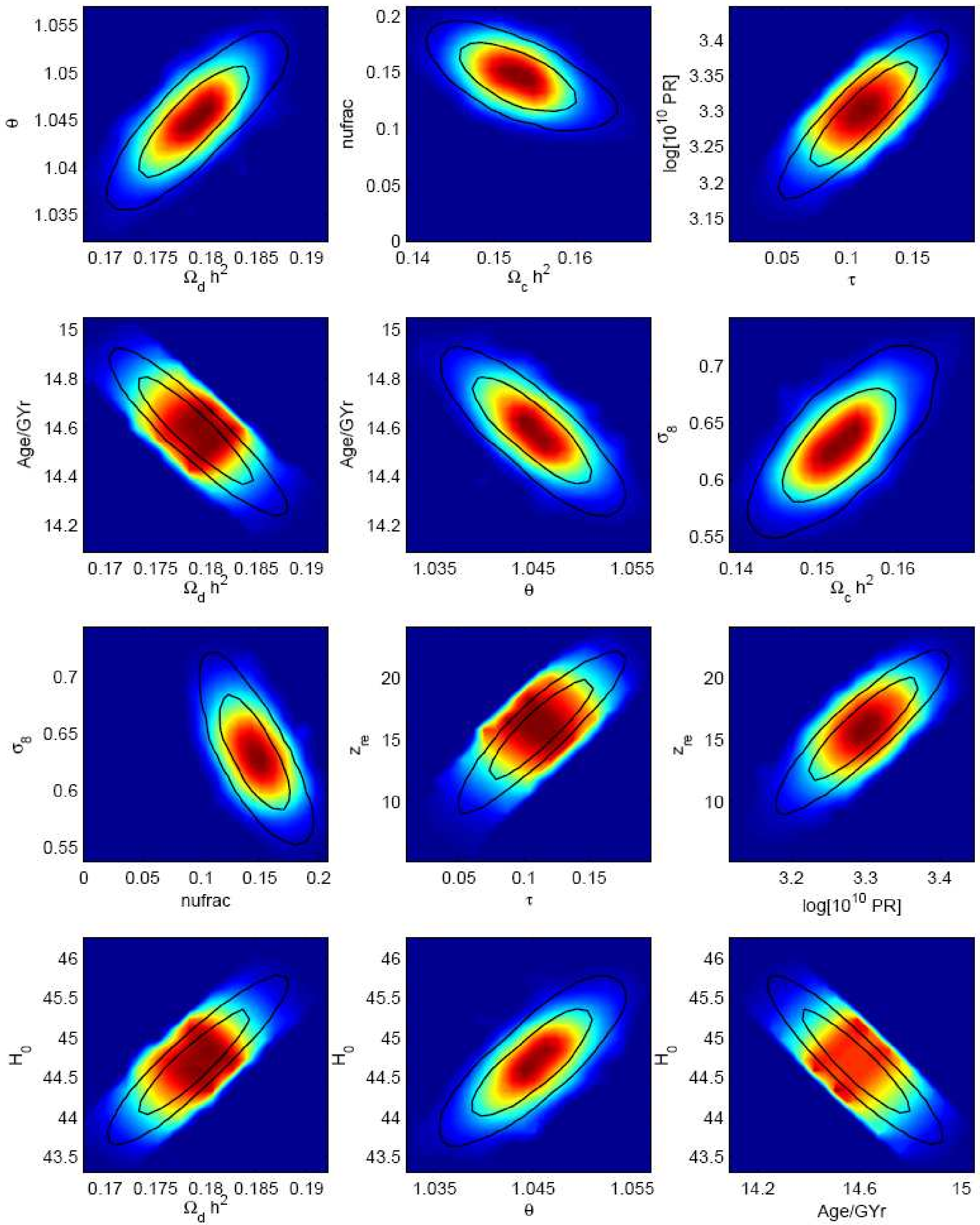
Fits to the *SDSS* galaxy power spectrum



Adding 3 νs of mass 0.8 eV ($\Rightarrow \Omega_\nu \approx 0.14$) gives *good* match to large-scale structure

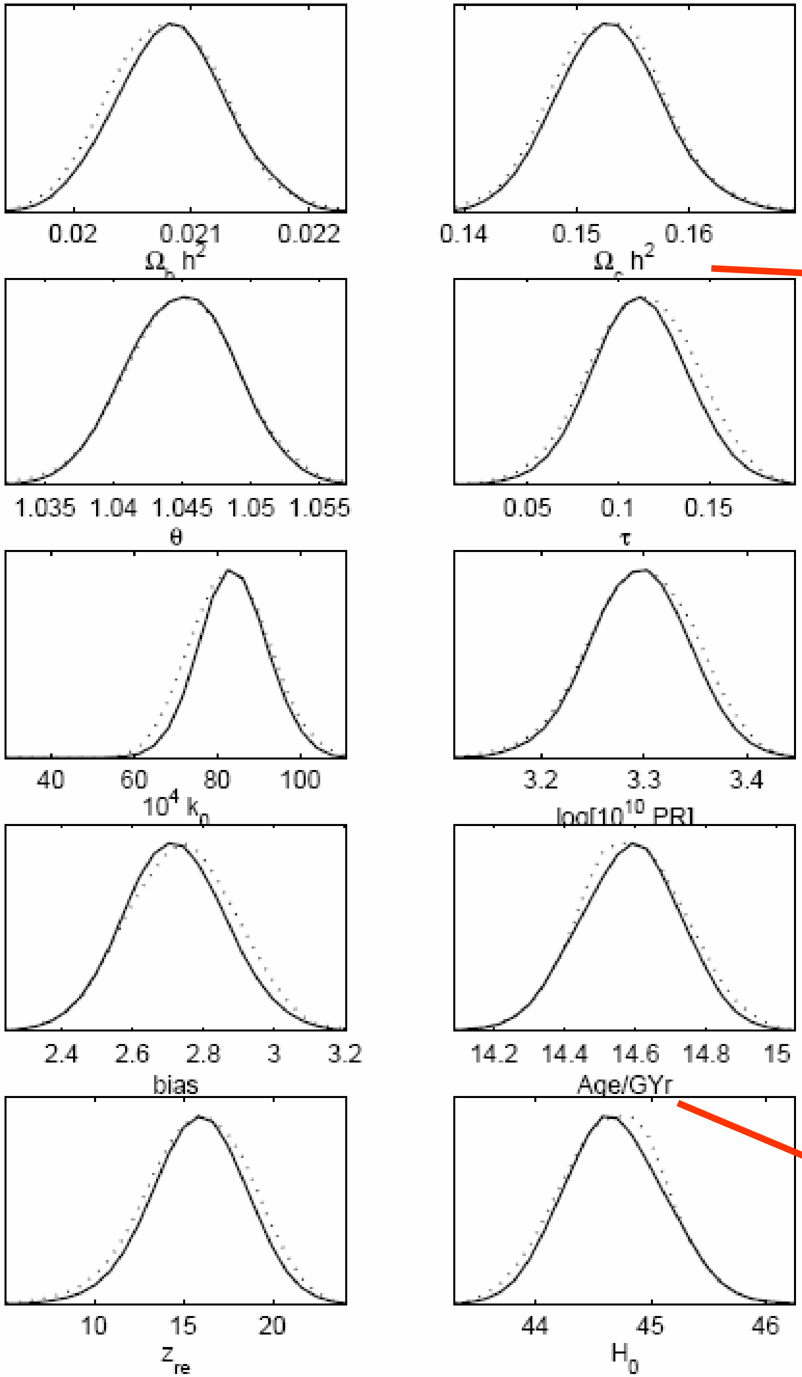
Fits give $\Omega_b h^2 \approx 0.021 \rightarrow$ BBN √ ⇒ baryon fraction in clusters predicted to be ~11% √

Parameter degeneracies - CHDM universe ('bump' spectrum)



Hunt & Sarkar
(2006)

MCMC likelihoods - CHDM universe (‘bump’ spectrum)



This is ~50% higher than the ‘WMAP value’ used widely for CDM abundance

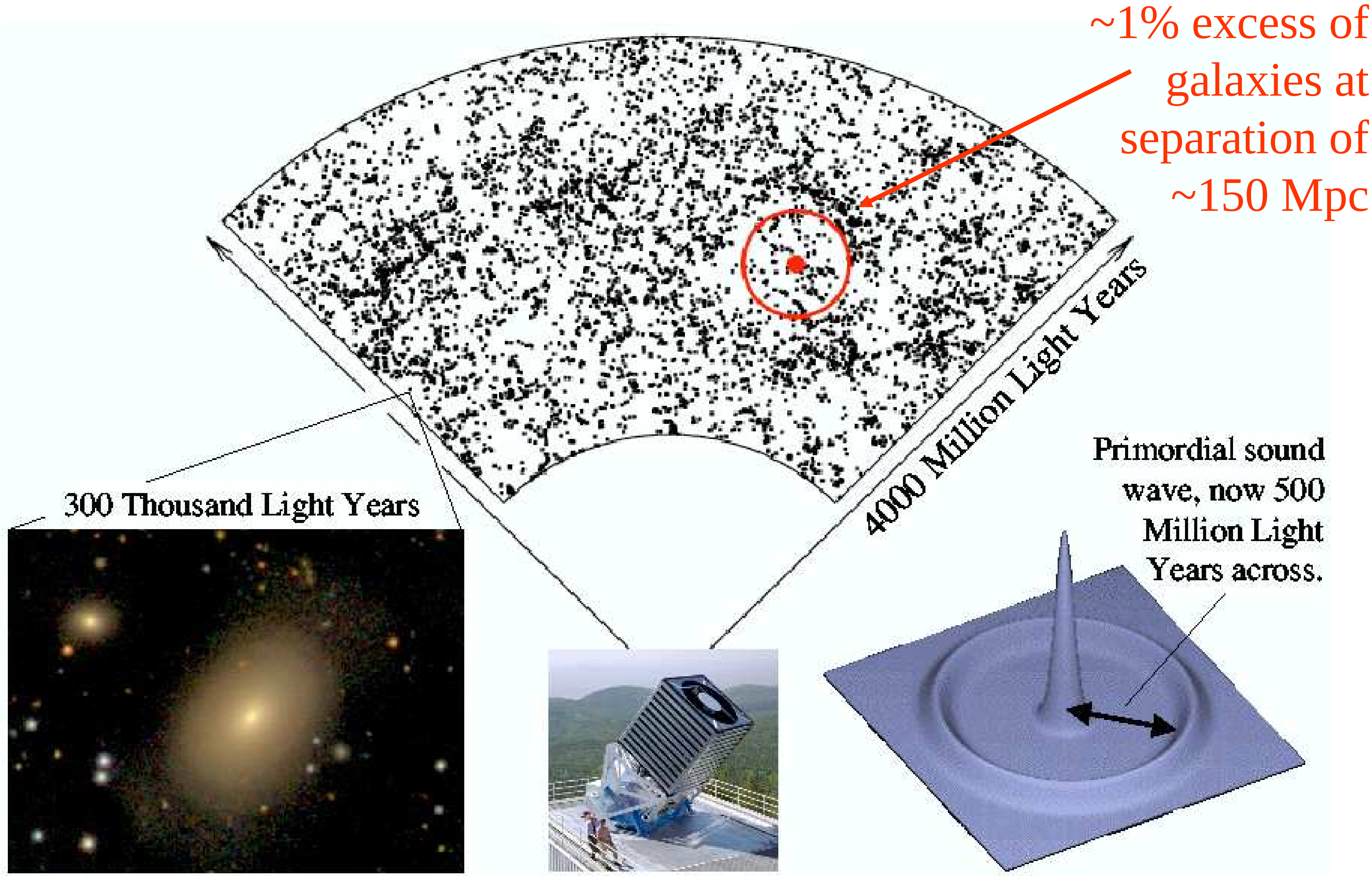
To fit the large-scale structure data *requires* ~eV mass neutrinos

Consistent with data on clusters and weak lensing

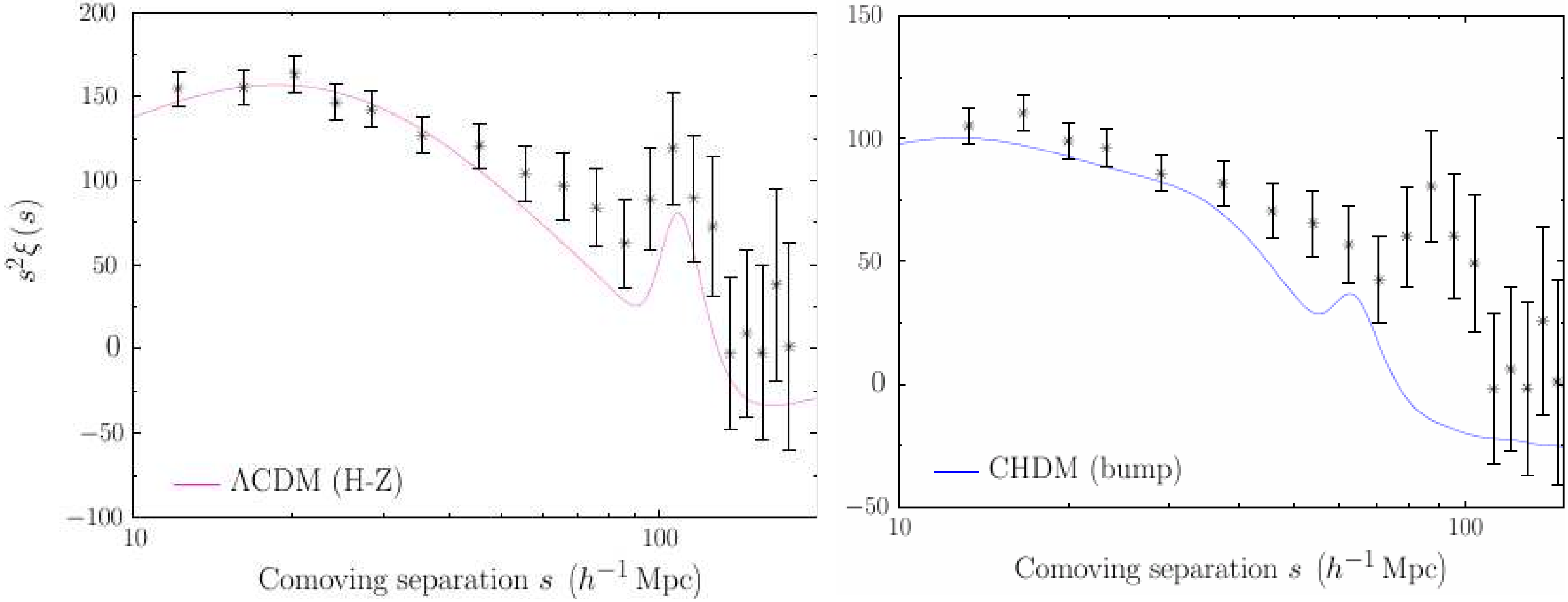
Consistent age for the universe

Hunt & Sarkar (2006)

New Test: Baryon Acoustic Peak in the Large-Scale Correlation Function of *SDSS* Luminous Red Galaxies



In the model with no dark energy, the baryon bump is at the ~same *physical* scale, but at a different location in observed (redshift) space



We *can* match the angular size of the 1st acoustic peak at $z \sim 1100$ by taking $h \sim 0.5$, but we *cannot* then also match the angular size of the baryonic feature at $z \sim 0.35$

***If* confirmed (@ > 5σ) this will rule out present alternative to ΛCDM**

(as will confirmation of LSS-CMB correlations due to 'late ISW effect')

Conclusions

WMAP is supposed to have confirmed the need for a dominant component of dark energy from precision observations of the CMB

➢However we cannot *simultaneously* determine both the primordial spectrum *and* the cosmological parameters from CMB (and LSS) data

We do not know the physics behind inflation hence it is not justified to *assume* that the generated scalar density perturbation is scale-free (and then conclude that CMB and LSS data *confirm* the ΛCDM model)

The data provides intriguing hints for features in the primordial spectrum … this has crucial implications for parameter extraction e.g. a 'bump' in the spectrum allows the data to be well-fitted *without* any dark energy!

➢Given the unacceptable degree of fine-tuning required to accommodate dark energy in fundamental theory, we should be uneasy about accepting its reality on 'concordance' arguments alone – insist on ***direct*** evidence!