Cosmologies with running parameters and dynamical cosmon

(Implications on the coincidence problem)

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GUIDELINES

- Dynamical Dark Energy
- Variable cosmological constant: Effective EOS. "DE picture" versus "CC picture"
- Effective transition " quintessence→phantom" without scalar fields.
- An application: a "running" cosmological "constant" model
- Adding a dynamical "cosmon" : ΛXCDM Model
- Implications on the Cosmological Coincidence Problem
- Conclusions.

Related works

This talk is mainly based on the results obtained in the recent works:

- J. Grande, JS, H. Stefancic, LXCDM: a cosmon model solution to the cosmological coincidence problem?, gr-qc/0604057

- JS, H. Stefancic, Dynamical dark energy versus variable cosmological constant, Mod. Phys. Lett. A21 (2006) 479.

- JS, H. Stefancic, Effective equation of state for dark energy: mimicking quintessence and phantom energy through a variable Λ Phys. Lett. B624 (2005) 147

Previous related works:

- I.L. Shapiro, JS, H. Stefancic, *JCAP* **0501** (2005) 012
- I.L. Shapiro, JS, C. España-Bonet, P. Ruiz-Lapuente *Phys. Lett.* **B** 574 (2003) 149; *JCAP* **0402** (2004) 6
- I.L. Shapiro, JS, *JHEP* **0202** (2002) 006
- I.L. Shapiro, JS, *Phys. Lett.* **B**475 (2000) 236

"Canonical" definition of Dynamical Dark Energy

Assume there is no "true" Λ but some **DE** entity with the following properties:

- The total energy-momentum tensor on the *r.h.s.* of Einstein eqs. is the sum

$$\tilde{T}_{\mu\nu} \equiv T^{M}_{\mu\nu} + T^{D}_{\mu\nu} \,.$$

- Both tensors are separately conserved, $\nabla^{\mu}\tilde{T}_{\mu\nu} = 0$ is equivalent to

$$\nabla^{\mu} T^{M}_{\mu\nu} = 0 \Longleftrightarrow \frac{d\rho}{dt} + 3\,H\,(\rho + p) = 0,$$

and (<u>unmixed</u> conservation laws)

$$\nabla^{\mu} T^{D}_{\mu\nu} = 0 \Longleftrightarrow \frac{d\rho_D}{dt} + 3\,H\,(\rho_D + p_D) = 0$$

One popular possibility of this is the idea of quintessence.

• Subsequently one introduces an "effective" equation of state

$$p_D = \omega_D \, \rho_D$$

to describe a phenomenological relationship between p_D and ρ_D.

• Finally one also assumes that at present:

$$\omega_D = \frac{p_D}{\rho_D} = \frac{\frac{1}{2}\xi\dot{\chi}^2 - V(\chi)}{\frac{1}{2}\xi\dot{\chi}^2 + V(\chi)} \begin{cases} \gtrsim -1 & \xi > 0 \text{ (quintessence)} \\ \lesssim -1 & \xi < 0 \text{ (phantom DE)} \end{cases}$$

• For Λ the only possible equation of state is

$$p_\Lambda = -\rho_\Lambda \, .$$

The solutions of the two conservation equations read

$$\rho_s(z) = \rho_s(0)\,(1+z)^{\alpha}$$

$$\alpha = 3(1+\omega_m)\ (\omega_m = 1/3 \text{ or } \omega_m = 0)$$

and

$$\rho_D(z) = \rho_D(0)\,\zeta(z)$$

$$\zeta(z) \equiv \exp\left\{3\int_0^z dz' \frac{1+\omega_D(z')}{1+z'}\right\}$$

Hence

$$\omega_D(z) = -1 + \frac{1}{3}\frac{1+z}{\zeta}\frac{d\zeta}{dz}$$

$$H_D^2(z) = H_0^2\left[\tilde{\Omega}_M^0\,(1+z)^{\alpha} + (1-\tilde{\Omega}_M^0)\,\zeta(z)\right]$$

(flat space)

$$(\Delta\Omega_M \equiv \Omega_M^0 - \tilde{\Omega}_M^0)$$

Variable Λ

- For variable Λ, the conserved quantity is not the matter energy-momentum tensor $T_{\mu\nu}$, but the sum

$$\tilde{T}_{\mu\nu} \equiv T_{\mu\nu} + g_{\mu\nu}\,\rho_\Lambda(t)\,, \qquad \nabla^\mu \tilde{T}_{\mu\nu} = 0\,.$$

By the Bianchi identities, Λ is constant $\Longleftrightarrow$ the matter $T_{\mu\nu}$ is individually conserved ($\nabla^\mu T_{\mu\nu} = 0$)– in particular, $\rho_\Lambda =$ const. if $T_{\mu\nu} = 0$ (e.g. during inflation).

- From FLRW metric

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1 - k\,r^2} + r^2\,d\theta^2 + r^2\,\sin^2\theta\,d\phi^2\right)\,,$$

we may compute explicitly the local energy-conservation law $\nabla^\mu \tilde{T}_{\mu\nu} = 0$. The result is an equation allowing transfer of energy between ordinary matter and the dark energy associated to the Λ term :

$$\frac{d\rho_\Lambda}{dt} + \frac{d\rho}{dt} + 3\,H\,(\rho + p) = 0,$$

(mixed conservation law!)

"DE picture" versus "CC picture"

- Observations leading to the **EOS** of the **DE** are sensitive to the function $H = H(z)$.

- We can describe a variable CC model with mixed energy-conservation law as if it would be a dynamical DE model with unmixed EC-law.

- Let us assume there is an underlying fundamental dynamics

$$\rho_\Lambda(z) = \rho_\Lambda(\rho(z), H(z), \ldots)\,, \quad G(z) = G(\rho(z), H(z), \ldots)$$

$$\Downarrow$$

$$H_\Lambda^2 = \frac{8\pi G}{3}(\rho + \rho_\Lambda)$$

H. Stefancic, J.S.
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$$\frac{d}{dt}\left[G(\rho+\rho_\Lambda)\right]+3\,G\,H_\Lambda\,(\rho+p)=0\,. \quad \text{(Bianchi identity)}$$

$$H_\Lambda^2(z)=H_0^2\,\left[\Omega_M^0\,f_M(z;r)(1+z)^\alpha+\Omega_\Lambda^0\,f_\Lambda(z;r)\right]$$

$$\xi_M(z)\equiv\frac{G(z)}{G_0}\rho_M(z)$$

$$f_M(z)\equiv\frac{\xi_M(z)}{\rho_M^0\,(1+z)^\alpha}$$

$$\xi_\Lambda(z)\equiv\frac{G(z)}{G_0}\rho_\Lambda(z)$$

$$f_\Lambda(z;r)=\frac{\xi_\Lambda(z)}{\rho_\Lambda^0}$$

Whatever it be their form, these functions must satisfy $f_M(0;r)=f_\Lambda(0;r)=1$ in order that the cosmic sum rule $\Omega_M^0+\Omega_\Lambda^0=1$ is fulfilled.

⇒ "Matching condition" of the two pictures:

$$H_D^2(z) = H_\Lambda^2(z)$$

$$H_0^2 \left[\tilde{\Omega}_M^0 \, (1+z)^\alpha + (1-\tilde{\Omega}_M^0)\, \zeta(z)\right] = H_\Lambda^2(z)$$

Matching generates an "effective **EOS**" for Λ:

$$\omega_{\rm eff}(z) = -1 + \frac{1}{3}\,\frac{1+z}{\zeta}\,\frac{d\zeta}{dz}$$

$$= -1 + \frac{\alpha}{3}\left(1 - \frac{\xi_\Lambda(z)}{\rho_D(z)}\right)$$

$$\rho_D(z) = \xi_\Lambda(z) - (1+z)^\alpha \int_{z^*}^{z} \frac{dz'}{(1+z')^\alpha} \frac{d\xi_\Lambda(z')}{dz'}$$

where z^* is a transition point where $\rho_D(z^*) = \xi_\Lambda(z^*)$

Theorem: z^* always exists near $z = 0$.

Proof: It follows from the relation

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$$\frac{d\zeta(z)}{dz} = \frac{\alpha\,(1+z)^{\alpha-1}}{1-\tilde{\Omega}_M^0}\left(\Omega_M^0\, f_M(z;r) - \tilde{\Omega}_M^0\right)$$

This relation can be proven using:

i) The matching condition of the two pictures;

ii) The Bianchi identity.

Then the Theorem follows from the fact that $f_M(0;r) = 1$ (q.e.d)

Running Λ from Planck Scale Physics

- One may expect that the **RGE** of Λ is totally dominated by sub-Planckian masses:

$$\frac{d\Lambda}{d\ln\mu} = \frac{1}{(4\pi)^2}\sum_i c_i\,\mu^2 M_i^2 + \ldots. = \frac{1}{(4\pi)^2}\sum_i c_i\,H^2 M_i^2 + \ldots$$

$$= \frac{1}{(4\pi)^2}\,\sigma\,H^2 M^2 + \ldots..$$

I.L.Shapiro, J.S.
JHEP 0202 (2002) 6

I.L.Shapiro, et al
Phys. Lett. B574 (2003) 149

with

$$M \equiv \sqrt{\sum_i c_i\,M_i^2}\,.$$

- Provides a natural explanation for the geometric mean puzzle:

$$\Lambda \simeq \sqrt{\rho_P\,\rho_H} = \sqrt{M_P{}^4\,H^4} = M_P{}^2\,H^2$$

A semiclassical FLRW with running Λ

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho + \Lambda) + H_0^2\,\Omega_K^0(1+z)^2$$

$$\frac{d\Lambda}{dt} + \frac{d\rho}{dt} + 3H(\rho + p) = 0\,.$$

$$\frac{d\Lambda}{d\ln\, H} = \frac{1}{(4\pi)^2}\sum_i c_i\, M_i^2\, H^2 + \ldots = \frac{3\nu}{4\pi}\, {M_P}^2 H^2.$$

$$\nu = \frac{\sigma\, M^2}{12\pi {M_P}^2}$$

$$\rho_\Lambda \equiv \Lambda = C_1 + C_2\, H^2\,.$$

Effective equation of state for the variable Λ as a function of the redshift: $\omega_{\rm eff} = \omega_{\rm eff}(z;\nu)$

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$\Delta\Omega_M \neq 0$ $\qquad (\Delta\Omega_M \equiv \Omega_M^0 - \tilde{\Omega}_M^0)$

$$\omega_{\rm eff}(z) = -1+(1-\nu)\,\frac{\Omega_M^0\,(1+z)^{3(1-\nu)} - \tilde{\Omega}_M^0\,(1+z)^3}{\Omega_M^0\,[(1+z)^{3(1-\nu)}-1] - (1-\nu)\,[\tilde{\Omega}_M^0\,(1+z)^3 - 1]}\,.$$

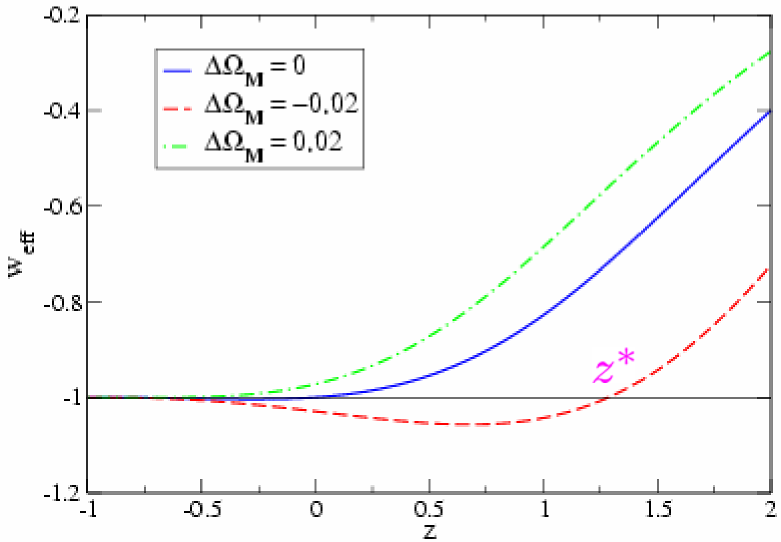
$\Delta\Omega_M = 0$

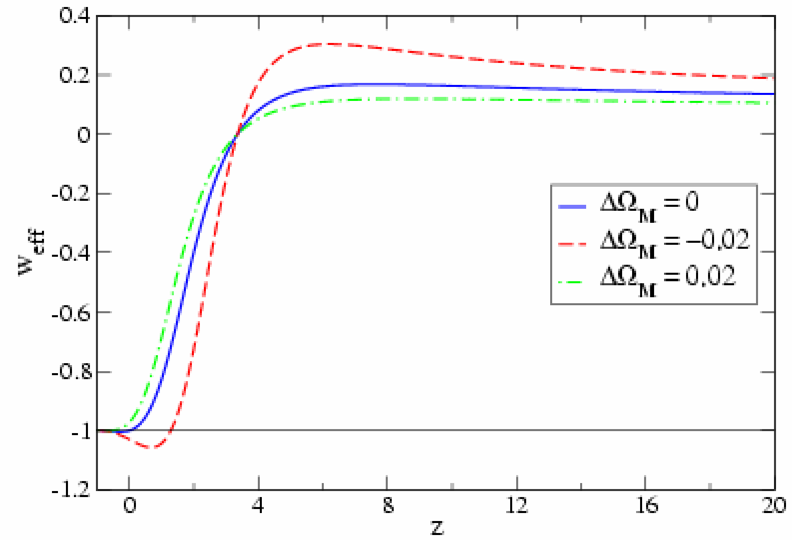
$$\omega_{\rm eff}(z) = -1+(1-\nu)\,\frac{\Omega_M^0\,(1+z)^3\,\left[(1+z)^{-3\nu}-1\right]}{1-\nu-\Omega_M^0+\Omega_M^0\,(1+z)^3\,\left[(1+z)^{-3\nu}-1+\nu\right]}\,.$$

$$\simeq -1 - 3\,\nu\frac{\Omega_M^0}{\Omega_\Lambda^0}\,(1+z)^3\,\ln(1+z)\,.$$

$$(\text{case } \nu < 0\,; \quad \nu = -\nu_0)$$

$\Delta\Omega_M = 0 \quad \Delta\Omega_M \neq 0$!!





$$\left(\nu_0 = \frac{1}{12\pi} \simeq 0.026\right)$$

H. Stefancic, J.S.
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Adding a dynamical cosmon: "ΛXCDM" model

(J. Grande, H. Stefancic, J.S. , gr-qc/0604057)

Effective EOS of a mixture of cosmic fluids:

$$w_e = \frac{p_D}{\rho_D} = \frac{\omega_1\,\rho_1 + \omega_2\,\rho_2 + \dots}{\rho_1 + \rho_2 + \dots}$$

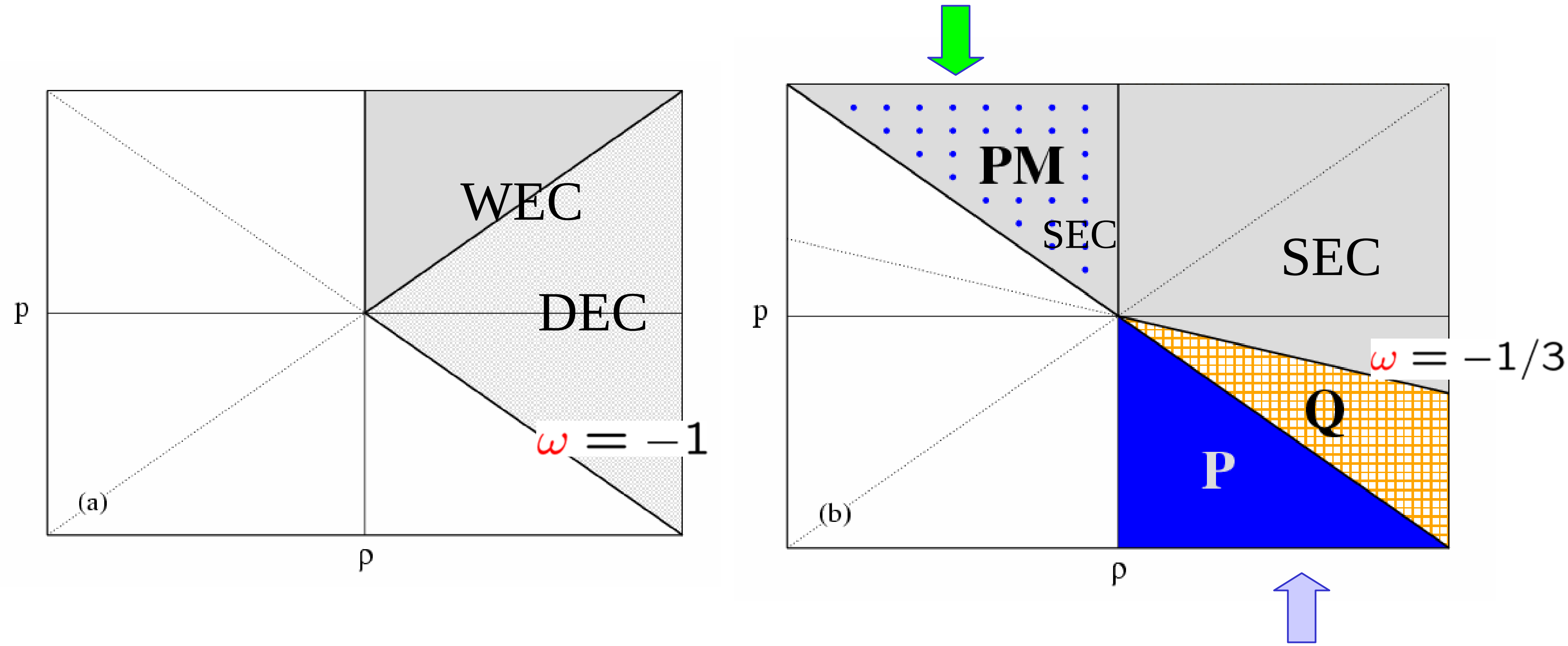
General Bianchi identity. Defining $\alpha_i \equiv 3(1+\omega_i)$

$$\frac{d}{dt}\left[G\left(\sum_i \rho_i\right)\right] + G\,H\,\sum_i \alpha_i\,\rho_i = 0$$

Energy conditions

$$p = \omega\,\rho$$

$$\omega < -1 \ (\rho < 0)$$



$$\omega < -1 \ (\rho > 0)$$

SEC ⇒ gravitational attraction

In the ΛXCDM case, with $G = const.$ and separate conservation of matter and DE:

$$\dot{\rho}_m + \alpha_m \, \rho_m \, H = 0 \,, \qquad \alpha_m = 3(1 + \omega_m)$$

$$\dot{\rho}_D + \alpha_D \, \rho_D \, H = 0 \,, \qquad \alpha_D = 3(1 + \omega_e)$$

with

$$\boxed{\rho_D = \rho_\Lambda + \rho_X}$$

cosmological "constant" contribution — cosmon contribution

$$\omega_e = \frac{p_\Lambda + p_X}{\rho_\Lambda + \rho_X} = -1 + \frac{1}{3} \, \frac{\alpha_X \, \rho_X}{\rho_D}$$

Equivalently, we have

$$\dot{\rho}_\Lambda + \dot{\rho}_X + \alpha_X \, \rho_X \, H = 0 \,, \qquad \alpha_X = 3(1 + \omega_X)$$

The ΛXCDM model satisfies the generalized cosmic sum rule

$$\Omega_m^0 + \Omega_\Lambda^0 + \Omega_X^0 + \Omega_K^0 = 1$$

For $\Omega_K^0 = 0 \Rightarrow$ autonomous system:

$$\frac{d\Omega_X}{d\zeta} = -\left[\nu\,\alpha_m + (1-\nu)\,\alpha_X\right]\Omega_X - \nu\,\alpha_m\,\Omega_\Lambda + \nu\,\alpha_m\,\Omega_c\,,$$
$$\frac{d\Omega_\Lambda}{d\zeta} = \nu\,(\alpha_m - \alpha_X)\,\Omega_X + \nu\,\alpha_m\,\Omega_\Lambda - \nu\,\alpha_m\,\Omega_c\,,$$
$$\frac{d\Omega_c}{d\zeta} = (\alpha_m - \alpha_X)\,\Omega_X + \alpha_m\,\Omega_\Lambda - \alpha_m\,\Omega_c$$

with $\zeta \equiv -\ln(1+z)$

$$\Omega_X(z) = \frac{\rho_m(z)}{\rho_c^0}\,,\quad \Omega_\Lambda(z) = \frac{\rho_\Lambda(z)}{\rho_c^0}\,,\quad \Omega_c(z) = \frac{\rho_c(z)}{\rho_c^0}$$

Eigenvalues:

$$\lambda_1 = -\alpha_X\,(1-\nu)\,, \quad \lambda_2 = -\alpha_m\,, \quad \lambda_3 = 0$$

Eigenvectors:

$$\mathbf{v_1} = \begin{pmatrix} 1-\nu \\ \nu \\ 1 \end{pmatrix}, \quad \mathbf{v_2} = \begin{pmatrix} \frac{-\nu\,\alpha_m}{\alpha_m-\alpha_X} \\ \nu \\ 1 \end{pmatrix}, \quad \mathbf{v_3} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Solution:

$$\mathbf{\Omega}(\zeta) = \begin{pmatrix} \Omega_X(\zeta) \\ \Omega_\Lambda(\zeta) \\ \Omega_c(\zeta) \end{pmatrix} = C_1\,\mathbf{v_1}\,e^{\lambda_1\,\zeta} + C_2\,\mathbf{v_2}\,e^{\lambda_2\,\zeta} + C_3\,\mathbf{v_3}$$

$$C_1 = \frac{1-\Omega_\Lambda^0}{1-\nu} - \frac{\Omega_m^0(\alpha_m-\alpha_X)}{\alpha_m-\alpha_X\,(1-\nu)}, \quad C_2 = \frac{\Omega_m^0(\alpha_m-\alpha_X)}{\alpha_m-\alpha_X\,(1-\nu)}, \quad C_3 = \frac{\Omega_\Lambda^0-\nu}{1-\nu}$$

Among the many possibilities (cf gr-qc/0604057), consider

$0 < \alpha_X < 2\,, \quad \nu = 0 \Leftrightarrow$ quintessence and $\Lambda = const.$

$(-1 < \omega_X < -1/3)$

$$\Omega_D(z) = \Omega_\Lambda^0 + \Omega_X^0\,(1+z)^{\alpha_X}$$

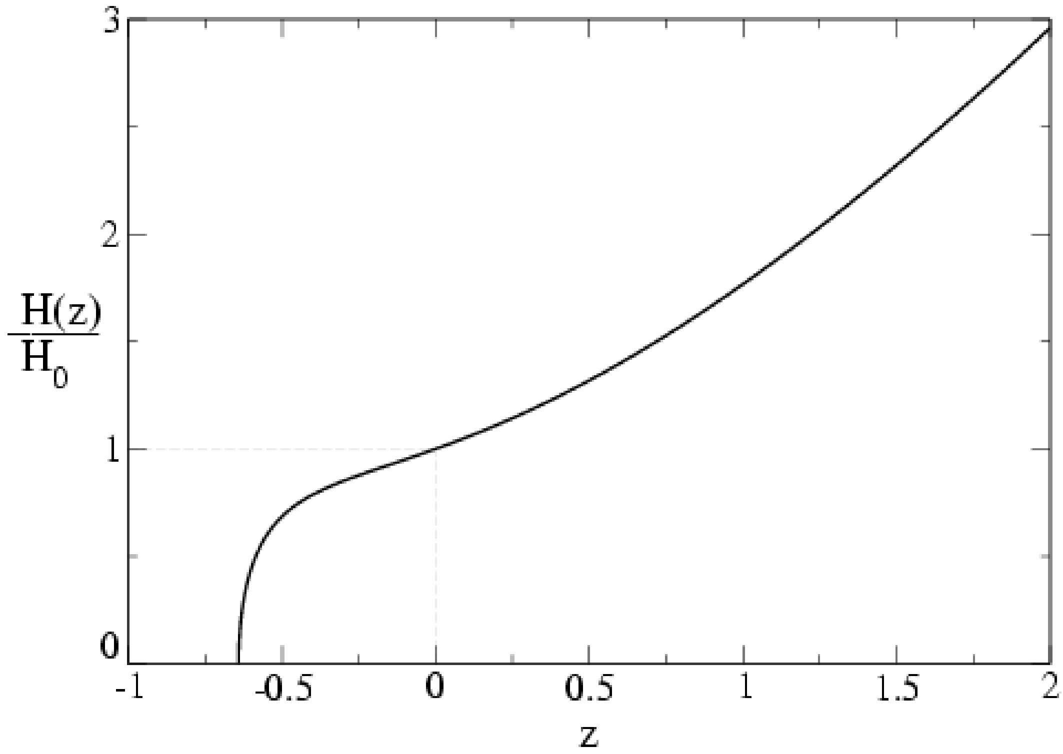
Since $\Omega_m^0 + \Omega_\Lambda^0 + \Omega_X^0 = 1 \;\Rightarrow\; \Omega_\Lambda^0 < 0$ is possible!

There can be stopping (turning point) of the evolution in the future!

However...

in the ΛXCDM model there are many other stopping possibilities !!

Example of Stopping



$$\Omega_\Lambda = 0.75,\ \omega_X = -1.85,\ \nu = -\nu_0$$

The next to simplest one is $-\delta < \alpha_X < 0\,, \quad \nu = 0 \quad (\delta > 0)$

The cosmon has $\omega_X < -1 \;\Rightarrow$ phantom behavior! ("standard" type)

Big Rip ?

Yes... except if $\Omega_X^0 < 0$ phantom matter! (non-standard !)

$$\Omega_D(z) = \Omega_\Lambda^0 + \Omega_X^0\,(1+z)^{\alpha_X} \quad \Rightarrow \quad \text{stopping with} \;\; \Omega_\Lambda^0 > 0$$

But in the ΛXCDM still many other stopping possibilities ...

$\nu \neq 0$!!

Nucleosynthesis Constraints

At temperatures $T \lesssim 0.1\, MeV$ the weak interactions (responsible for neutrons and protons to be in equilibrium) freeze-out. The expansion rate is sensitive to the amount of DE, hence primordial nucleosynthesis can place stringent bounds on the parameters of the ΛXCDM model

Define $$r = \frac{\rho_D}{\rho_m} = \frac{\rho_\Lambda + \rho_X}{\rho_m}$$

$$\Downarrow \quad (\Omega_K^0 = 0)$$

$$\tilde{\Omega}_D = \frac{r}{1+r}$$

Then $$\tilde{\Omega}_D \lesssim 10\% \iff r \lesssim 10\%$$

$$\Rightarrow \quad \boxed{\epsilon \equiv \nu\,(1 + \omega_X) \lesssim 10\%}$$

Parameter Space

$$(\Omega_m, \Omega_\Lambda, \Omega_X, \omega_X, \nu)$$

Priors and constraints

$$\begin{cases} \Omega_m = 0.3\,, \quad \Omega_K^0 = 0 \\ \Omega_m^0 + \Omega_\Lambda^0 + \Omega_X^0 = 1 \end{cases}$$

$$\Downarrow$$

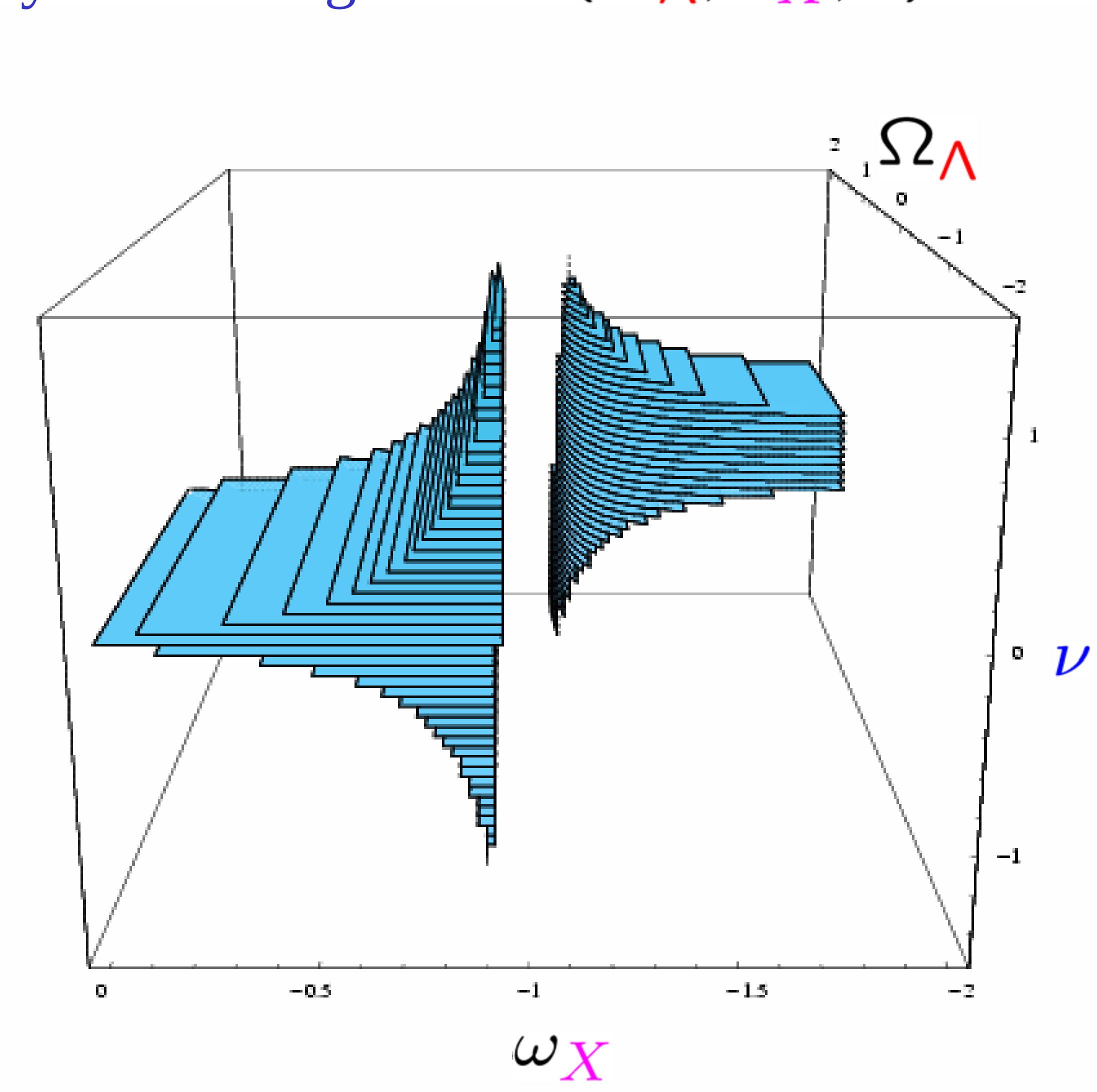
$$(\Omega_\Lambda, \omega_X, \nu)$$

Physical region?

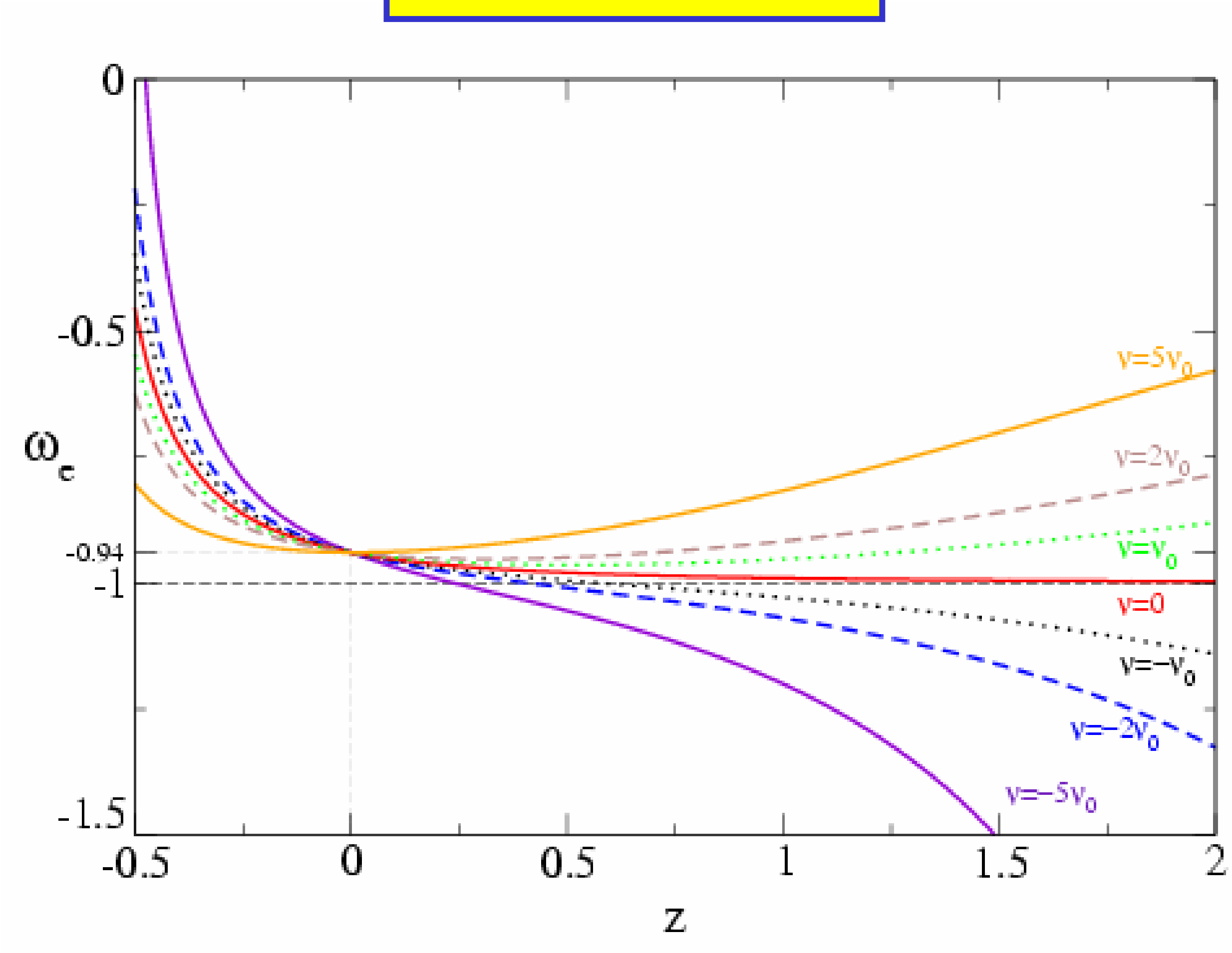
Subspace satisfying:

- i) Nucleosynthesis bound
- ii) Stopping condition
- iii) $r \equiv \rho_D/\rho_m < 10$

Physical subregion of $(\Omega_\Lambda, \omega_X, \nu)$



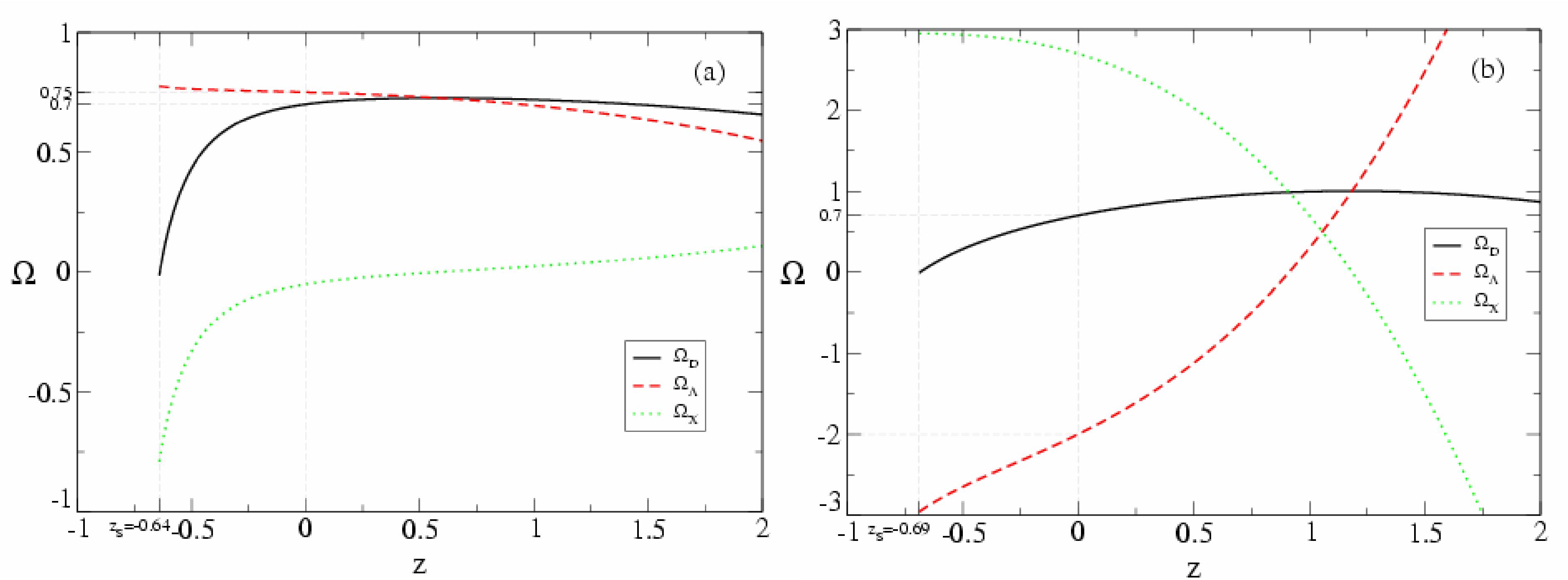
Effective EOS



Comparison of the effective EOS parameter of the ΛXCDM model, ω_e, for fixed values $\omega_X = -1.85$, $\Omega_\Lambda^0 = 0.75$, and different values of ν in units of ν_0. All curves give $\omega_e(0) = -0.94$ at the present time.

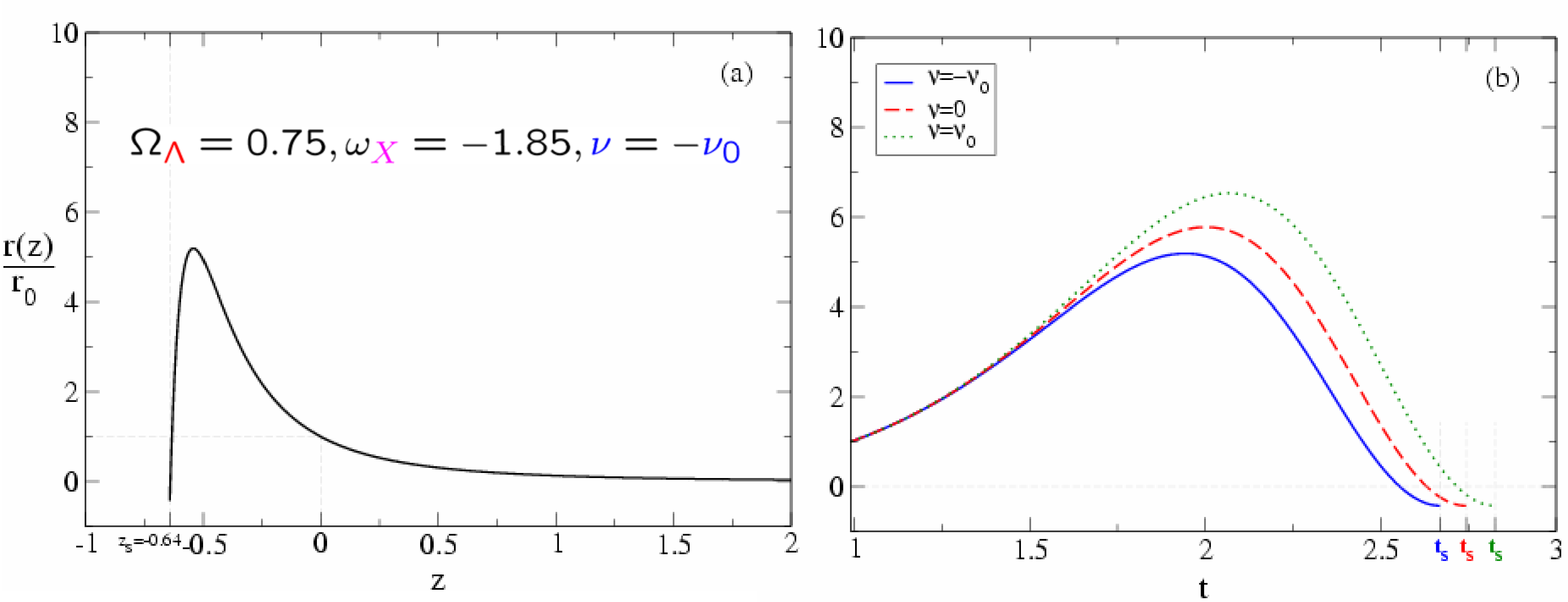
$$\left(\nu_0 = \frac{1}{12\pi} \simeq 0.026\right)$$

Evolution of the DE Densities



Total and individual DE densities for a cosmon barotropic index of phantom-type ($\omega_X < -1$) and of quintessence-type ($\omega_X \gtrsim -1$) respectively: **(a)** $\omega_X = -1.85$, $\Omega_\Lambda = 0.75$, $\nu = -\nu_0$; **(b)** $\omega_X = -0.93$, $\Omega_\Lambda = -2$, $\nu = 0.96$.

Evolution of the Ratio $r = \rho_D/\rho_m$

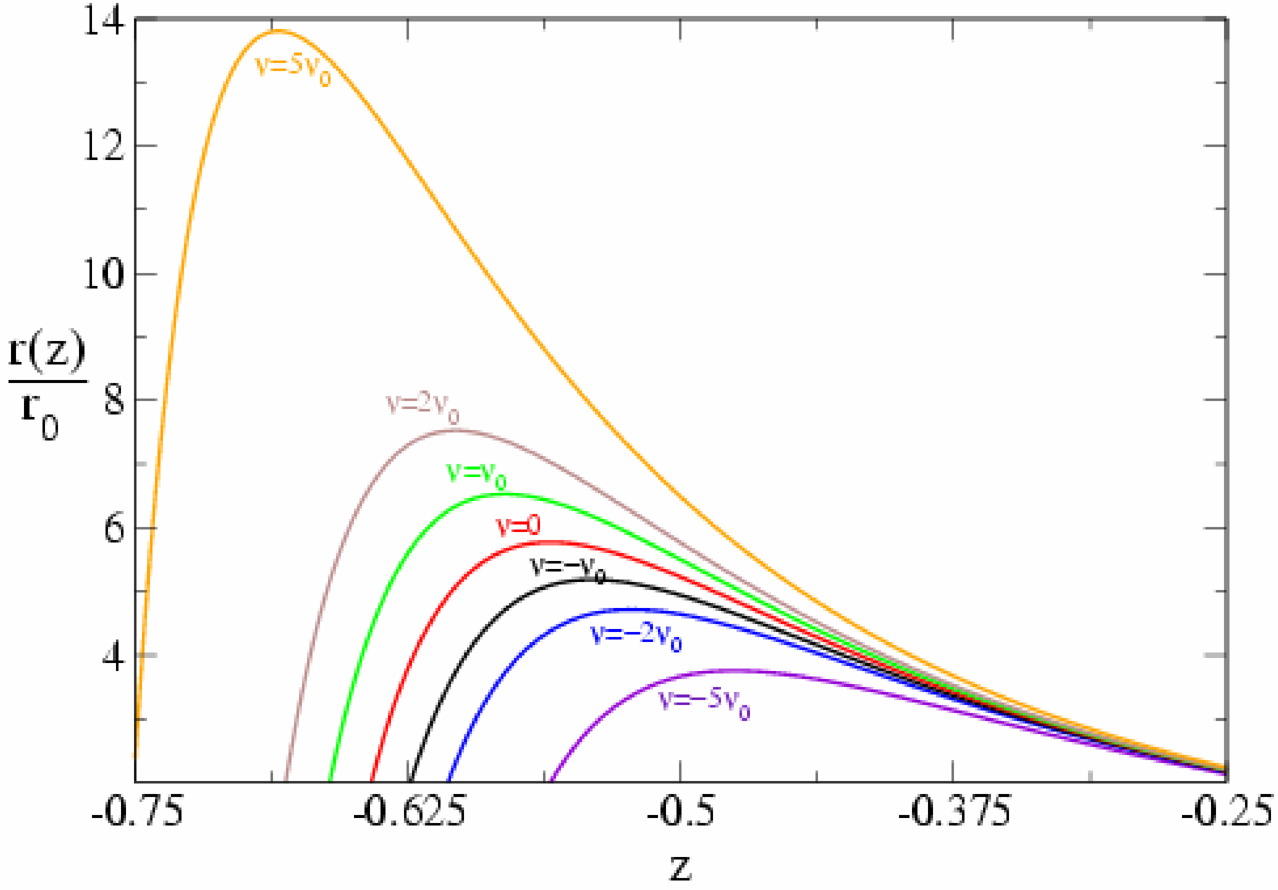


(redshift)

(in Hubble times)

r as function of redshift z and cosmic time t

Evolution of the Ratio $r = \rho_D/\rho_m$



Evolution of r for $\omega_X = -1.85$, $\Omega_\Lambda = 0.75$ and different ν

Asymptotic regime of the EOS in the past

$$\Omega_D(z \gg 1) = \begin{cases} -\frac{\epsilon}{\omega_m - \omega_X + \epsilon}\,\Omega_m^0\,(1+z)^{\alpha_m}, & \text{for } \nu \neq 0 \\ \Omega_\Lambda^0 & \text{for } \nu = 0, \quad \alpha_X < 0 \\ \Omega_X^0\,(1+z)^{\alpha_X} & \text{for } \nu = 0, \quad \alpha_X > 0\,. \end{cases}$$

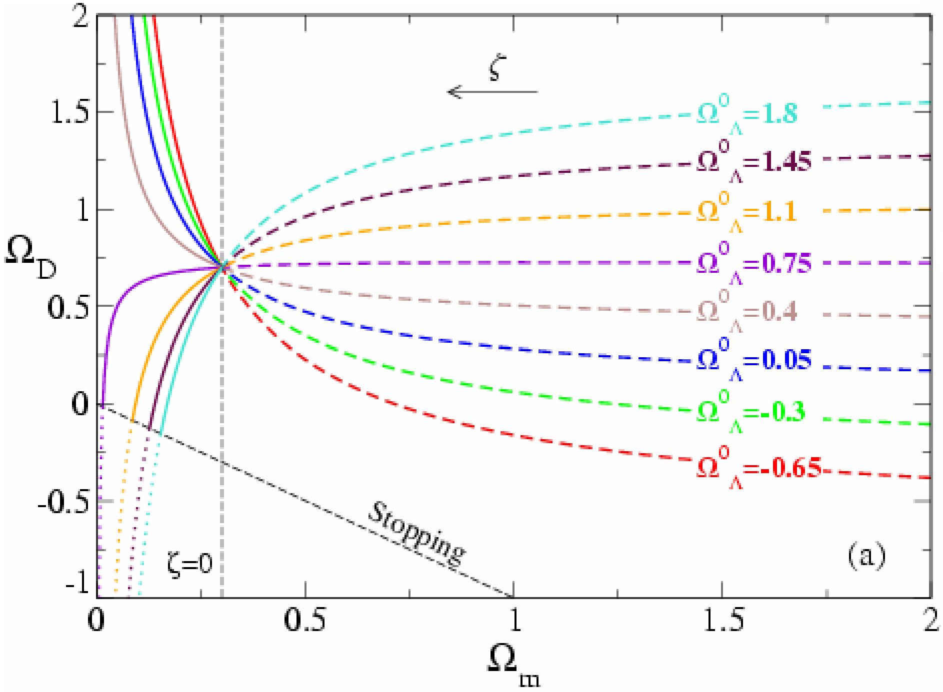
$$\omega_e(z \gg 1) = -1 + (1+\omega_X)\,\frac{\Omega_X(z \gg 1)}{\Omega_D(z \gg 1)} = \begin{cases} \omega_m \;! & \text{for } \nu \neq 0 \\ -1 & \text{for } \nu = 0, \quad \alpha_X < 0 \\ \omega_X & \text{for } \nu = 0, \quad \alpha_X > 0\,. \end{cases}$$

Possible observable effect,
"renormalization" of Ω_m^0 :

$$\Omega_m^0 \rightarrow \Omega_m^0 \left(1 - \frac{\epsilon}{\omega_m - \omega_X + \epsilon}\right) \Rightarrow \frac{\delta\Omega_m^0}{\Omega_m^0} \simeq 10\%$$

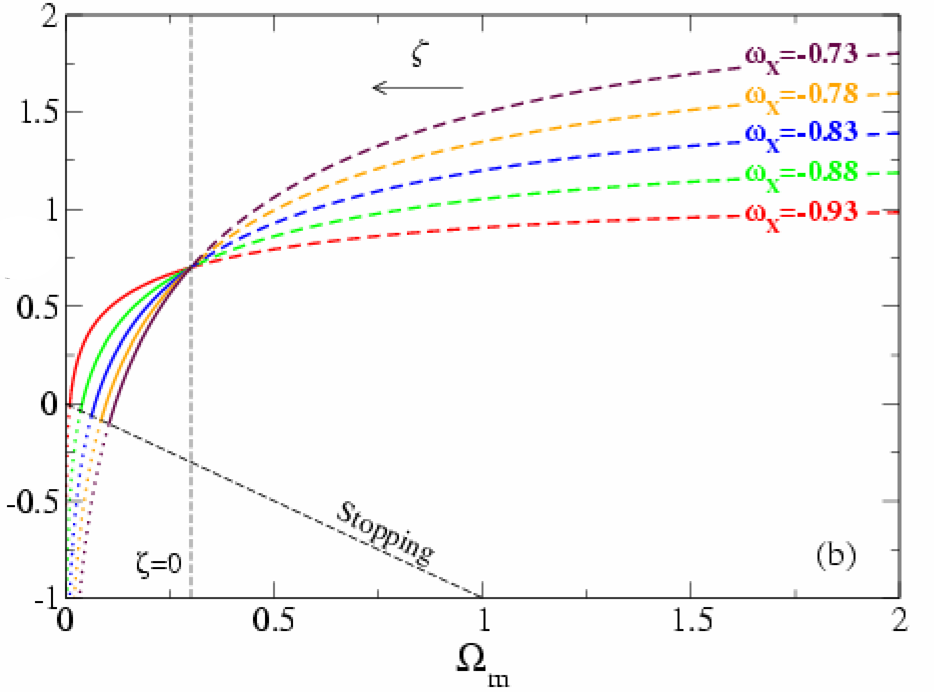
Phase trajectories of the cosmological system

$(\omega_X = -1.85,\ \nu = -\nu_0)$



saddle point

$(\Omega_\Lambda = -2,\ \nu = 0.96)$



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Conclusions

- **Dynamical dark energy** ρ_D can be mimicked by a variable Λ;
- In QFT we generally expect Λ/G to be variable: $\Lambda = \Lambda(z)$; its value should have run in the course of the Universe evolution due to quantum effects;
- A variable Λ/G model can be mapped unambiguously to an **effective** "**DE picture**" where matter and **DE** are conserved separately;
- In the **DE picture** the variable Λ-model has an **effective EOS** $\boxed{\omega_{\rm eff} = \omega_{\rm eff}(z)}$ which can be of **quintessence** and **phantom** type;
- This scenario could naturally explain the possibly observed crossing of the $\boxed{\omega_{\rm eff} = -1}$ barrier near our time, without resorting to scalar fields;
- Adding a **cosmon** ("**ΛXCDM model**") $\Rightarrow$ similar **effective EOS** features while explaining the **Cosmological Coincidence Problem**;

- **Observable effect** :**renormalization** of $\Omega_{\rm m}$ when comparing intermediate redshift data (from supernovae) and high-z (from CMB);

$$\Downarrow$$

- **Moral**: high precision cosmology experiments in the near future, like **SNAP** and **PLANCK**, should bear in mind this possibility!!

Decoupling and Λ running

◇ We expect from dimensional analysis, decoupling theorem and general covariance, that the **RGE** for the physical Λ may take in principle the form

$$(4\pi)^2 \frac{d\Lambda}{d \ln \mu} = \beta_\Lambda = \sum_{n=0}^{\infty} \sum_i \alpha_{in}\, \mu^{2n}\, \mathcal{M}_i^{4-2n}$$

$$= \sum_i A_i\, m_i^4 + \mu^2 \sum_j B_j M_j^2 + \mu^4 \sum_j C_j + \mu^6 \sum_j \frac{D_j}{M_j^2} + \ldots$$

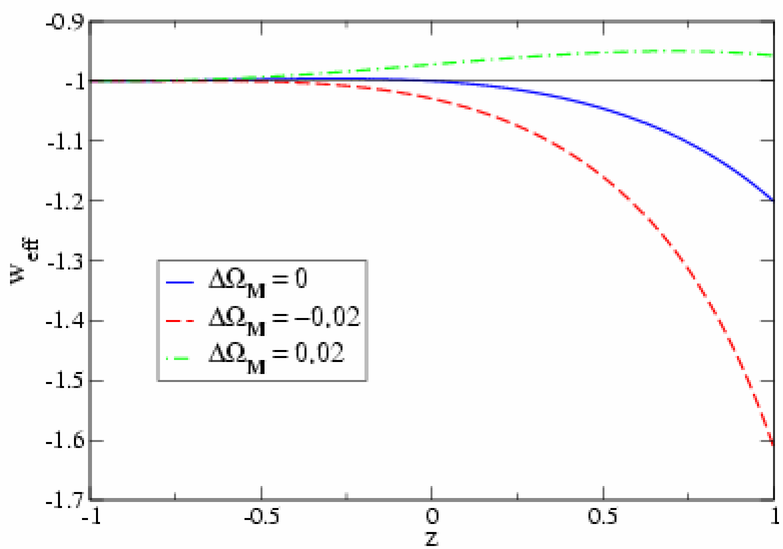
◇ Dimensional analysis not enough to explain β_Λ structure. The fact that only even powers of μ are involved stems from the covariance of the effective action and the identification $\mu \sim H$ (I.L.Shapiro & JS, 2001).

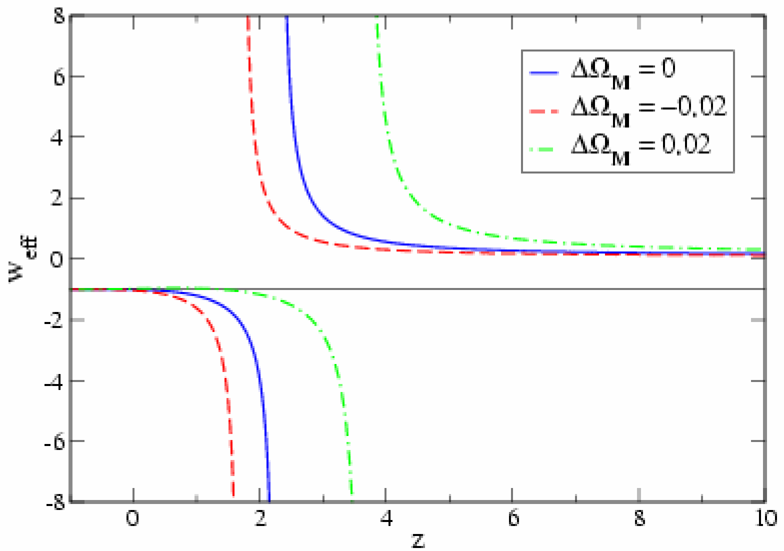
- In the FLRW cosmological framework: $R \sim G T_\mu^\mu \Rightarrow$

$$\mu \sim R^{1/2} \sim H(t)$$

Effective equation of state for the variable Λ as a function of the redshift: $\omega_{\text{eff}} = \omega_{\text{eff}}(z;\nu)$

$$(\text{case } \nu > 0\,; \quad \nu = \nu_0 \equiv 1/12\pi \simeq 0.026)$$

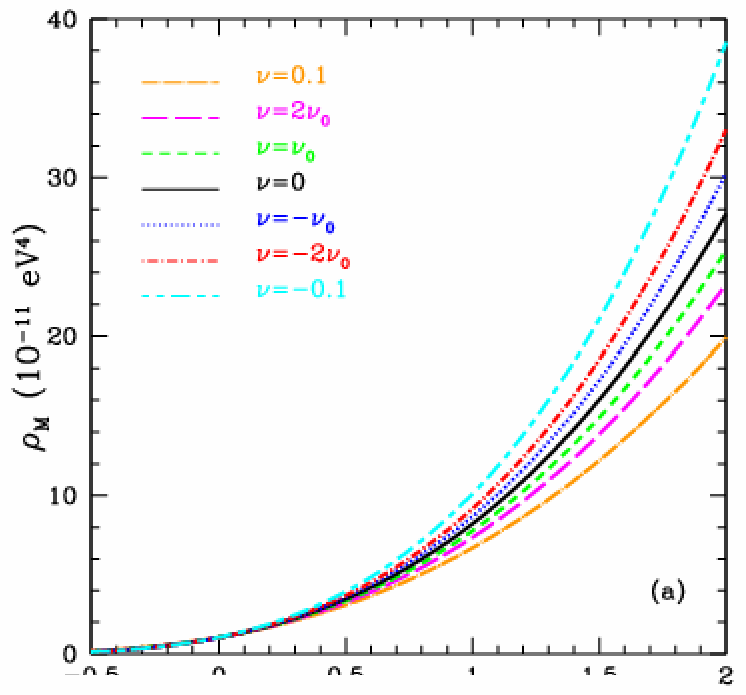


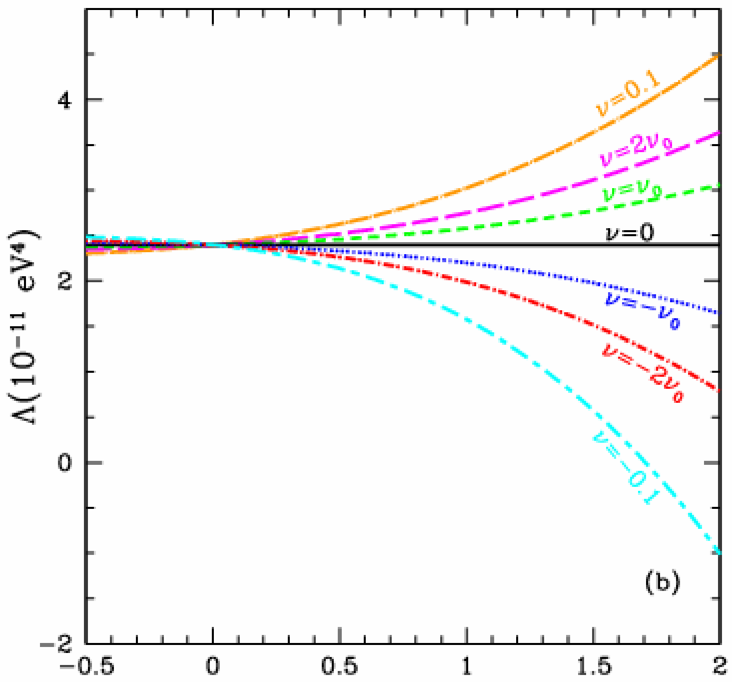


$$(\Omega_M^0 = 0.3\,,\ \Omega_\Lambda^0 = 0.7\,, \quad \Delta\Omega_M = \Omega_M^0 - \tilde{\Omega}_M^0)$$

H. Stefancic, J.S. astro-ph/0505133
Phys. Lett. B (to appear)

Effects on ρ_{M} and Λ, for $\Omega_M^0 = 0.3$, $\Omega_\Lambda^0 = 0.7$



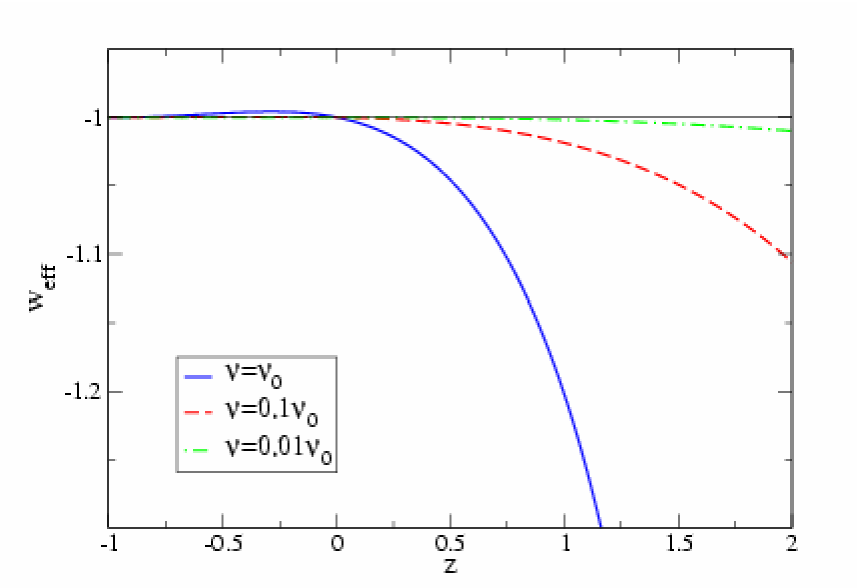


$$\nu = \frac{\sigma\, M^2}{12\pi M_P{}^2}$$

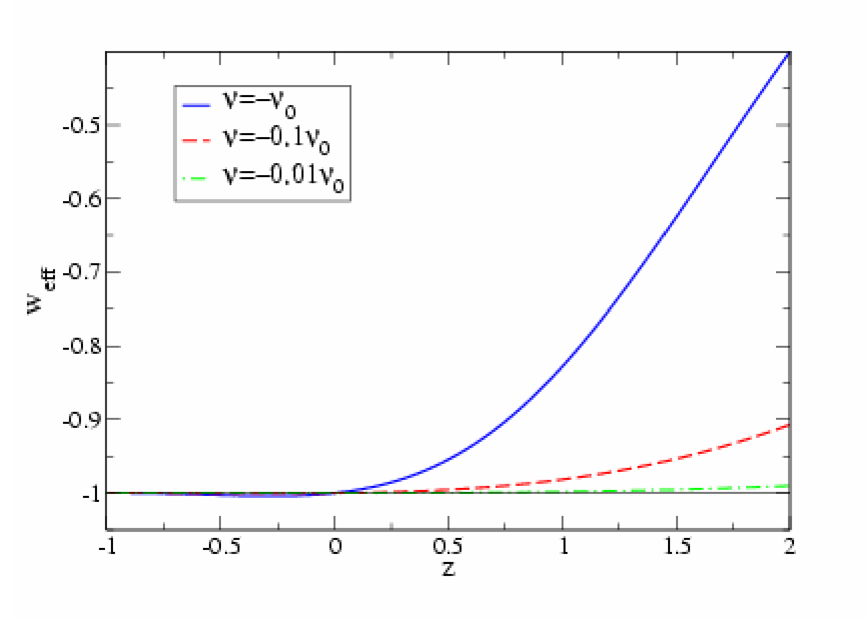
$$\nu_0 \equiv \frac{1}{12\,\pi} \simeq 2.6 \times 10^{-2}$$

C. España-Bonet et al., *JCAP* **0402** (2004) 6.

three values $\nu > 0$:



three values $\nu < 0$:



$$(\Omega_M^0 = 0.3\,,\ \Omega_\Lambda^0 = 0.7)$$

$$\Delta\Omega_M = 0 \quad !!$$

$$(\Delta\Omega_M = \Omega_M^0 - \tilde{\Omega}_M^0)$$

H. Stefancic, J.S. `astro-ph/0505133`
Phys. Lett. B (to appear)

Using matching condition and the redshift variable, Bianchi identity can be written

$$(1+z)\ d(\rho_s+\rho_D) = \alpha\ (\rho_s+\rho_D-\xi_\Lambda)\ dz$$

and from standard EC-law of $\rho_s \Rightarrow$

$$\boxed{\frac{d\rho_D(z)}{dz} = \alpha\,\frac{\rho_D(z)-\xi_\Lambda(z)}{1+z}}$$

Notice:

- $\rho_D(z) > \xi_\Lambda(z)$ (quintessence)
- $\rho_D(z) < \xi_\Lambda(z)$ (phantom)

Transition:
$\rho_D(z^*) = \xi_\Lambda(z^*)$

$\Rightarrow$ The effective **EOS** for Λ is:

$$\omega_{\text{eff}}(z) = -1 + \frac{1}{3}\frac{1+z}{\rho_D}\frac{d\rho_D}{dz} \quad \Rightarrow \quad \omega_{\text{eff}}(z) = -1 + \frac{\alpha}{3}\left(1-\frac{\xi_\Lambda(z)}{\rho_D(z)}\right)$$

For $\alpha = 3$ (MDE) $\Rightarrow$

$$\omega_{\text{eff}}(z) = -\frac{\xi_\Lambda(z)}{\rho_D(z)}$$

Finally, we note that this kind of scenario can also be considered for a variable $\Lambda = \Lambda(z)$ with a standard EC-law for matter.

$$\rho_s(z) = \rho_s(0)\,(1+z)^{\alpha}$$

Bianchi ident. $\Rightarrow \quad d\xi_{\Lambda}/dt = -(\rho_s/G_0)\,dG/dt$

$$\frac{d\rho_D(z_1)}{dz} = -\alpha\,(1+z_1)^{\alpha-1}\int_{z^*}^{z_1}\frac{dz'}{(1+z')^{\alpha}}\frac{d\xi_{\Lambda}(z')}{dz'}$$

$$= \alpha(1+z_1)^{\alpha-1}\,(\rho_s(0)/G_0)[G(z_1)-G(z^*)]\,.$$

In this case the quintessence $\rightarrow$ phantom completely controled by a variable $G = G(z)$

if G is asymptotically free $\Rightarrow$ quintessence for $z_1 \leqslant z \leqslant z^*$; if G is "IR free" $\Rightarrow$ phantom

Consider the previous Theorem and compute

$$\frac{d\rho_D(z_1)}{dz} = -\alpha\,(1+z_1)^{\alpha-1}\int_{z^*}^{z_1}\frac{dz'}{(1+z')^{\alpha}}\frac{d\xi_\Lambda(z')}{dz'}$$

Naive expectation: for increasing/decreasing ξ_Λ with redshift $\Rightarrow \omega_{\rm eff}(z) \gtrsim -1/\omega_{\rm eff}(z) \lesssim -1$.

(**False** in general !)

e.g. if $d\xi_\Lambda(z)/dz < 0$ and $z^* < z_1$, the observer at z_1 will see quintessence (counterintuitive!)

But if $z_1 < z^*$ he/she will see phantom DE. If $z_1 = 0$ this case could just correspond to the present observational data !!