(Some) Inflation Models (Q. Shaf)

- WMAP 1
  - Minimal (susy hybrid + variations)
  - Non-minimal ('New', Sneutrino)

- WMAP 3
  - Non-minimal
  - Quartic (CW) Potential ('84)

- Monopoles & Inflation
Based on work done in various collaborations. Special thanks to

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Reviews: Linde's book
  Lyth, Riotto
  Lazarides
To be called 'successful' an inflation model should satisfy (at least) the following requirements:

- Solve flatness & horizon problems;
  number of e-foldings $N \sim 50 - 60$
  (can be lower for low scale inflation)

- Predictions for $n_s, r, \alpha (\equiv n_s/d\ln k)$ should be consistent with observations.

- Explain the observed baryon asymmetry.

- Consistent with nucleosynthesis, etc.

- Resolution of the monopole problem (in case model has them)
Fig. 14.— Joint two-dimensional marginalized contours (68% and 95% confidence levels) for inflationary parameters \((r_{0.002}, n_s)\) predicted by monomial potential models, \(V(\phi) \propto \phi^n\). We assume a power-law primordial power spectrum, \(dn_s/d\ln k = 0\), as these models predict the negligible amount of running index, \(dn_s/d\ln k \approx -10^{-3}\). (Upper left) WMAP only. (Upper right) WMAP+SDSS. (Lower left) WMAP+2dFGRS. (Lower right) WMAP+CBI+VSA. The dashed and solid lines show the range of values predicted for monomial inflaton models with 50 and 60 e-folds of inflation (equation (13), respectively. The open and filled circles show the predictions of \(m^2\phi^2\) and \(\lambda\phi^4\) models for 50 and 60 e-folds of inflation. The rectangle denotes the scale-invariant Harrison-Zel'dovich-Peebles (HZ) spectrum \((n_s = 1, r = 0)\). Note that the current data prefers the \(m^2\phi^2\) model over both the HZ spectrum and the \(\lambda\phi^4\) model by likelihood ratios greater than 50.

\[
\begin{align*}
\text{WMAP3: } n_s &= 0.95 \pm 0.02 \\
\text{Other: } n_s &= 0.965 \pm 0.02(?)
\end{align*}
\]

\[
\begin{align*}
\text{WMAP1: } n_s &= 0.99 \pm 0.04 \\
\text{Other Analyses: } n_s &= 0.98 \pm 0.02
\end{align*}
\]
Supersymmetric Hybrid Inflation

'Minimal' models based on

- superpotential \( W = k S ( \bar{\phi} \phi - M^2 ) \)
  and some extensions;

- minimal Kähler \( ( K = \phi^+ \phi + S^+ S + \bar{\phi}^+ \bar{\phi} ) \)

These models

- possess close ties to GUTs; \( \delta T/T \sim (M/M_p)^2 \)
- resolve MSSM \( \mu \) problem;
- solve monopole problem;
- Reheat temperature \( T_r \sim 10^6 - 10^9 \) GeV;
- \( n_B/s \) from \( n_L/s \).
Avoid $\eta$ problem ($\eta \propto \frac{M^2_v V''}{V}$)

In SUGRA

$$V_F = e^{K/m_p^2} \left[ K_{ij} D_{z_i} W D_{z_j}^* W^* - 3 m_p^2 |W|^2 \right]$$

$$D_{z_i} W = \frac{\partial W}{\partial z_i} + m_p^2 \frac{\partial K}{\partial z_i} W$$

$$K_{ij} = \frac{\partial^2 K}{\partial z_i \partial z_j^*}$$

And one often finds that the inflaton picks up contributions $H^2 \phi^2$ which spoils inflation.
With minimal Kähler potential
the inflationary potential is given by

\[ V = k^2 M_{B-L}^4 \left[ 1 + \frac{151^4}{2 m_p^4} + \alpha m_3 \frac{k^2 M_{B-L}^2}{151} \right. \]

\[ + \frac{k^2 N}{32\pi^2} \left( 2 \ln \frac{k^2 151^2}{\Lambda^2} + (2+1)^2 \ln \left( 1 + \frac{K^2}{\Lambda^2} \right) \right. \]

\[ + \left. (2-1)^2 \ln \left( 1 - \frac{K^2}{\Lambda^2} \right) \right] \]

\[ z = 151^2 / M^2 \]
\[ N = 1 \ (\text{for} \ U(1)_{B-L}) \]
\[ |\alpha m_3| \sim \text{TeV} \]

$151 \ll \text{Mplanck}$
during inflation
For $S > S_c$, (and $k$ not too small)

$$V_{	ext{eff}}(S) \sim k^2 \mu^4 \left[ 1 + \frac{N_k^2}{32\pi^2} \ln\left( \frac{k^2 S^2}{\Lambda^2} \right) \right]$$

Use to calculate

$$\alpha \left( \frac{M}{M_p} \right) \leftarrow \delta T/T, n, S/T \left( n = 1 - \frac{1}{N_e} \right) \approx 0.98 \frac{1}{N_e}$$
For $10^3 \leq k \leq 10^4$, the radiative corrections dominate, so that

\[ \frac{\delta T}{T} = \left( \frac{N_e}{45N} \right)^{1/2} \left( \frac{M}{M_p} \right)^2 \]

(no. of e-foldings)

\[ (=1 \text{ for } U(1)_{B-L}) \]

Taking $\frac{\delta T}{T} \approx 6 \times 10^{-5}$, say,

\[ M \approx (6 \times 10^{15}) \sqrt{N} \text{ GeV} \]

Knowing $M$, one could 'predict' $\delta T/T$. 
FIG. 3: The spectral index $n_s$ vs. the allowed range of $\kappa$, for SUSY hybrid inflation with $N = 1$ (blue), with $N = 2$ (green), and for shifted hybrid inflation with $M_S = m_P$ (red). The dashed segments denote the range of $\kappa$ for which the change in $\text{arg} S$ is significant.
Shifted Hybrid Inflation (topological defects such as monopoles)

Superpotential $W = W_{\text{minimal}} + \text{higher order}$
FIG. 6: The spectral index $n_s$ as a function of the gauge symmetry breaking scale $M$ for smooth hybrid inflation (dashed line—without SUGRA correction, solid line—with SUGRA correction).

**Smooth Hybrid Inflation**

$$W = S ( -v^2 + \frac{(\phi \phi)^2}{M_*^2} )$$

$$V_{\text{infl}} \approx v^4 \left[ 1 - \frac{M_4^4}{541514} + \frac{1514}{2 M_p^4} \right], \quad (S \gg M)$$

$$M = (v M_*)^{\frac{1}{2}} .$$

(Cf.: Brane Inflation)
FIG. 5: The tensor to scalar ratio $r$ vs. the allowed range of $\kappa$, for SUSY hybrid inflation with $N = 1$ (blue), with $N = 2$ (green), and for shifted hybrid inflation with $M_S = m_P$ (red). The dashed segments denote the range of $\kappa$ for which the change in $\arg S$ is significant.

$$r = \frac{(\Delta T/T)_T^2}{(\Delta T/T)_S^2};$$

$$(\Delta T/T)_T^2 \sim \frac{V}{M_P^4} \sim \frac{H^2}{M_P^2}$$
FIG. 4: $dn_s/d \ln k$ vs. the allowed range of $\kappa$, for SUSY hybrid inflation with $N = 1$ (blue), with $N = 2$ (green), and for shifted hybrid inflation with $M_S = m_P$ (red). The dashed segments denote the range of $\kappa$ for which the change in $\arg S$ is significant.
For 'regular' and 'shifted' hybrid inflation, one finds

\[ n_s \geq 0.98 \]

For 'smooth' inflation

\[ n_s \geq 0.97 \]

Consistent with WMAP 1.

Not with WMAP 3?
Non-minimal Models

\[ K = |S|^2 + |\phi|^2 + |\bar{\phi}|^2 + |N|^2 \]

\[ + K_s \frac{|S|^4}{4m_p^2} + K_s \phi \frac{|S|^2 |\phi|^2}{m_p^2} + K_{SN} |S|^2 |N|^2 + \ldots \]

- 'regular' hybrid inflation with \( \phi \) & \( N \) at origin during inflation, but Vinge picks up a term

\[ (-K_s) K^2 M^4 \frac{S^2}{m_p^2} \], such that

\[ n_s \approx 1 - 2\delta - 2K_s \]

\[ \uparrow \text{radiative corrections} \]
\[ \uparrow \text{non-minimal contribution} \]
‘New’ inflation

Here $S$ and $N$ stay at zero during inflation now driven by $\phi$ \( W = S(-\mu^2 + \frac{(\phi \phi)^m}{M_*^{2m-2}}) \)

During inflation,

\[ V \approx \mu^4 (1 - \frac{\beta}{2} \frac{\phi^2}{m_p^2} + \ldots) \]

(with $\beta \equiv Ks_\phi - 1 \geq 0$.)

\[ n_s \approx 1 - 2\beta, \text{ for } \beta \gg 1 / [(2m-2)N_e] \]

\[ \approx 1 - \left[ \frac{2(2m-1)}{(2m-2)N_e} \right], \text{ for } \beta \approx 0. \]
FIG. 1: The spectral index $n_s$ vs. $\beta$, for $m = 2$ (green), $m = 3$ (blue), $m = 4$ (orange) and $m = 5$ (purple).
Sneutrino Hybrid Inflation

Here the right-handed sneutrino behaves as inflaton.

\[ n_s \approx 1 - 2\phi \quad (\phi \equiv K_{SN-1}) \]

\[ \tau \ll \phi^2 \]

\[ \frac{dn_s}{d\ln k} \leq -\tau \left( \frac{N^2}{m_p^2} \right) \]
Quartic (CW) Potential

$$V(\phi) = A \phi^4 \left[ \ln \left( \frac{\phi}{M} \right) - \frac{1}{4} \right] + \frac{A M^4}{4} \uparrow$$

Gauge singlet

$$V(\phi = M) = 0 ; \quad V(\phi = 0) = A M^4/4 = V_0$$

$$V(\phi \ll M) \approx \frac{A M^4}{4} - b \phi^4$$

- For $$V_0^{1/4} < 10^{16} \text{ GeV}$$, $$\phi < M_P (\approx 2.4 \times 10^{18} \text{ GeV})$$

Model behaves as for $$V \approx V_0 (1 - (\phi/\mu)^4)$$

$$n_s \approx 1 - \frac{3}{N_0}$$, \quad $$\alpha \approx (n_s - 1)/N_0$$

$$V_0^{1/4} > 10^5 \text{ GeV} \text{ to avoid conflict with WMAP }$$

$$< e^{-\text{folds}}$$

$$k_0 = 0.002 \text{ Mpc}^{-1}$$
For $V_0^{1/4} \geq 10^{16}$ GeV, $\phi > m_p$
during observable inflation. Predictions approach that of
$\phi^2$ potential, with

$$N_s = 1 - \frac{2}{N_0} \approx 0.96$$

$$\gamma \approx 0.13$$

$$\alpha \approx -0.6 \times 10^{-3}$$

? Where does $\phi$ come from
Breaks global $U(1)_{B-L}$ (SM)
$U(1)_{PQ}$
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Table 1: The Silukhary parameters for the Silukhary model (w = 1)
FIG. 1: $1 - n_s$ and $r$ vs. $\log_{10}[V_0^{1/4} \text{(GeV)}]$ for Coleman-Weinberg potential.
FIG. 2: $\alpha$ vs. $n_s$ for the Coleman-Weinberg potential.
FIG. 1: $1 - n_s$ and $r$ vs. $\log[V_0^{1/4} \text{ (GeV)}]$ for the Coleman-Weinberg potential.
Monopoles & Inflation

Consider

$$SO(10) \rightarrow SU(4) \times SU(2) \times SU(2)$$

$M_G \rightarrow M_I \rightarrow SM$

- First breaking yields superheavy monopoles with one unit of Dirac charge.
- Second breaking gives rise to 'lighter' ($\sim 10M_I$) monopoles with two units of Dirac charge.
MACRO experiment has put an upper bound $\sim 10^{-16} \text{ cm}^{-2}\text{ s}^{-1}\text{ sr}^{-1}$ on the flux of monopoles with mass $\sim 10^{13} \text{ GeV}$.

'Lighter' monopoles arise when

$$\phi \sim \phi_x = V(\phi_0)^{1/2} \frac{M}{M_I}$$

(Quartic coupling $-\phi^2 X^+X$)

breaks

4-2-2
$(10^{13}\text{ GeV})$

$M_G \sim 10^6\text{ GeV}$

$$N_x \geq \ln (H/T_r) + 20$$

$\sim 20-30$ yields flux close to the observable level.
Conclusions

• Quartic (CW) potential yields predictions in good agreement with WMAP3.

• When applied to SO(10) broken, say, via SU(4) x SU(2) x SU(2), one finds that doubly charged monopoles may exist close to MACRO limits.

• Leptogenesis is automatic.

• SUSY hybrid inflation models are nicely related to SUSY GUTS. WMAP3 appears to prefer non-minimal Kähler potential. Nice properties are retained.
Much rests on Planck (07)!