(Some) Inflation Models (Q. Shaf

- WMAP 1 → Minimal (susy hybrid + variations)
  → Non-minimal ('New', Sneutrino)

- WMAP 3 → Non-minimal
  → Quartic (CW) Potential ('84)

- Monopoles & Inflation

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Reviews: Linde' book
Lyth, Riotto
Lazarides
⋮

To be called 'successful' an inflation model should satisfy (at least) the following requirements:

- Solve flatness & horizon problems; no. of e-foldings $N \sim 50-60$ (can be lower for low scale inflation)
- Predictions for $n_s, r, \alpha (\equiv dn_s/d\ln k)$ should be consistent with observations.
- Explain the observed baryon asymmetry.
- Consistent with nucleosynthesis, etc.
- Resolution of the monopole problem (in case model has them)

WMAP 3 : $n_s \approx 0.95 \pm 0.02$

Other : $n_s \approx 0.965 \pm 0.02$ (?)

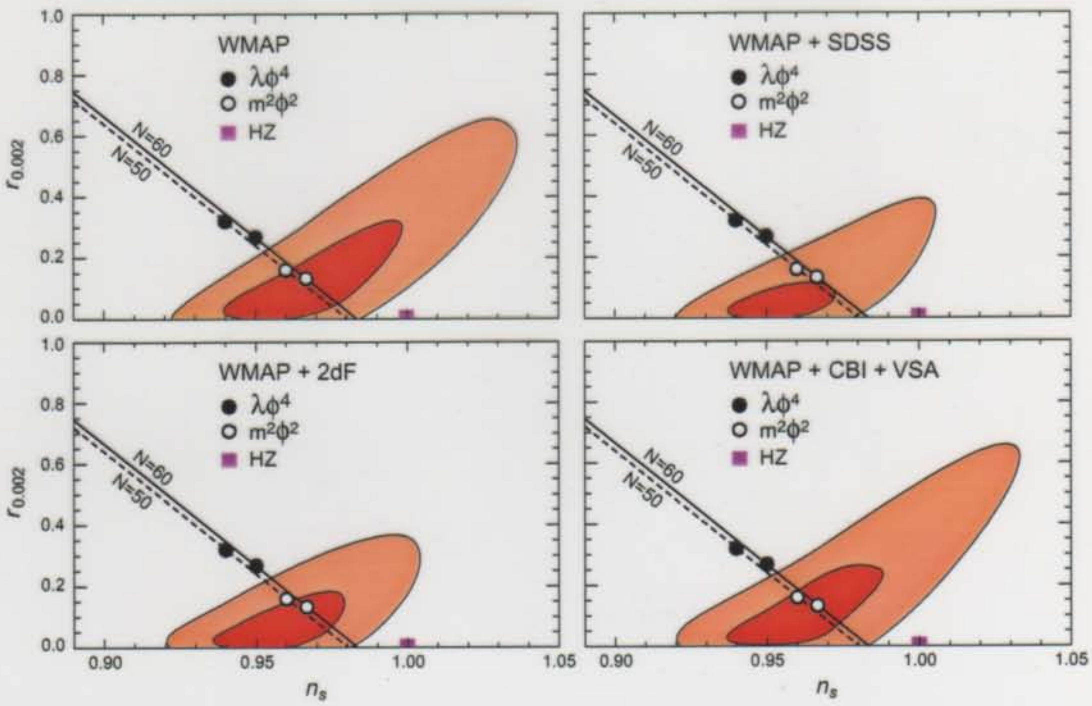


Fig. 14.— **Joint two-dimensional marginalized contours (68% and 95% confidence levels) for inflationary parameters ($r_{0.002}$, n_s) predicted by monomial potential models, $V(\phi) \propto \phi^n$. We assume a power-law primordial power spectrum, $dn_s/d\ln k = 0$, as these models predict the negligible amount of running index, $dn_s/d\ln k \approx -10^{-3}$. (*Upper left*) WMAP only. (*Upper right*) WMAP+SDSS. (*Lower left*) WMAP+2dFGRS. (*Lower right*) WMAP+CBI+VSA. The dashed and solid lines show the range of values predicted for monomial inflaton models with 50 and 60 e-folds of inflation (equation (13), respectively. The open and filled circles show the predictions of $m^2\phi^2$ and $\lambda\phi^4$ models for 50 and 60 e-folds of inflation. The rectangle denotes the scale-invariant Harrison-Zel'dovich-Peebles (HZ) spectrum ($n_s = 1, r = 0$). Note that the current data prefers the $m^2\phi^2$ model over both the HZ spectrum and the $\lambda\phi^4$ model by likelihood ratios greater than 50.**

(WMAP1 : $n_s \approx 0.99 \pm 0.04$)

(Other Analyses : $n_s \approx 0.98 \pm 0.02$)

Supersymmetric Hybrid Inflation

Copeland et al
Dvali et al
⋮

'Minimal' models based on

- superpotential $W = \kappa S(\bar{\phi}\phi - M^2)$

and some extensions;

- minimal Kähler ($K = \phi^\dagger\phi + S^\dagger S + \bar{\phi}^\dagger\bar{\phi}$)

These models

- possess close ties to GUTs; $\delta T/T \propto \left(\frac{M}{M_P}\right)^2$
- resolve MSSM μ problem;
- solve monopole problem;
- Reheat temperature $T_r \sim 10^6 - 10^9$ GeV;
- n_B/s from n_L/s.

- Avoid η problem $\left(\eta \propto \frac{M_p^2 V''}{V}\right)$

In SUGRA

$$V_F = e^{K/m_p^2}\left[K_{ij}^{-1} D_{z_i} W \, D_{z_j^*} W^* - 3\, m_p^{-2} |W|^2 \right]$$

$$D_{z_i} W \equiv \frac{\partial W}{\partial z_i} + m_p^{-2} \frac{\partial K}{\partial z_i} W$$

$$K_{ij} \equiv \frac{\partial^2 K}{\partial z_i \, \partial z_j^*}$$

And one often finds that the inflaton picks up contributions $H^2\phi^2$ which spoils inflation.

With minimal Kähler potential the inflationary potential is given by

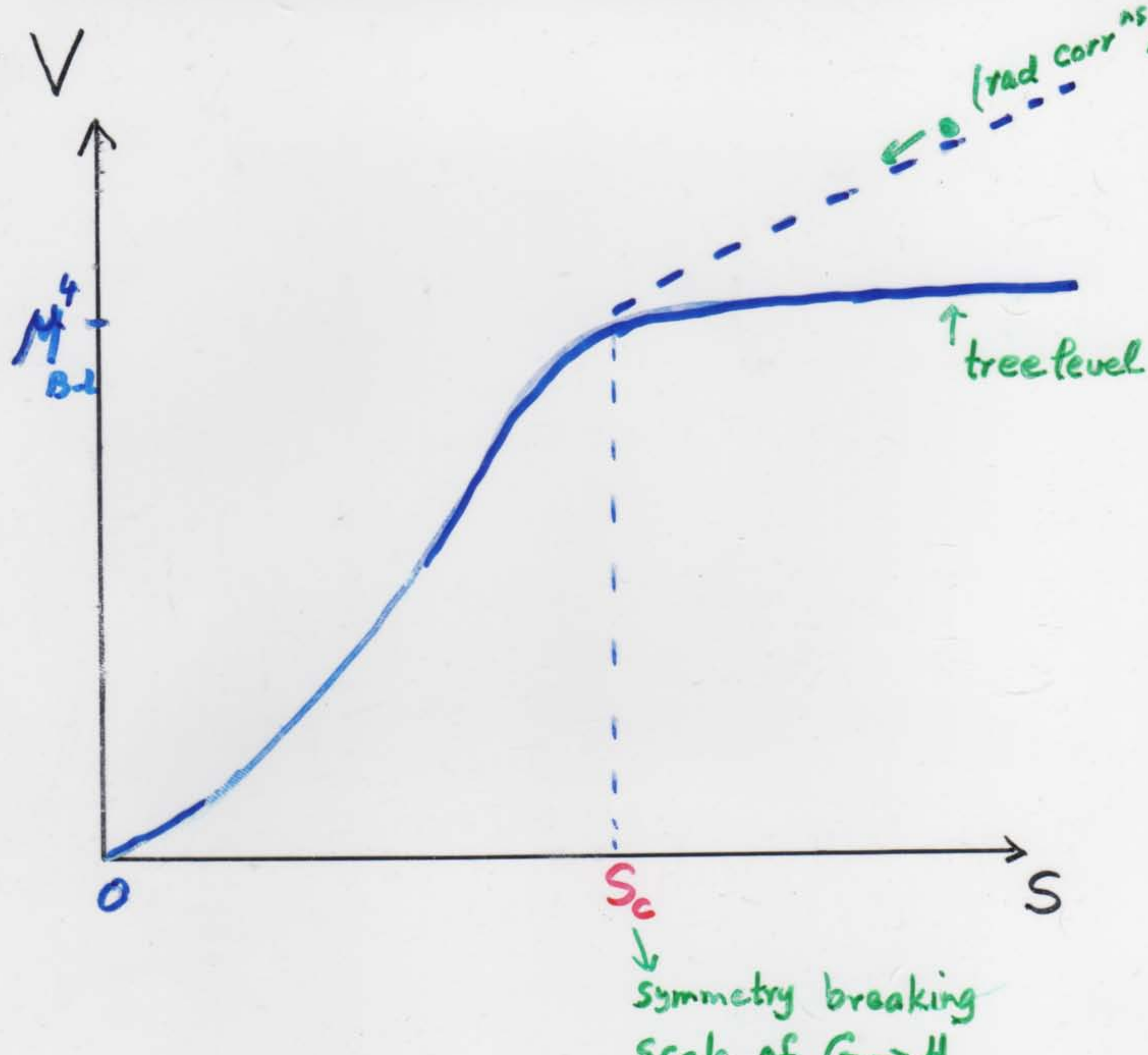
$$V = \kappa^2 M_{B-L}^4 \left[1 + \frac{|S|^4}{2 m_p^4} + a m_{3/2} \kappa M_{B-L}^2 |S| + \frac{\kappa^2 \mathcal{N}}{32\pi^2} \left(2 \ln \frac{\kappa^2 |S|^2}{\Lambda^2} + (z+1)^2 \ln(1+z^{-1}) + (z-1)^2 \ln(1-z^{-1}) \right) \right],$$

$z = |S|^2/M^2$

$\mathcal{N} = 1$ (for $U(1)_{B-L}$)

$|a m_{3/2}| \sim \text{TeV}$

$|S| \ll M_{\text{planck}}$ during inflation



For $S > S_c$, (& κ not too small)

$$V_{eff}(S) \sim \kappa^2 M^4 \left[1 + \frac{\mathcal{N}\kappa^2}{32\pi^2} \ln\left(\frac{\kappa^2 S^2}{\Lambda^2}\right) \right]$$

Use to calculate

$\propto (M/M_P)^2 \leftarrow \delta T/T$, n, S/T $\left(n \simeq 1 - \frac{1}{N_\ell}\right)$

$\simeq 0.98$

For $10^{-3} \lesssim \kappa \lesssim 10^{-1}$, the radiative corrections dominate, so that

no. of e foldings

$$\frac{\delta T}{T} \approx \left(\frac{N_\ell}{45\mathcal{N}}\right)^{1/2} \left(\frac{M}{M_p}\right)^2$$

(=1 for $U(1)_{B-L}$)

Taking $\frac{\delta T}{T} \approx 6 \times 10^{-5}$, say,

$$M \simeq (6 \times 10^{15})\, \mathcal{N}^{1/4} \text{ GeV}$$

- Knowing M, one could 'predict' $\delta T/T$.

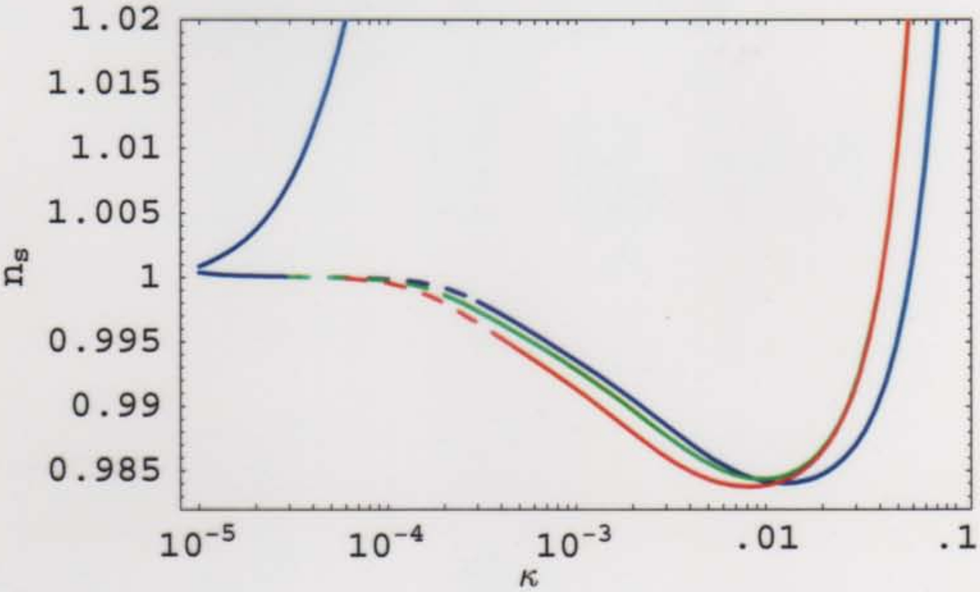
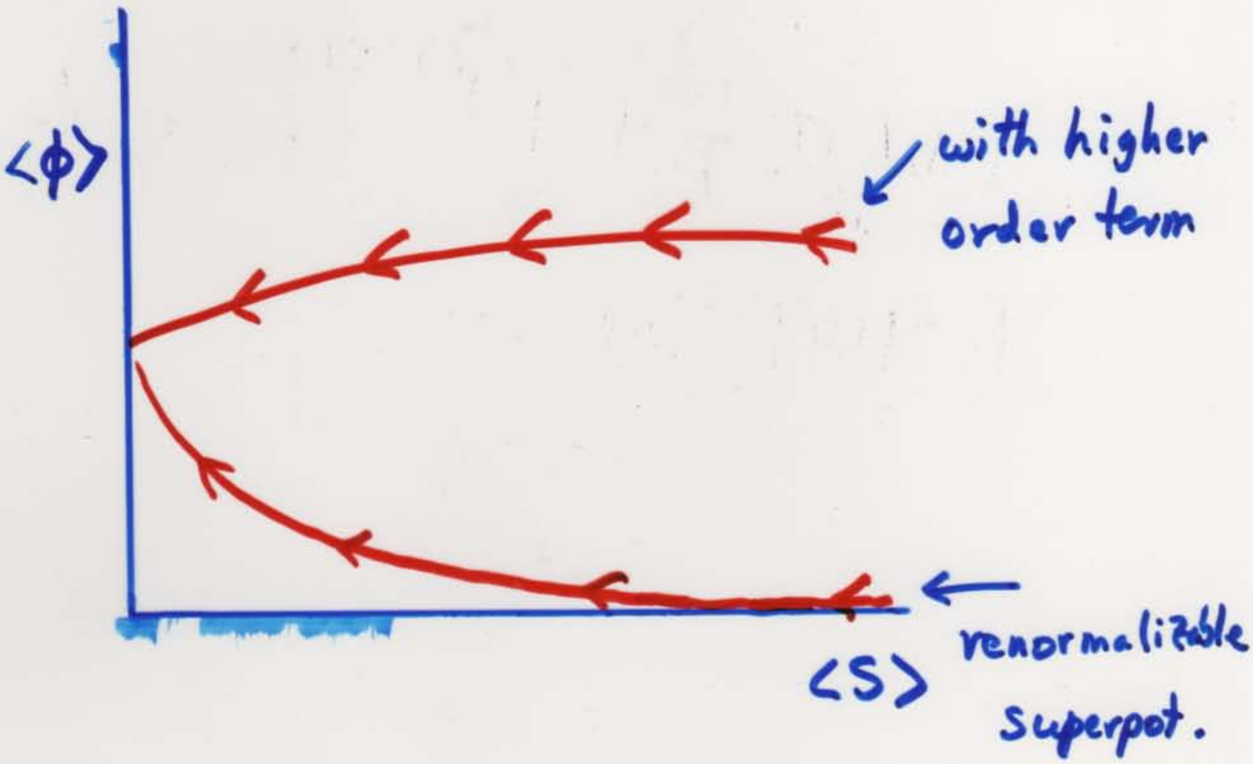


FIG. 3: The spectral index n_s vs. the allowed range of κ, for SUSY hybrid inflation with $\mathcal{N} = 1$ (blue), with $\mathcal{N} = 2$ (green), and for shifted hybrid inflation with $M_S = m_P$ (red). The dashed segments denote the range of κ for which the change in $\arg S$ is significant.

Shifted Hybrid Inflation (to take care of topological defects such as monopoles)



Superpotential $W = W_{minimal} + \text{higher order}$

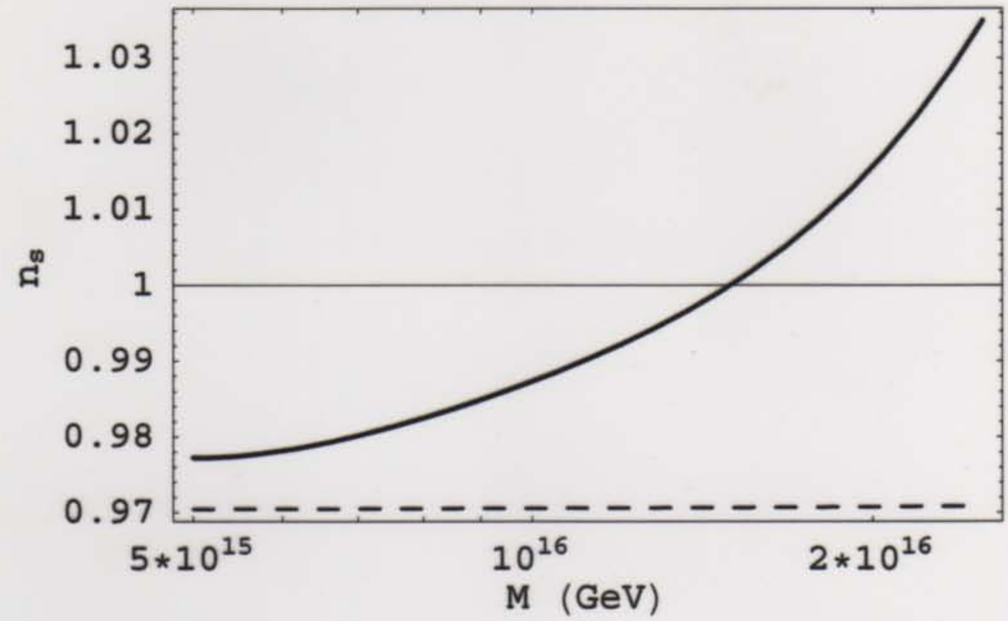


FIG. 6: The spectral index n_s as a function of the gauge symmetry breaking scale M for smooth hybrid inflation (dashed line–without SUGRA correction, solid line–with SUGRA correction).

Smooth Hybrid Inflation

$$W = S\left(-v^2 + \frac{(\bar{\Phi}\phi)^2}{M_*^2}\right)$$

$$V_{infl} \approx v^4\left[1 - \frac{M^4}{54|S|^4} + \frac{|S|^4}{2M_P^4}\right], \ (S \gg M)$$

$$M = (vM_*)^{1/2}.$$

(Cf: Brane Inflation)

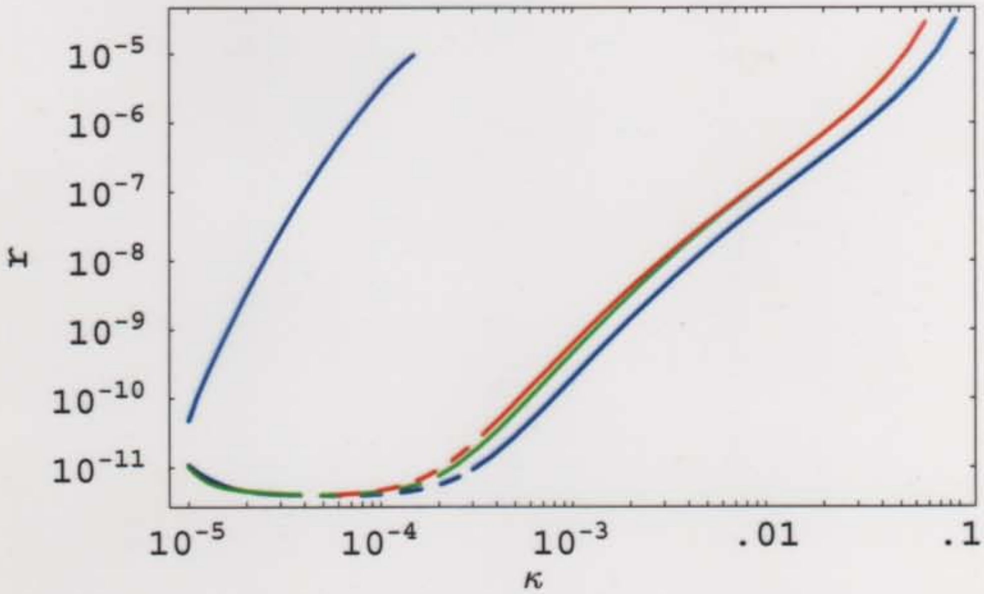


FIG. 5: The tensor to scalar ratio r vs. the allowed range of κ, for SUSY hybrid inflation with $\mathcal{N}=1$ (blue), with $\mathcal{N}=2$ (green), and for shifted hybrid inflation with $M_S = m_P$ (red). The dashed segments denote the range of κ for which the change in $\arg S$ is significant.

$$r \equiv (\Delta T/T)_T^2 \big/ (\Delta T/T)_S^2 \; ;$$

$$(\Delta T/T)_T^2 \simeq \frac{V}{M_P^4} \sim \frac{H^2}{M_P^2}$$

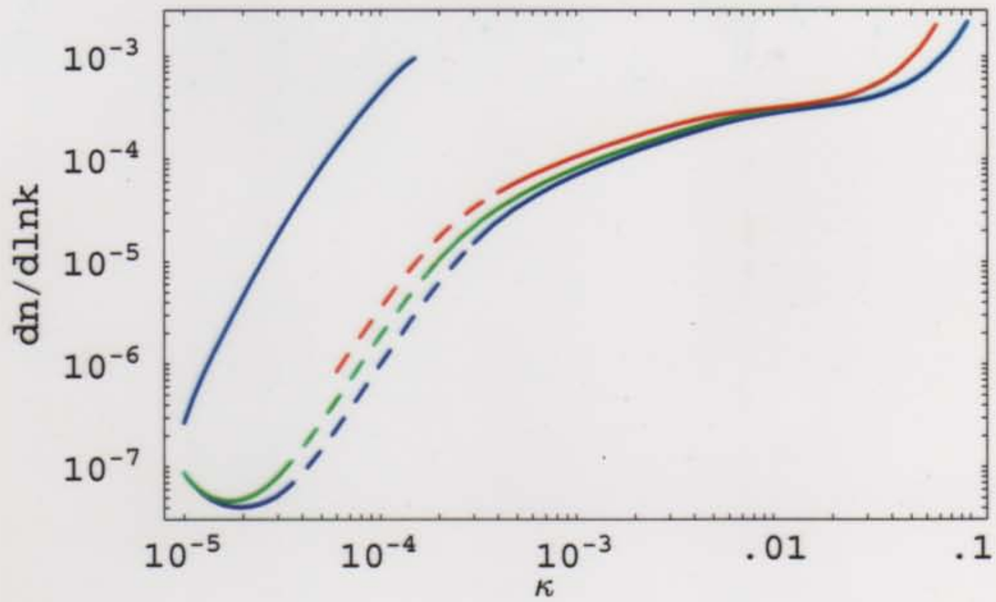


FIG. 4: $dn_s/d\ln k$ vs. the allowed range of κ, for SUSY hybrid inflation with $\mathcal{N} = 1$ (blue), with $\mathcal{N} = 2$ (green), and for shifted hybrid inflation with $M_S = m_P$ (red). The dashed segments denote the range of κ for which the change in $\arg S$ is significant.

- For 'regular' and 'shifted' hybrid inflation, one finds

$$n_s \gtrsim 0.98$$

- For 'smooth' inflation

$$n_s \gtrsim 0.97$$

Consistent with WMAP 1.

Not with WMAP 3?

Non-minimal Models

right handed sneutrino

$$K = |S|^2 + |\phi|^2 + |\bar{\phi}|^2 + |N|^2$$

$$+ \kappa_S \frac{|S|^4}{4m_P^2} + \kappa_{S\phi} \frac{|S|^2|\phi|^2}{m_P^2} + \kappa_{SN} |S|^2|N|^2$$

$$+ \cdots$$

- 'regular' hybrid inflation with ϕ & N at origin during inflation, but V_{infl} picks up a term

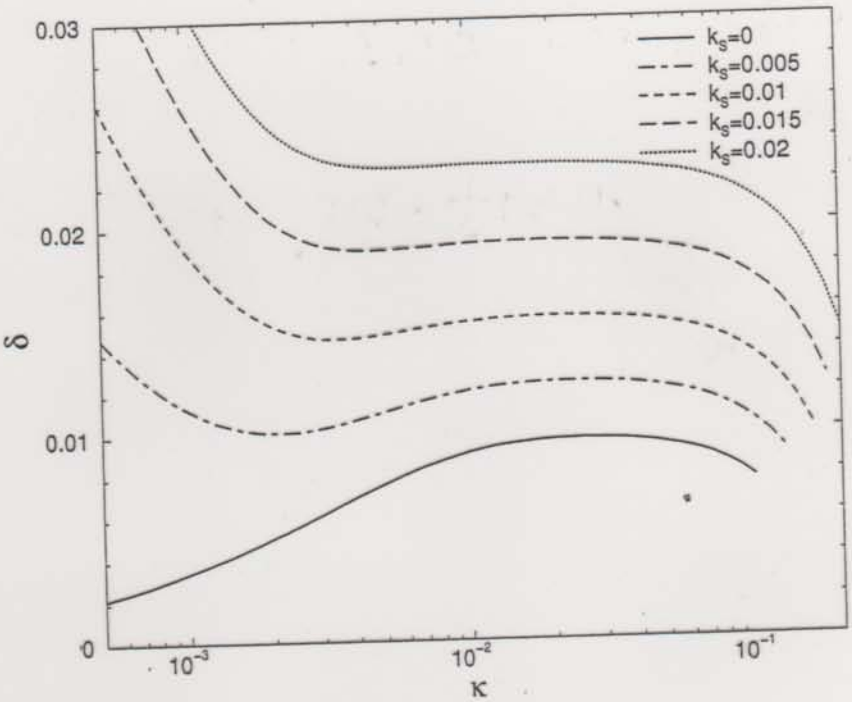
$$(-\kappa_S)\, \kappa^2 M^4 \frac{S^2}{m_P^2}$$

, such that

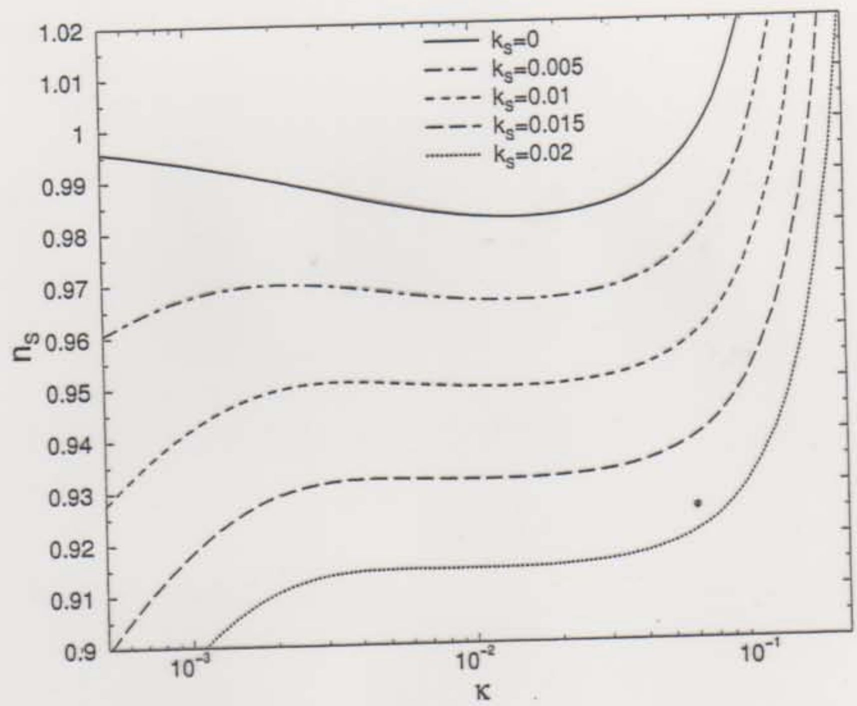
$$n_s \simeq 1 - 2\delta - 2\kappa_s$$

radiative corrections

non-minimal contribution



k_s=0
k_s=0.005
k_s=0.01
k_s=0.015
k_s=0.02
0.03
0.02
0.01
0
δ
10^{-3}
10^{-2}
10^{-1}
κ



k_s=0
k_s=0.005
k_s=0.01
k_s=0.015
k_s=0.02
n_s
1.02
1.01
1
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
0.91
0.9
10^{-3}
10^{-2}
10^{-1}
κ

Kawasaki et al
Senoguz, QS
⋮

- ## 'New' inflation

Here S and N stay at zero during inflation now driven by ϕ ($W = S\left(-\mu^2 + \frac{(\bar{\phi}\phi)^m}{M_*^{2m-2}}\right)$)

During inflation,

$$V \simeq \mu^4\left(1 - \frac{\beta}{2}\frac{\phi^2}{m_p^2} + \cdots\right)$$

(with $\beta \equiv \kappa_{s\phi} - 1 \geq 0$.)

$$n_s \simeq 1 - 2\beta \text{, for } \beta \gg 1/[(2m-2)N_\ell]$$

$$\simeq 1 - [2(2m-1)/(2m-2)N_\ell] \text{, for } \beta \simeq 0 .$$

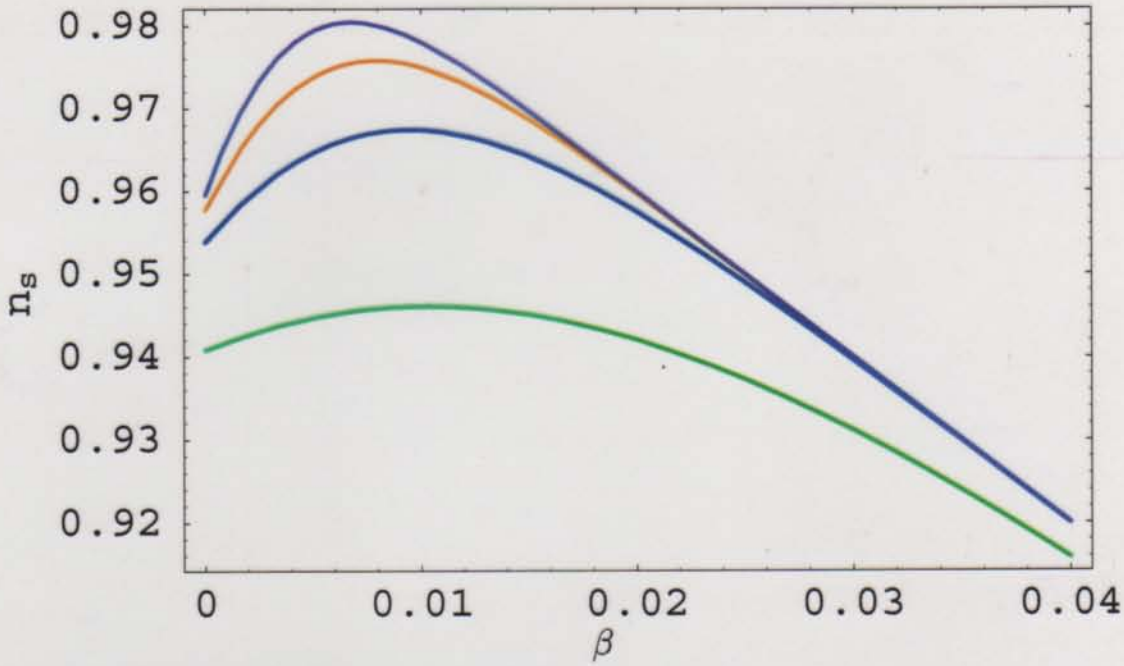


FIG. 1: The spectral index n_s vs. β, for $m = 2$ (green), $m = 3$ (blue), $m = 4$ (orange) and $m = 5$ (purple).

• Sneutrino Hybrid Inflation

Antusch, Bastero-Gil et al

Here the right-handed sneutrino behaves as inflaton.

$$n_s \simeq 1 - 2\gamma \qquad (\gamma \equiv \kappa_{SN} - 1)$$

$$r \ll \gamma^2$$

$$dn_s/d\ln k \lesssim -\gamma(N^2/m_p^2)$$

Quartic (CW) Potential

$$V(\phi) = A\phi^4\left[\ln\left(\frac{\phi}{M}\right) - \frac{1}{4}\right] + \frac{AM^4}{4}$$

gauge singlet

$$V(\phi = M) = 0 \; ; \; V(\phi = 0) = AM^4/4 \equiv V_0$$

$$V(\phi \ll M) \approx \frac{AM^4}{4} - b\phi^4$$

- For $V_0^{1/4} < 10^{16}$ GeV, $\phi < m_p$ ($\simeq 2.4 \times 10^{18}$ GeV)

Model behaves as for $V \simeq V_0(1 - (\phi/\mu)^4)$

$$n_s \simeq 1 - \frac{3}{N_0} \, , \quad \alpha \simeq (n_s - 1)/N_0$$

e-folds for $k_0 = 0.002$ Mpc^{-1}

($V_0^{1/4} > 10^5$ GeV to avoid conflict with WMAP)

- For $V_0^{1/4} \gtrsim 10^{16}$ GeV, $\phi > m_P$ during observable inflation.

Predictions approach that of ϕ^2 potential, with

$$n_s = 1 - \frac{2}{N_0} \simeq 0.96$$

$$r \simeq 0.13$$

$$\alpha \simeq -0.6 \times 10^{-3}$$

? Where does ϕ come from

Breaks global $U(1)_{B-L}$ (SM)

$U(1)_{PQ}$

Table 1: The inflationary parameters for the Shafi-Vilenkin model ($m_P = 1$)

$V_0^{1/4}$(GeV)	$A(10^{-14})$	M	ϕ_e	ϕ_0	$V(\phi_0)^{1/4}$(GeV)	n_s	α $(-)10^{-3}$	r
10^{13}	1.0	0.018	0.010	3.0×10^{-6}	$\approx V_0^{1/4}$	0.938	1.4	9×10^{-15}
5×10^{13}	1.2	0.088	0.050	7.5×10^{-5}	$\approx V_0^{1/4}$	0.940	1.3	5×10^{-12}
10^{14}	1.3	0.17	0.10	3.0×10^{-4}	$\approx V_0^{1/4}$	0.940	1.2	9×10^{-11}
5×10^{14}	1.9	0.79	0.51	7.5×10^{-3}	$\approx V_0^{1/4}$	0.941	1.2	5×10^{-8}
10^{15}	2.3	1.5	1.1	0.030	$\approx V_0^{1/4}$	0.941	1.2	9×10^{-7}
5×10^{15}	4.8	6.2	5.1	0.71	$\approx V_0^{1/4}$	0.942	1.0	5×10^{-4}
1×10^{16}	5.2	12	10	3.2	9.9×10^{15}	0.952	1.0	8×10^{-3}
2×10^{16}	1.1	36	35	23	1.7×10^{16}	0.966	0.6	0.07
3×10^{16}	.17	86	85	72	1.9×10^{16}	0.967	0.6	0.11

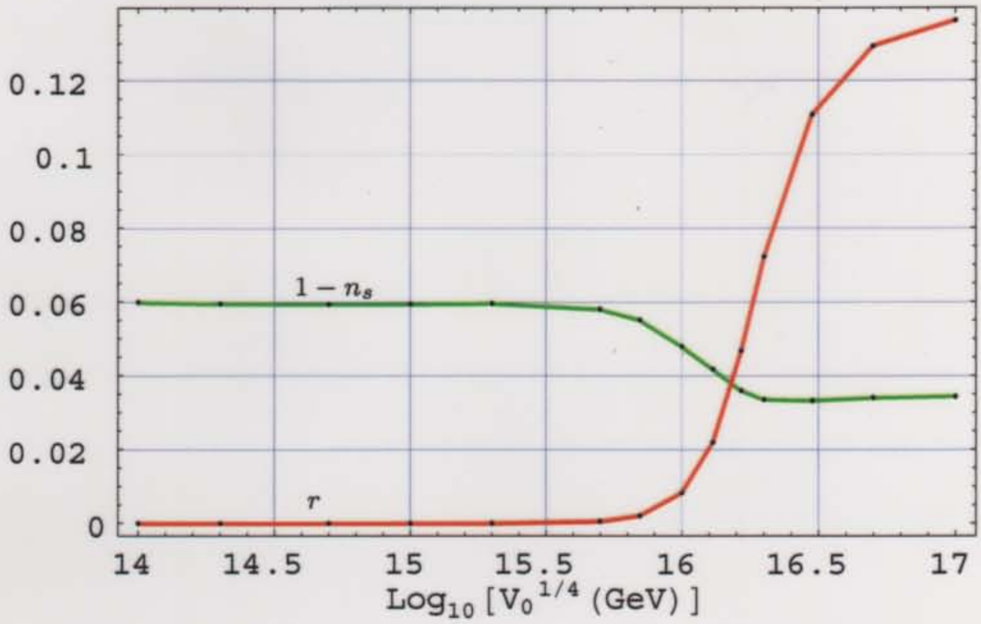


FIG. 1: $1-n_s$ and r vs. $\log[V_0^{1/4}\,(\mathrm{GeV})]$ for Coleman-Weinberg potential.

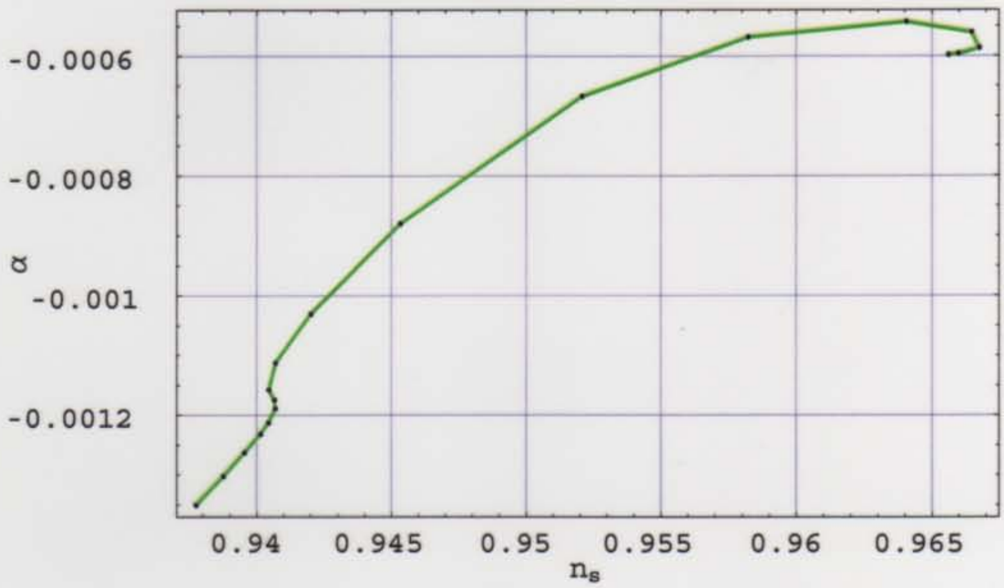


FIG. 2: α vs. n_s for the Coleman-Weinberg potential.

? Lower Bound on V_0

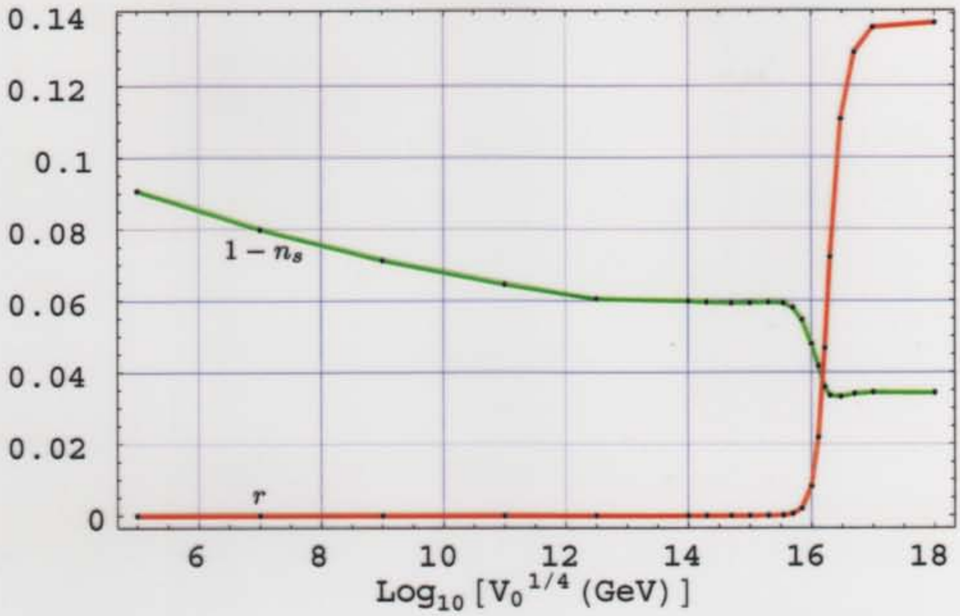


FIG. 1: $1-n_s$ and r vs. $\log[V_0^{1/4}\,(\mathrm{GeV})]$ for the Coleman-Weinberg potential.

Monopoles & Inflation

Consider

$$\begin{array}{c} SO(10) \xrightarrow[M_G]{} SU(4) \times SU(2) \times SU(2) \\ \downarrow M_I \\ SM \end{array}$$

- First breaking yields Superheavy monopoles with one unit of Dirac charge;

- Second breaking gives rise to 'lighter' ($\sim 10 M_I$) monopoles with two units of Dirac charge.

MACRO experiment has put an upper bound $\sim 10^{-16}\ cm^{-2} s^{-1} sr^{-1}$ on the flux of monopoles with mass $\sim 10^{13}$ GeV.

'Lighter' monopoles arise when

$$\phi \sim \phi_X \approx V(\phi_0)^{1/2} \frac{M}{M_I}$$

(quartic coupling $- \phi^2 X^+ X$) $\hookrightarrow$ breaks 4-2-2 (10^{13} GeV) $M_G \sim 10^{16}$ GeV

$$N_X \gtrsim \ln (H/T_r) + 20$$

$\sim$ 20 - 30 yields flux close to the observable level.

Conclusions

- Quartic (CW) potential yields predictions in good agreement with WMAP 3.
  - When applied to $SO(10)$ broken, say, via $SU(4) \times SU(2) \times SU(2)$, one finds that doubly charged monopoles may exist close to MACRO limits
  - Leptogenesis is automatic.

- susy hybrid inflation models are nicely related to susy GUTS. WMAP 3 appears to prefer non-minimal Kähler potential. Nice properties are retained.

- Much rests on

  Planck (07) !