Dark energy equation of state: how far can we go from Lambda ?

Hrvoje Štefančić
Departament ECM
Universitat de Barcelona, Barcelona

The Dark Side of the Universe 2006,
Madrid, June 21, 2006

Acceleration of the universe

- Dark energy
  - Dark energy equation of state $$p_d = w\rho_d$$
  - CMB+2dFGRS+SDSS+SNIa
    $$w = -1.062^{+0.128}_{-0.079}$$
    Spergel at al., astro-ph/0603449
- Dynamical dark energy "close" to Lambda (equation of state (EOS) description)
  - "Expansion" around the vacuum EOS
  - Crossing of the cosmological constant boundary

“Expansion” around the vacuum EOS

S. Nojiri, S.D. Odintsov,
Phys. Rev. D 70 (2004)103522

Dark energy equation of state

$$p_d = -\rho_d - A\rho_d^{\alpha}$$

Models deviation
from the CC EOS

Dark energy density scaling

$$\rho_d = \rho_{d,0}\left(1 + 3\tilde{A}(1-\alpha)\ln\frac{a}{a_0}\right)^{1/(1-\alpha)}$$

H. Š., Phys. Rev. D 71 (2005) 084024

Scale factor evolution

Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}(\rho_d + \rho_m)$$

Scale factor evolution

$$\tilde{A} = A\rho_{d,0}^{\alpha-1}$$

$$\alpha \neq 1/2$$

$$\left(1 + 3\tilde{A}(1-\alpha)\ln\frac{a_1}{a_0}\right)^{\frac{1-2\alpha}{2(1-\alpha)}} - \left(1 + 3\tilde{A}(1-\alpha)\ln\frac{a_2}{a_0}\right)^{\frac{1-2\alpha}{2(1-\alpha)}}$$

$$= \frac{3}{2}\tilde{A}(1-2\alpha)\Omega_{d,0}^{1/2}H_0(t_1 - t_2)\,.$$

$$\alpha = 1/2$$

$$\ln\frac{1 + \frac{3}{2}\tilde{A}\ln\frac{a_1}{a_0}}{1 + \frac{3}{2}\tilde{A}\ln\frac{a_2}{a_0}} = \frac{3}{2}\tilde{A}\Omega_{d,0}^{1/2}H_0(t_1 - t_2)$$

Parametric space

$$\tilde{A} > 0, \ \alpha > 1$$

Singularity at finite time and finite scale factor

$$\tilde{A} > 0, \ 1/2 < \alpha < 1$$

Finite time singularity – "standard" big rip

$$\tilde{A} > 0, \ \alpha = 1/2$$

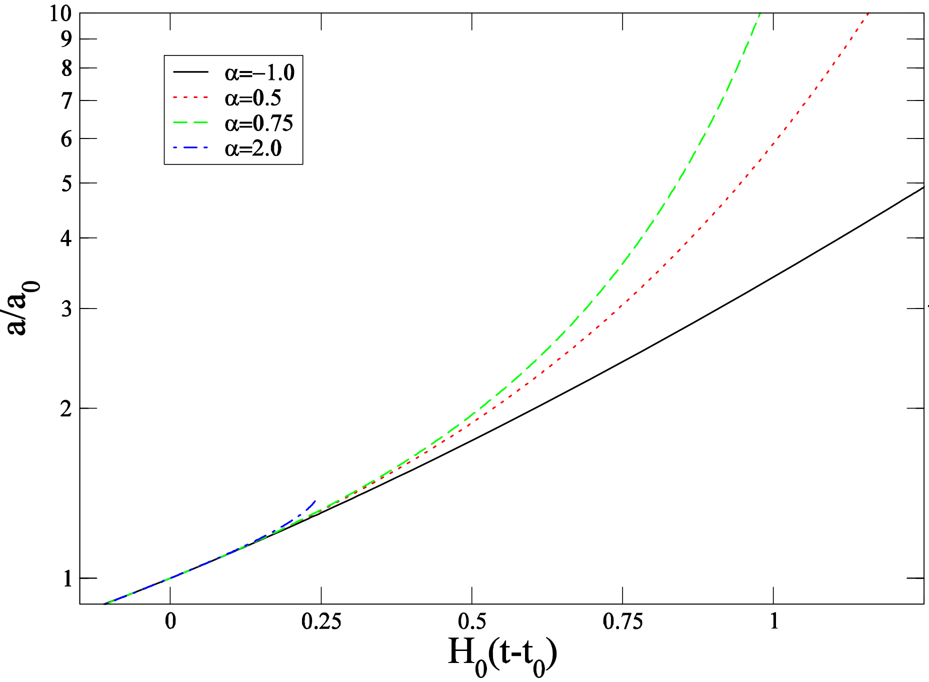
No singularity

$$\tilde{A} > 0, \ \alpha < 1/2$$

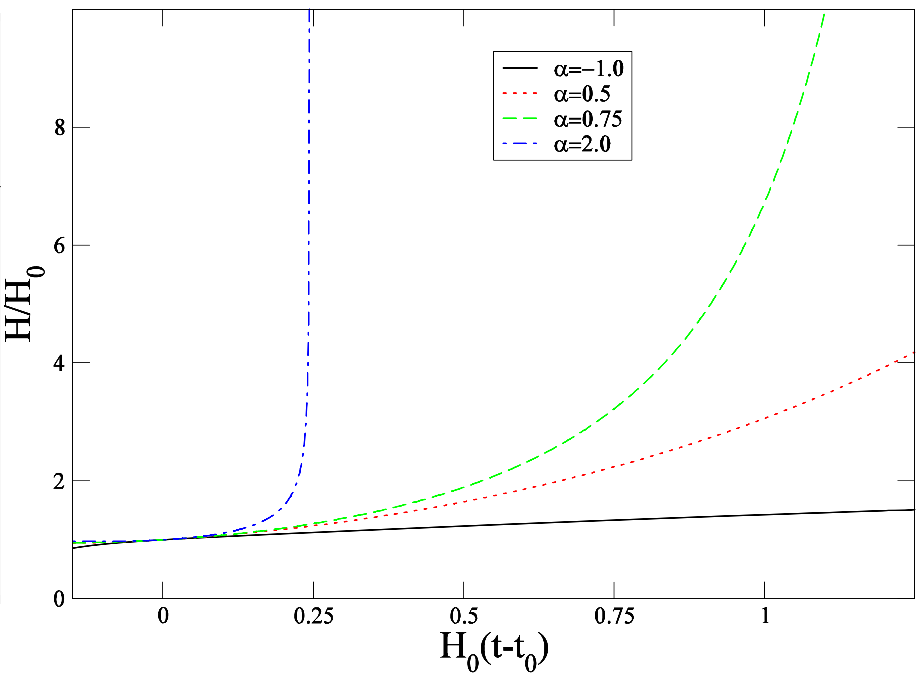
No singularity

Numerical solutions

$$\tilde{A} = 1,\ \Omega_{d,0} = 0.7,\ \Omega_{m,0} = 0.3$$



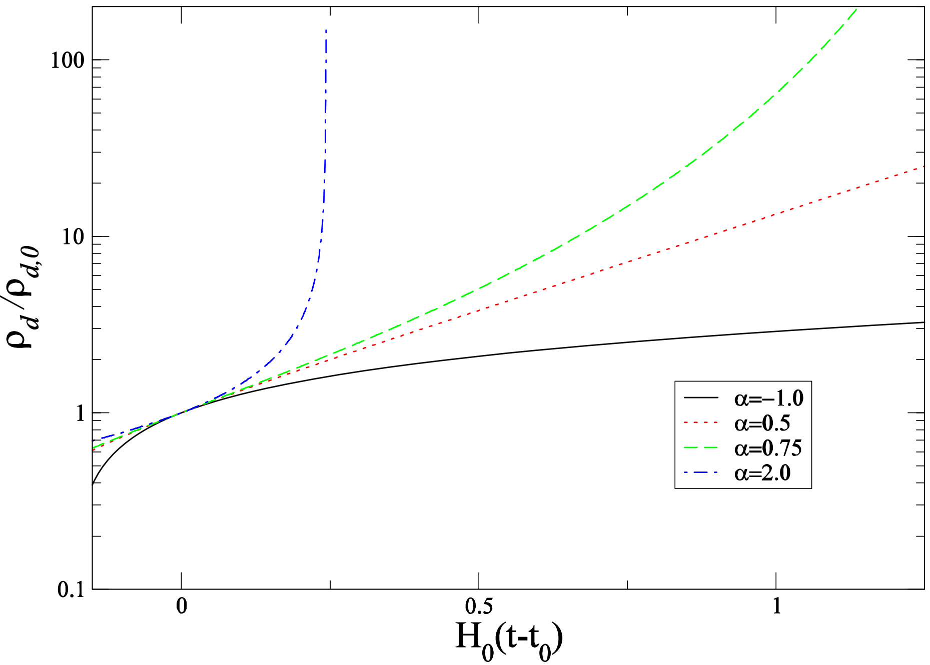
scale factor

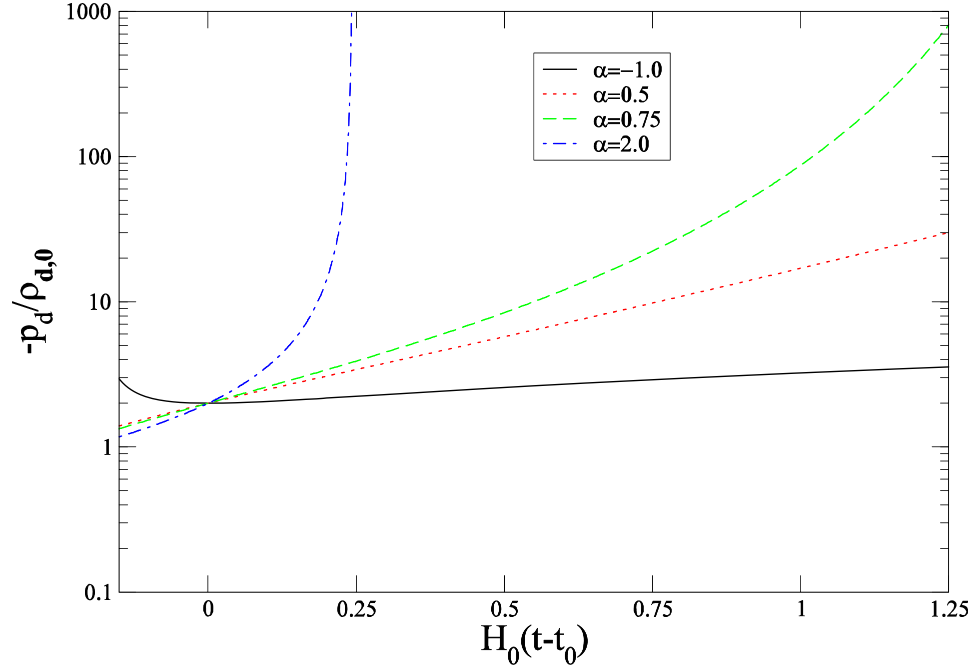


Hubble parameter

Numerical solutions

$$\tilde{A} = 1,\ \Omega_{d,0} = 0.7,\ \Omega_{m,0} = 0.3$$





dark energy density

-dark energy pressure

Parametric space:

$$\tilde{A} < 0, \ \alpha < 0$$

Singularity at: finite time, finite scale factor,

finite dark energy and total energy density

Divergence of the dark energy pressure

and acceleration

$$\ddot{a} \rightarrow -\infty \,, \qquad p_d \rightarrow +\infty$$

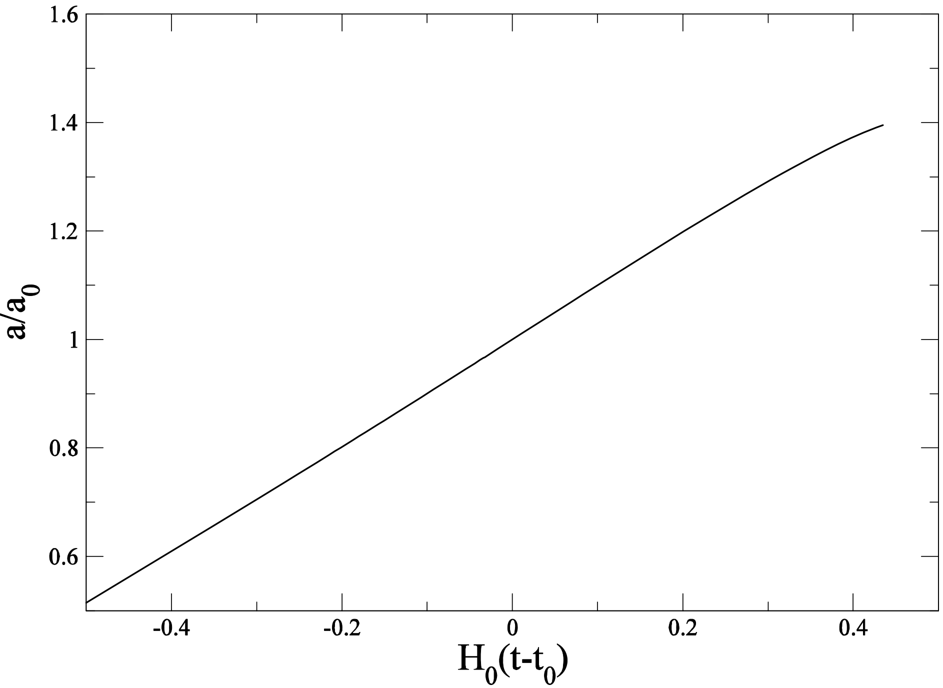
Barrow (2004)

$$\tilde{A} < 0 \,, \qquad \alpha < 1$$

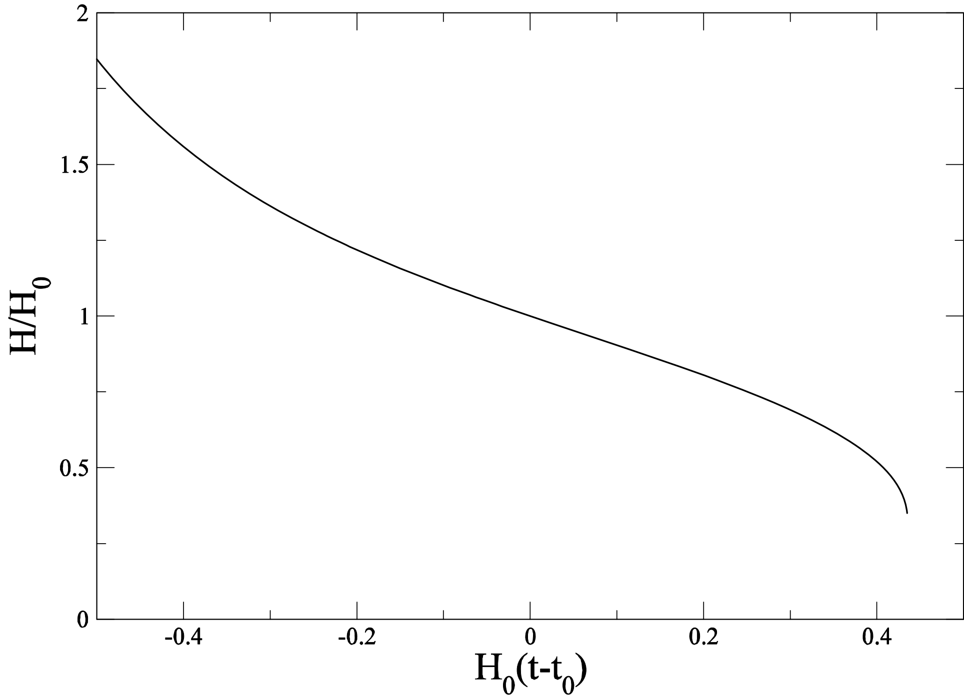
Transient acceleration

Numerical solutions

$\Omega_{d,0} = 0.7,\ \Omega_{m,0} = 0.3,\ \tilde{A} = -0.5$ and $\alpha = -1$



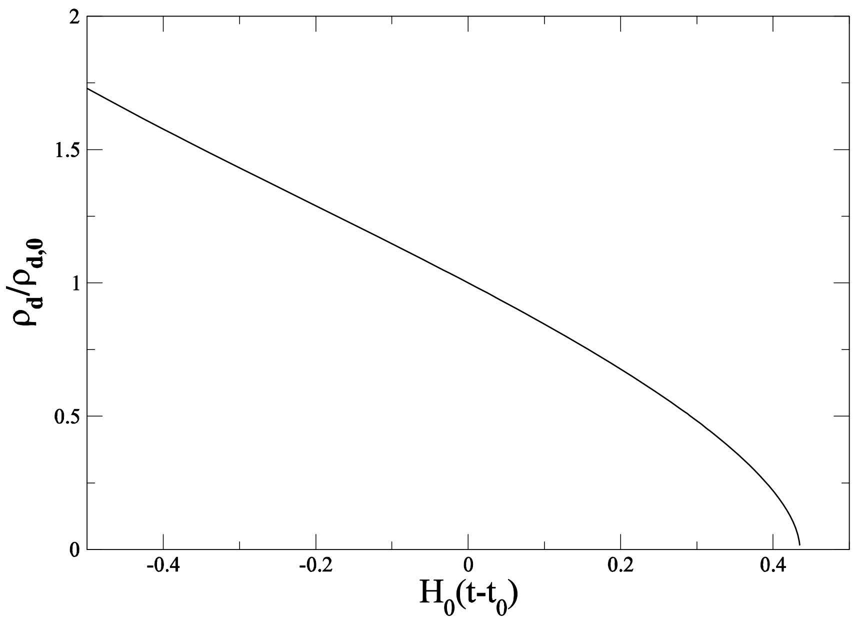
scale factor



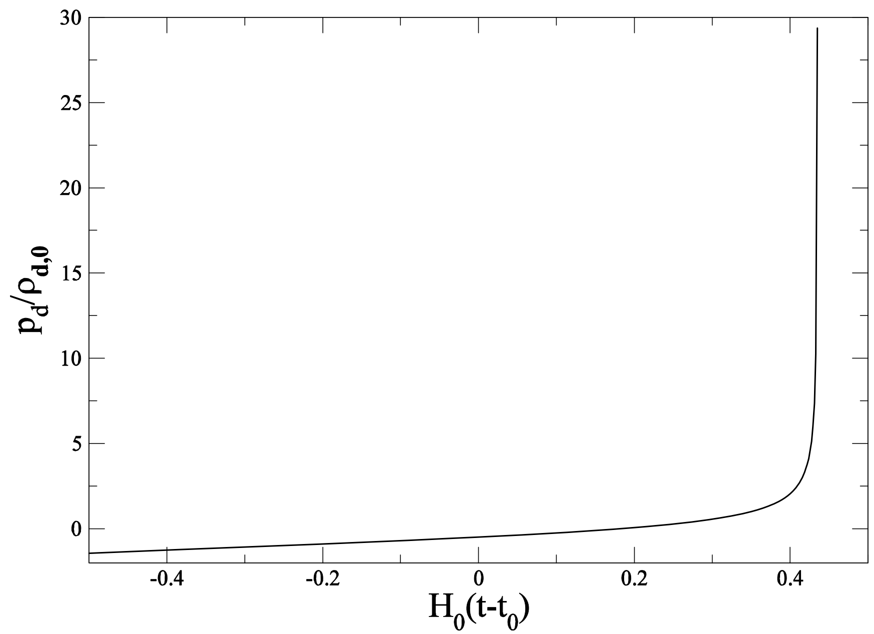
Hubble parameter

Numerical calculations

$$\Omega_{d,0} = 0.7,\ \Omega_{m,0} = 0.3,\ \tilde{A} = -0.5 \text{ and } \alpha = -1$$

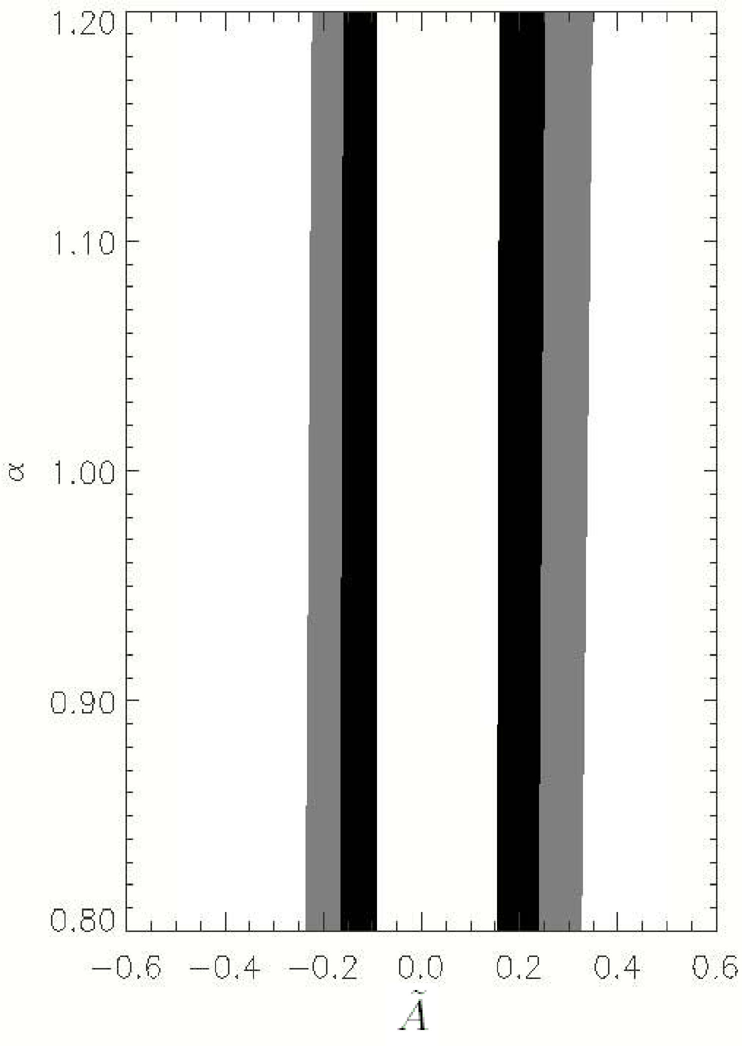


dark energy density



dark energy pressure

Observational constraints



- J.S. Alcaniz, H.Š., astro-ph/0512622
- WMAP3+SDSS+SNIa

Dark energy equation of state description of the CC boundary crossing - goals

- Dark energy equation of state $(\rho(t), p(t))$
- The form of EOS which allows the transition
- The mechanism behind the transition
- The conditions on the (generalized) model parameters

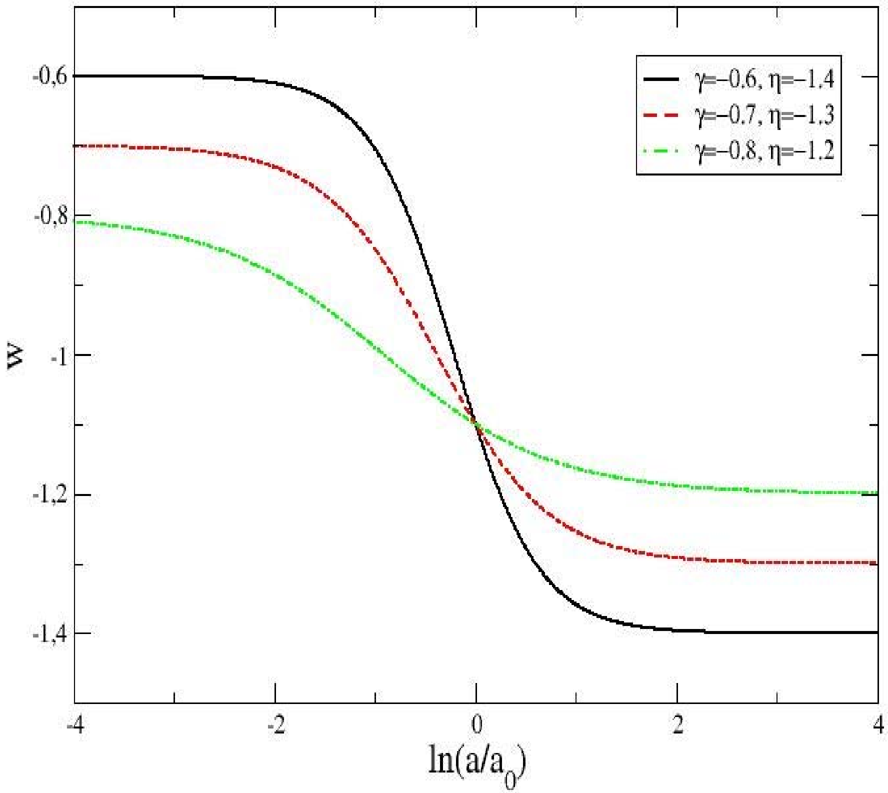
A simple model of transition – EOS

$$\rho = C_1 \left(\frac{a}{a_0}\right)^{-3(1+\gamma)} + C_2 \left(\frac{a}{a_0}\right)^{-3(1+\eta)}$$

$$d\rho + 3(\rho + p)\frac{da}{a} = 0$$

$$p = \gamma C_1 \left(\frac{a}{a_0}\right)^{-3(1+\gamma)} + \eta C_2 \left(\frac{a}{a_0}\right)^{-3(1+}$$

$$w = \frac{\gamma + \eta \frac{\gamma - w_0}{w_0 - \eta} \left(\frac{a}{a_0}\right)^{3(\gamma-\eta)}}{1 + \frac{\gamma - w_0}{w_0 - \eta} \left(\frac{a}{a_0}\right)^{3(\gamma-\eta)}}$$



H. Štefančić, Phys. Rev. D 71 (2005)124036

A simple model of transition – equation of state

$$\left(\frac{a}{a_0}\right)^{-3} = \left(\frac{\gamma\rho - p}{(\gamma-\eta)C_2}\right)^{1/(1+\eta)} = \left(\frac{p - \eta\rho}{(\gamma-\eta)C_1}\right)^{1/(1+\gamma)}$$

$$\frac{p - \eta\rho}{(\gamma-\eta)C_1} = \left(\frac{\gamma\rho - p}{(\gamma-\eta)C_2}\right)^{(1+\gamma)/(1+\eta)}$$

implicitly defined equation of state

what is the mechanism of crossing?

Generalization of the model

$$A\rho + Bp = (C\rho + Dp)^{\alpha} \qquad \alpha \neq 1$$

$$\rho = \frac{(C + Dw)^{\alpha/(1-\alpha)}}{(A + Bw)^{1/(1-\alpha)}}$$

$$E = A/B \qquad F = C/D$$

$$\left(\frac{\alpha}{(F+w)(1+w)} - \frac{1}{(E+w)(1+w)}\right) dw = 3(\alpha - 1)\frac{da}{a}$$

Generalized model - solutions

$$E \neq 1,\ F \neq 1$$

$$\left|\frac{w+F}{w_0+F}\right|^{\alpha/(1-F)} \left|\frac{w+E}{w_0+E}\right|^{-1/(1-E)} \left|\frac{1+w}{1+w_0}\right|^{1/(1-E)-\alpha/(1-F)} = \left(\frac{a}{a_0}\right)^{3(\alpha-1)}$$

$$\alpha_{\rm cross} = (1-F)/(1-E)$$

CC boundary crossing

$$\left|\frac{w+F}{w_0+F}\right| \left|\frac{w+E}{w_0+E}\right|^{-1} = \left(\frac{a}{a_0}\right)^{3(E-F)}$$

$$\frac{w+\frac{\alpha E-F}{\alpha-1}}{(F+w)(E+w)(1+w)}dw = 3\frac{da}{a}$$

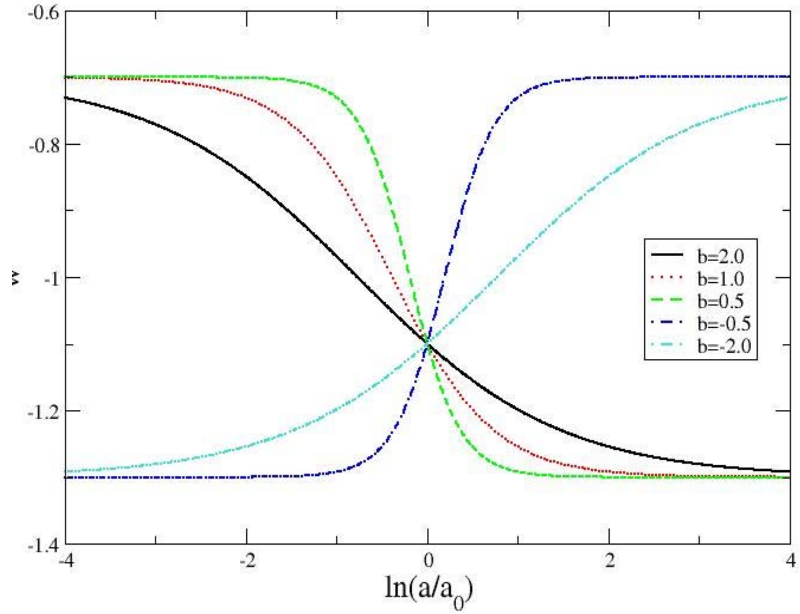
Elimination of boundaries - cancellation

$$w_* = -(\alpha E - F)/(\alpha - 1)$$

A simple and a more general model

$$\rho = \left(C_1 \left(\frac{a}{a_0} \right)^{-3(1+\gamma)/b} + C_2 \left(\frac{a}{a_0} \right)^{-3(1+\eta)/b} \right)^b$$

$$\frac{p - \eta\rho}{(\gamma - \eta)C_1} = \rho^{((1-b)(\gamma-\eta))/(b(1+\eta))} \left(\frac{\gamma\rho - p}{(\gamma - \eta)C_2} \right)^{(1+\gamma)/(1+\eta)}$$



Generalized EOS

$$A\rho + Bp = (C\rho + Dp)^{\alpha}(M\rho + Np)^{\beta}$$

Nontrivial EOS exhibiting the CC boundary transition

$$A\rho^{2n+1} + Bp^{2n+1} = (C\rho^{2n+1} + Dp^{2n+1})^{\alpha}$$

General class of EOS exhibiting the transition

$$p = p(w)\,, \qquad \rho = \rho(w)$$

class of EOS

$$\frac{1}{\rho}\frac{d\rho}{dw} = (1+w)g(w)$$

condition

$$\lim_{w\to -1} g(w) = \mathrm{finite(nonzero)}$$

Dynamics of w

$$g(w)dw = -3\frac{da}{a}$$

Conclusions

- The model based on the “expansion” around the vacuum EOS provides a variety of phenomena (“fate of the universe”, sudden future singularities, transient acceleration) in different parameter regimes
- Observational constraints – currently do not provide a clear selection of the parameter regime
- The crossing of the CC boundary described in terms of the implicitly defined dark energy EOS
- Mechanism of the CC boundary crossing - cancellation of the term associated with the CC
- CC boundary crossing as an effective phenomenon (indication of an alternative mechanism of the accelerated expansion)?

Auxilliary slides

Parametric space: $\tilde{A} > 0,\ \alpha > 1$

Dark energy density

$$\rho_d = \rho_{d,0} \left(1 - 3\tilde{A}(\alpha - 1) \ln \frac{a}{a_0} \right)^{-1/(\alpha-1)}$$

Maximal value of the scale factor

$$a_{max} = a_0 e^{1/(3\tilde{A}(\alpha-1))}$$

Singularity at finite time and finite scale factor

$$a = a_0 exp \left[\frac{1}{3\tilde{A}(\alpha - 1)} \left(1 - \left[\frac{3}{2} \tilde{A}(2\alpha - 1) \Omega_{d,0}^{1/2} (t_{max} - t) \right]^{\frac{2(\alpha-1)}{2\alpha-1}} \right) \right]$$

Parametric space: $\tilde{A} > 0,\ 1/2 < \alpha < 1$

Minimal value of the scale factor

$$a_{min} = a_0 e^{-1/(3\tilde{A}(1-\alpha))}$$

Evolution of the scale factor

$$\left(1 + 3\tilde{A}(1-\alpha)\ln\frac{a}{a_0}\right) = \left[\left(1 + 3\tilde{A}(1-\alpha)\ln\frac{a_*}{a_0}\right)^{-\frac{2\alpha-1}{2(1-\alpha)}} - \frac{3}{2}\tilde{A}(2\alpha-1)\Omega_{d,0}^{1/2}H_0(t-t_*)\right]^{-\frac{2(1-\alpha)}{2\alpha-1}}$$

Finite time singularity – “standard” big rip

$$t_{rip} = t_* + \frac{2}{3\tilde{A}(2\alpha-1)\Omega_{d,0}^{1/2}H_0}\left(1 + 3\tilde{A}(1-\alpha)\ln\frac{a_*}{a_0}\right)^{-\frac{2\alpha-1}{2(1-\alpha)}}$$

Parametric space: $\tilde{A} > 0, \ \alpha = 1/2$

No singularity

$$a = a_0 exp\left[\frac{2}{3\tilde{A}}\left[\left(1 + \frac{3}{2}\tilde{A}\ln\frac{a_*}{a_0}\right) exp\left(\frac{3}{2}\tilde{A}\Omega_{d,0}^{1/2} H_0(t - t_*)\right) - 1\right]\right]$$

Parametric space: $\tilde{A} > 0, \ \alpha < 1/2$

$$a = a_0 exp\left[\frac{1}{3\tilde{A}(1-\alpha)}\left(\left[\left(1+3\tilde{A}(1-\alpha)\ln\frac{a_*}{a_0}\right)^{\frac{1-2\alpha}{2(1-\alpha)}} + \frac{3}{2}\tilde{A}(1-2\alpha)\Omega_{d,0}^{1/2}H_0(t-t_*)\right]^{\frac{2(1-\alpha)}{1-2\alpha}} - 1\right)\right]$$

Parametric space: $\tilde{A} < 0, \ \alpha > 1$

Dark energy density

$$\rho_d = \rho_{d,0}\left(1 + 3\tilde{A}(1-\alpha)\ln\frac{a}{a_0}\right)^{-1/(\alpha-1)}$$

Evolution of the scale factor

$$a = a_0 exp\left[\frac{1}{3\tilde{A}(1-\alpha)}\left(\left[\left(1 + 3\tilde{A}(1-\alpha)\ln\frac{a_*}{a_0}\right)^{\frac{1-2\alpha}{2(1-\alpha)}} + \frac{3}{2}\tilde{A}(1-2\alpha)\Omega_{d,0}^{1/2}H_0(t-t_*)\right]^{\frac{2(1-\alpha)}{1-2\alpha}} - 1\right)\right]$$

Parametric space: $\tilde{A} < 0 \quad \alpha < 1$

Dark energy density vanishes at

$$a_{NULL} = a_0 e^{-1/(3\tilde{A}(1-\alpha))}$$

Cosmology containing only the dark energy component

$\tilde{A} < 0,\ 1/2 < \alpha < 1$ $\quad a_{NULL}$

$\tilde{A} < 0,\ \alpha = 1/2$

reached in infinite time

$$\tilde{A} < 0,\ 0 \leq \alpha < 1/2 \Rightarrow t_{NULL} = -2/(3\tilde{A}(1-2\alpha)\Omega_d^{1/2} H_0)$$

Composition of the universe

- WMAP

  ~ 73 % dark energy

  ~ 23 % dark matter

  ~ 4 % baryonic matter

- Accelerated expansion of the universe

Energy conditions

- Null energy condition: $\rho + p \geq 0$
- Weak energy condition: $\rho \geq 0 \quad \rho + p \geq 0$
- Strong energy condition: $\rho + 3p \geq 0 \quad \rho + p \geq 0$
- Dominant energy condition: $\rho \geq \mid p \mid$

Microscopic models

- Models with negative kinetic term

$$\rho_\phi = -\frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p_\phi = -\frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$w = \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 + V(\phi)}{\frac{1}{2}\dot{\phi}^2 - V(\phi)}$$

- Strong constraints on the applicability of the quantum microscopis models - low effective theory cut-off scale

S.M. Carroll, M. Hoffman, M. Trodden, Phys. Rev. D68 (2003) 023509

J.M. Cline, S. Jeon, G.D. Moore, hep-ph/0311312

Microscopic models

- k-essence models

$$\mathcal{L} = f(\phi)g(X) - V(\phi) \qquad X \equiv \dot{\phi}^2/2$$

A. Melchiorri, L. Mersini, C.J. Ödman, M. Trodden, Phys. Rev. D68 (2003) 043509

GR notation

$$\left\{ \begin{array}{c} \alpha \\ \mu\nu \end{array} \right\} = \frac{1}{2} g^{\alpha\beta} \left[\frac{\partial g_{\mu\beta}}{\partial x^{\nu}} + \frac{\partial g_{\nu\beta}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\beta}} \right]$$

Christoffel symbols of the second kind

Riemann tensor

$$R^{\alpha}_{\eta\beta\gamma} = \left\{ \begin{array}{c} \alpha \\ \beta\eta \end{array} \right\}_{|\gamma} - \left\{ \begin{array}{c} \alpha \\ \eta\gamma \end{array} \right\}_{|\beta} + \left\{ \begin{array}{c} \alpha \\ \tau\gamma \end{array} \right\} \left\{ \begin{array}{c} \tau \\ \beta\eta \end{array} \right\} - \left\{ \begin{array}{c} \alpha \\ \tau\beta \end{array} \right\} \left\{ \begin{array}{c} \tau \\ \gamma\eta \end{array} \right\}$$

Ricci tensor

$$R_{\eta\gamma} = R^{\alpha}_{\eta\alpha\gamma}$$

Ricci scalar

$$R = R^{\eta}_{\eta}$$

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = -8\pi G T^{\mu\nu}$$

Einstein equation

Dark energy

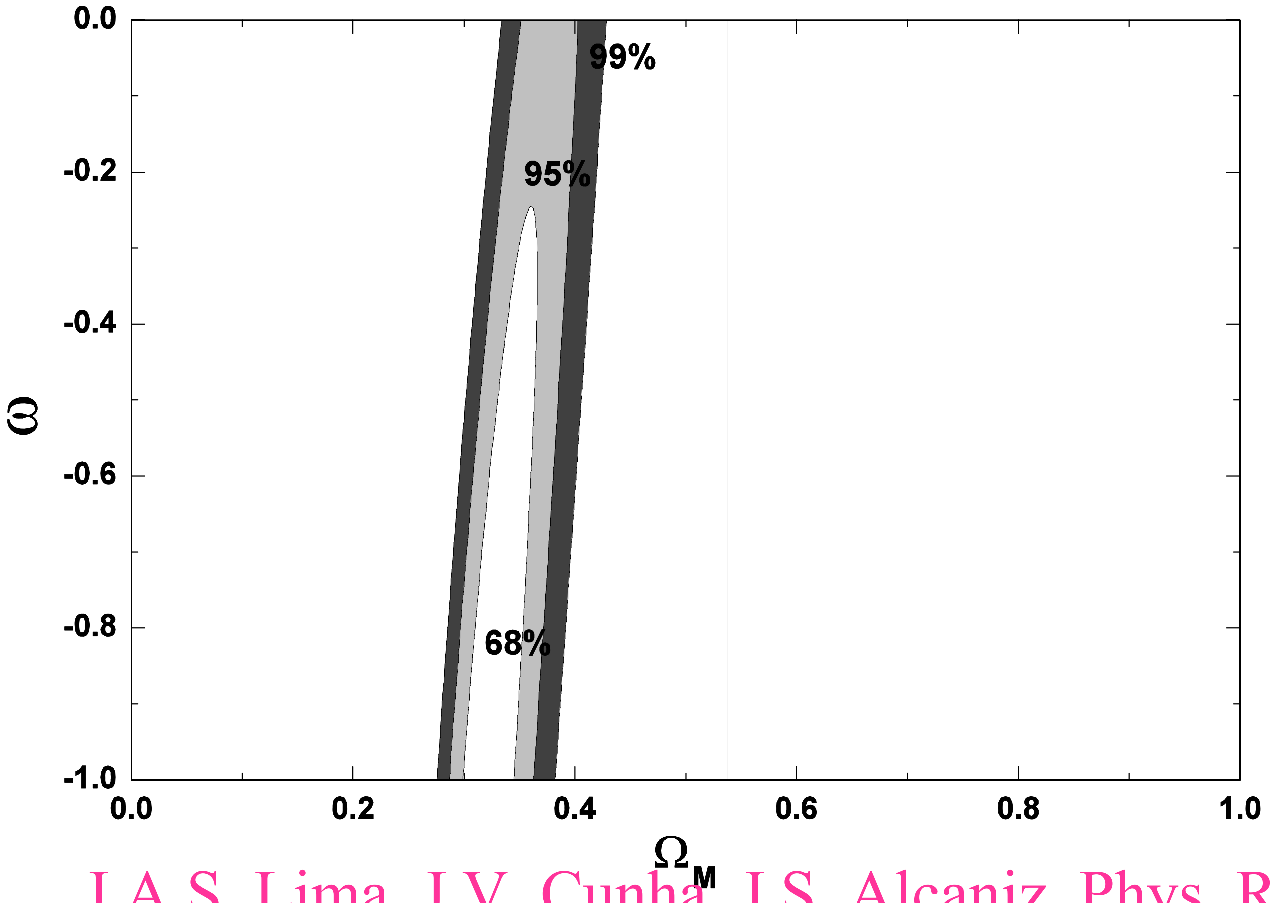
- Accelerated expansion of the universe
- Equation of state

$$p_d = w\rho_d$$

- w > -1: quintessence, Chaplygin gas ...
- w = -1: cosmological term
- w < -1: phantom energy

R.R. Caldwell, Phys. Lett. B545 (2002) 23

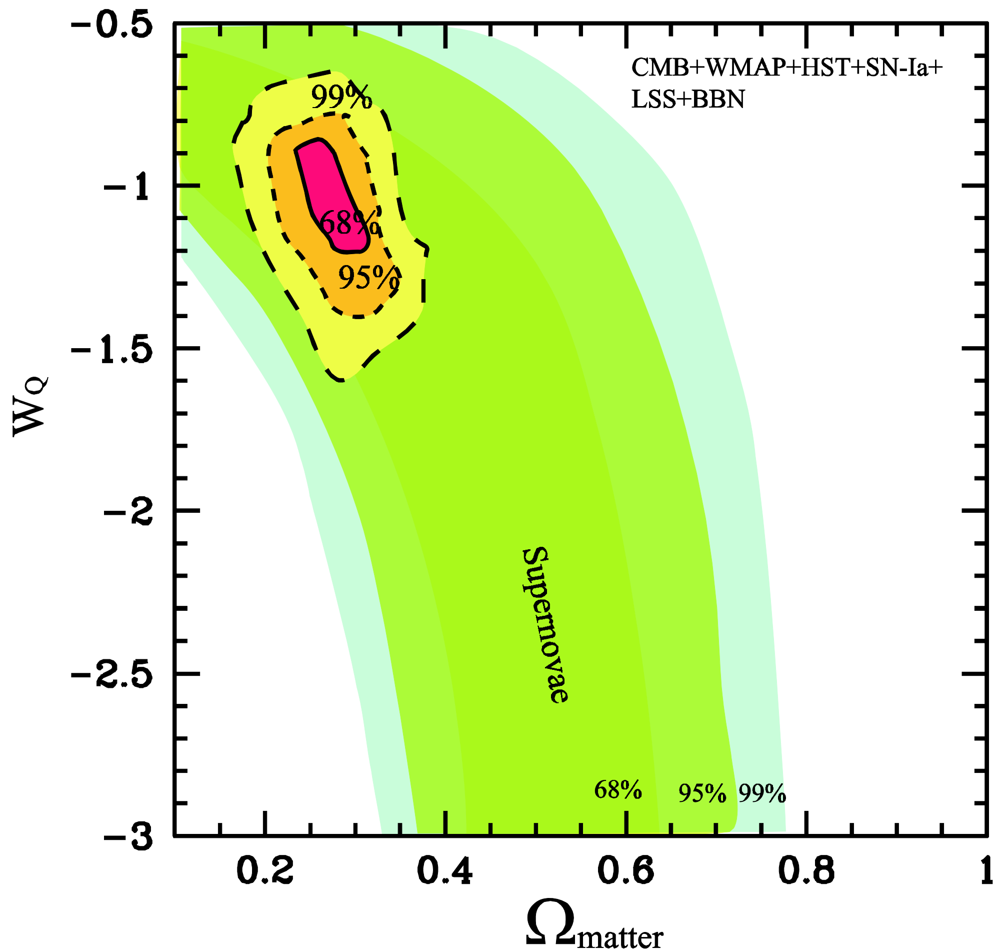
Cosmological observations



Luminosity of galactic clusters in X ray region + baryonic mass density measurments + HST

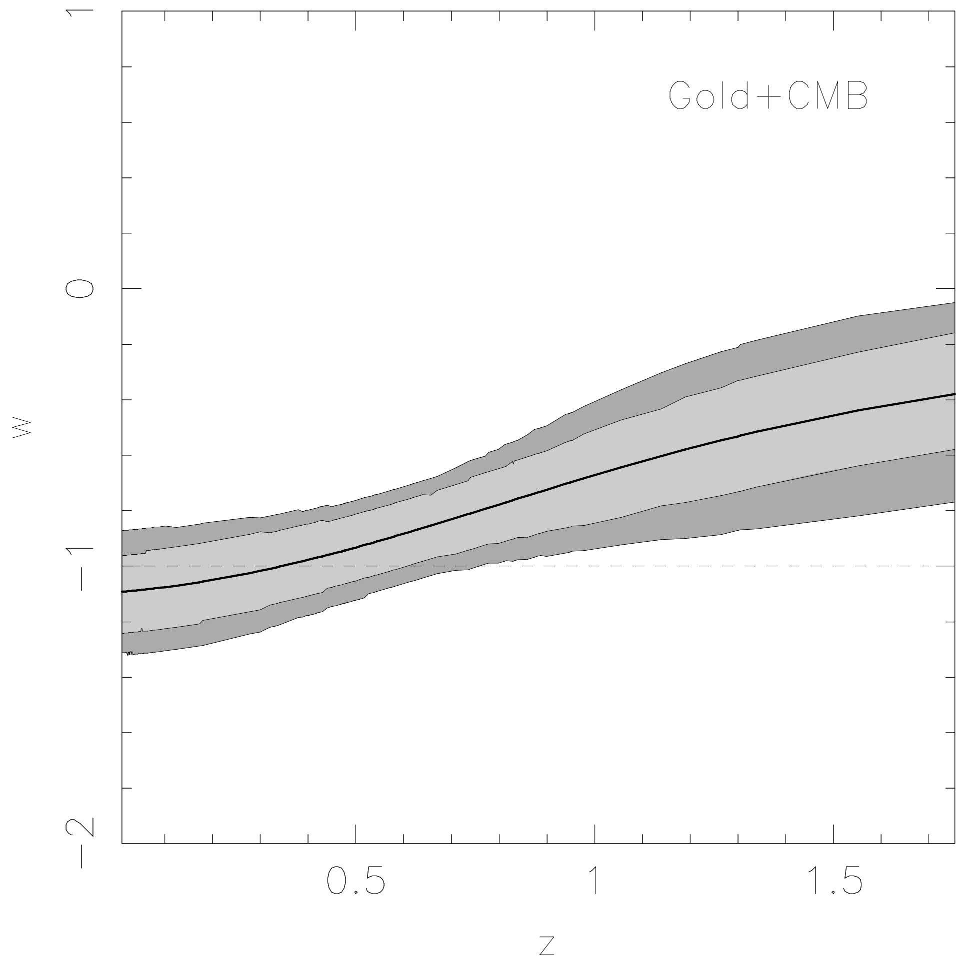
J.A.S. Lima, J.V. Cunha, J.S. Alcaniz, Phys. Rev. D68 (2003) 023510

Cosmological observations



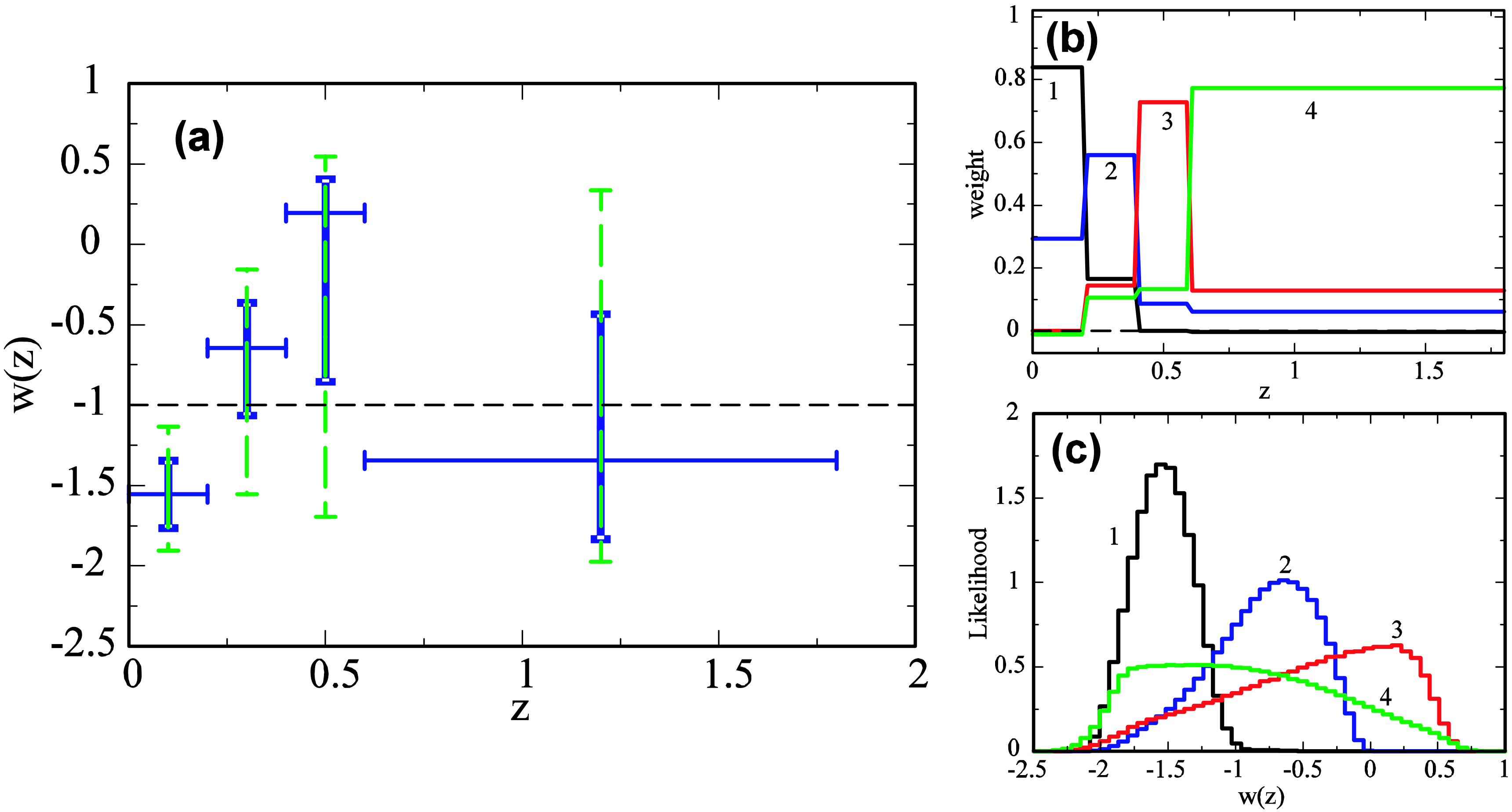
A. Melchiorri, L. Mersini, C.J. Ödman, M. Trodden, Phys. Rev. D68 (2003) 043509

Cosmological observations



U. Alam, V. Sahni, A.A. Starobinsky, JCAP 0406 (2004) 008

Cosmological observations

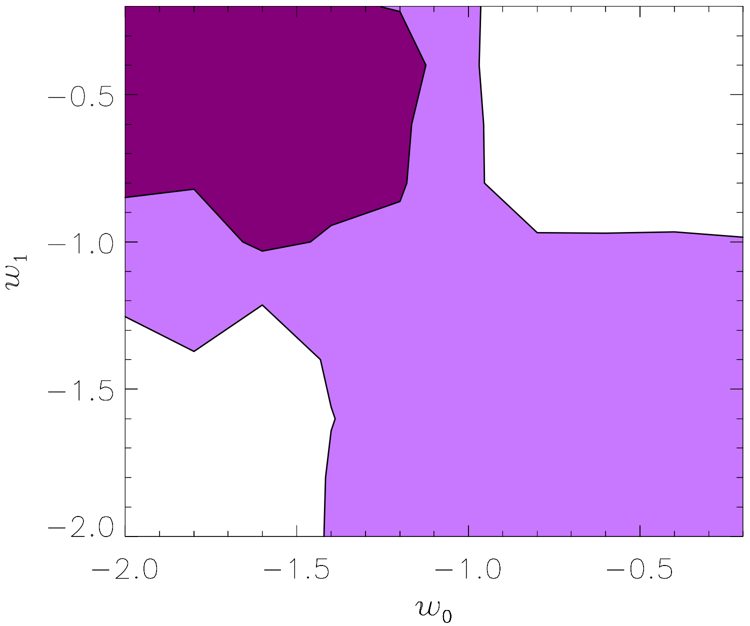


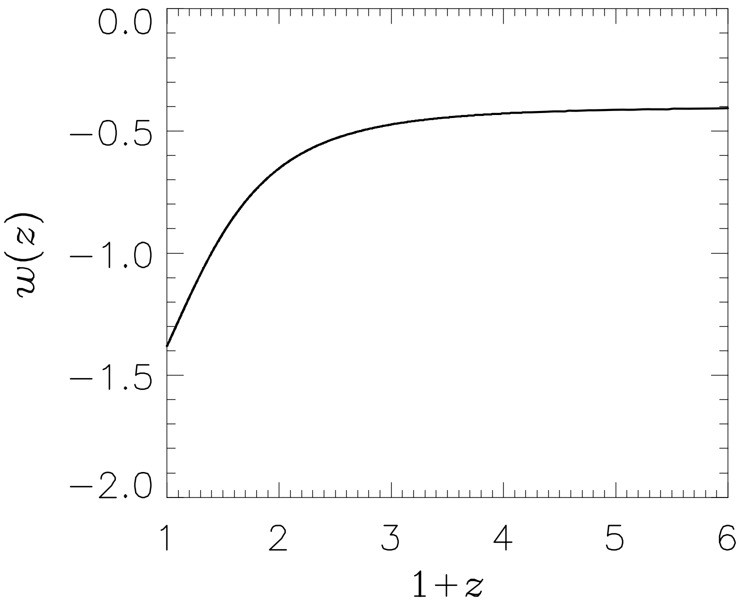
D. Huterer, A. Cooray, astro-ph/0404062

w variable with redshift

S. Hannestad and E. Mortsell,
JCAP 0409 (2004) 001

SNIa+ CMB+LSS





$$w(a)=w_0 w_1(a^q + a_s^q)/(a^q w_1+a_s^q w_0)$$

Phantom energy

- Phantom energy density grows in time
- Singularity reached in finite time – "big rip"
- $w = -1-\kappa_0 = \text{const}$

$$H(t) = \frac{H(t_0)}{1 - \frac{3}{2}H(t_0)\kappa_0(t - t_0)}$$

$$a(t) = a(t_0)\left(1 - \frac{3}{2}H(t_0)\kappa_0(t - t_0)\right)^{-\frac{2}{3\kappa_0}}$$

Phantom energy – “big rip”

- Decomposition of all bound systems in the universe
- Gravitationally bound systems

$$G(\rho_d + 3p_d)R^3 \quad \text{vs.} \quad GM$$

- Nongravitationally bound systems

$$G(\rho_d + 3p_d)R^3$$

R.R. Caldwell, M. Kamionkowski, N.N. Weinberg, Phys. Rev. Lett. 91 (2003) 071301

Phantom energy - controversies

- Violation of energy conditions
- Problematic microscopic formulation - negative kinetic terms
  - strong phenomenological constraints

S.M. Carroll, M. Hoffman, M. Trodden, Phys. Rev. D68 (2003) 02

J.M. Cline, S. Jeon, G.D. Moore, hep-ph/0311312

HOWEVER!!

- Observational support

Mimicking phantom energy?

- Is it possible that the non-phantom components of the universe produce the expansion of the universe characteristic of cosmologies with phantom energy?

i.e.

- Is phantom energy a mirrage mimicked by the non-phantom components?

H.Š. Eur. Phys. J. C 36 (2004) 523

Topics

Mimicking of phantom energy
by nonphantom components

Dark energy related singularities
in expanding cosmologies

Model 1

Two components

Ordinary matter component

$$p_m = \gamma(a)\rho_m$$

$$\gamma(a) \geq 0$$

$$\rho_m = \rho_{m,0}\, e^{-3\int_{a_0}^{a}(1+\gamma(a'))\frac{da'}{a'}}$$

Dark energy component → Nonconserved energy-mometum tensor

$$p_d = \eta(a)\rho_d\,, \qquad \eta(a) \geq -1$$

$$(G(t)T^{\mu\nu})_{;\nu} = 0$$

Variable G

Total energy-momentum tensor

Model 1

$$d(G\rho_d) + \rho_m dG + 3G\rho_d(1+\eta(a))\frac{da}{a} = 0$$

Assumption

$$G\rho_d = G_0\rho_{d,0}\left(\frac{a}{a_0}\right)^{-3(1+w(a))} \quad \longrightarrow \quad G = G_0\left(1 - \frac{\rho_{d,0}}{\rho_{m,0}}\frac{\eta - w}{\gamma - w}\left[\left(\frac{a}{a_0}\right)^{-3(w-\gamma)} - 1\right]\right)$$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi}{3}G(\rho_m + \rho_d)$$

$$G\rho_m = G_0\left(\rho_{m,0} + \rho_{d,0}\frac{\eta - w}{\gamma - w}\right)\left(\frac{a}{a_0}\right)^{-3(1+\gamma)} - G_0\rho_{d,0}\frac{\eta - w}{\gamma - w}\left(\frac{a}{a_0}\right)^{-3(1+w)}$$

$$G(\rho_m + \rho_d) = G_0\left(\rho_{m,0} + \rho_{d,0}\frac{\eta - w}{\gamma - w}\right)\left(\frac{a}{a_0}\right)^{-3(1+\gamma)} + G_0\rho_{d,0}\frac{\gamma - \eta}{\gamma - w}\left(\frac{a}{a_0}\right)^{-3(1+w)}$$

Model 2

- Two components, G nonvariable
- Dark energy

$$p_d = \eta(a)\rho_d\,, \qquad \eta \geq -1$$

- "Ordinary" matter

$$p_m = \gamma(a)\rho_m\,, \qquad \gamma \geq 0$$

- Conservation of total energy-momentum tensor

$$T^{\mu\nu} = T_m^{\mu\nu} + T_d^{\mu\nu}$$

$$d\rho_m + 3\rho_m(1+\gamma(a))\frac{da}{a} =$$

$$-d\rho_d - 3\rho_d(1+\eta(a))\frac{da}{a}$$

Model 2

Assumption

$$\rho_d = \rho_{d,0} \left(\frac{a}{a_0}\right)^{-3(1+w(a))}$$

$$\rho_m = \left(\rho_{m,0} + \rho_{d,0}\frac{\eta - w}{\gamma - w}\right)\left(\frac{a}{a_0}\right)^{-3(1+\gamma)} - \rho_{d,0}\frac{\eta - w}{\gamma - w}\left(\frac{a}{a_0}\right)^{-3(1+w)}$$

$$\rho = \left(\rho_{m,0} + \rho_{d,0}\frac{\eta - w}{\gamma - w}\right)\left(\frac{a}{a_0}\right)^{-3(1+\gamma)} + \frac{\gamma - \eta}{\gamma - w}\rho_{d,0}\left(\frac{a}{a_0}\right)^{-3(1+w)}$$

Model 2 - a more detailed formulation

$$\mathcal{L} = \frac{\dot{\phi}^2}{2} + \frac{\dot{\psi}^2}{2} - V(\phi, \psi)$$

$$\begin{aligned} \dot{\phi}^2 + \dot{\psi}^2 &= (1+\eta)\rho_d + (1+\gamma)\rho_m \\ 2V(\phi, \psi) &= (1-\eta)\rho_d + (1-\gamma)\rho_m \end{aligned}$$

Subsequent similar approaches

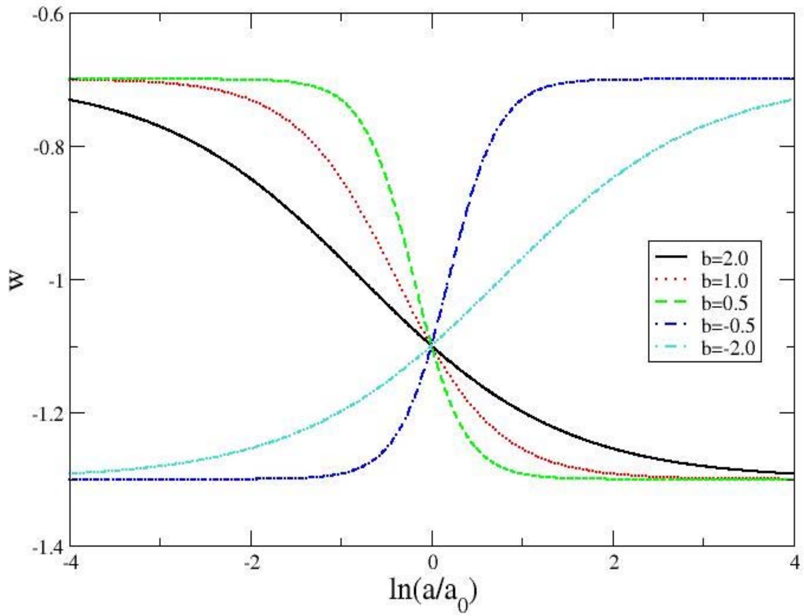
- S.M. Carroll, A. De Felice and M. Trodden, “Can we be tricked into thinking that w is less than -1?”, astro-ph/0408081
- A. Lue and G.D. Starkman, “How a brane cosmological constant can trick us into thinking that w < -1”, Phys.Rev. D70 (2004) 101501
- C. Csaki, N. Kaloper and J. Terning, “Exorcising w < -1”, astro-ph/0409596

A simple and a more general model

$$\rho = \left(C_1 \left(\frac{a}{a_0} \right)^{-3(1+\gamma)/b} + C_2 \left(\frac{a}{a_0} \right)^{-3(1+\eta)/b} \right)^b$$

$$p\rho^{(1-b)/b} = \gamma C_1 \left(\frac{a}{a_0} \right)^{-3(1+\gamma)/b} + \eta C_2 \left(\frac{a}{a_0} \right)^{-3(1+\eta)/b}$$

$$w = \frac{\gamma + \eta \frac{\gamma - w_0}{w_0 - \eta} \left(\frac{a}{a_0} \right)^{3(\gamma-\eta)/b}}{1 + \frac{\gamma - w_0}{w_0 - \eta} \left(\frac{a}{a_0} \right)^{3(\gamma-\eta)/b}}$$



$$\frac{p - \eta\rho}{(\gamma - \eta)C_1} = \rho^{((1-b)(\gamma-\eta))/(b(1+\eta))} \left(\frac{\gamma\rho - p}{(\gamma - \eta)C_2} \right)^{(1+\gamma)/(1+\eta)}$$

Generalization of the more general model

$$A\rho + Bp = (C\rho + Dp)^{\alpha}(M\rho + Np)^{\beta}$$

$$\rho = \left(\frac{(C + Dw)^{\alpha}(M + Nw)^{\beta}}{A + Bw}\right)^{1/(1-\alpha-\beta)}$$

$$\left(\frac{\alpha D}{C + Dw} + \frac{\beta N}{M + Nw} - \frac{B}{A + Bw}\right)\frac{dw}{1 + w} = 3(\alpha + \beta - 1)\frac{da}{a}$$

$$\left|\frac{C + Dw}{C + Dw_0}\right|^{-\alpha D/(C-D)} \left|\frac{M + Nw}{M + Nw_0}\right|^{-\beta N/(M-N)} \left|\frac{A + Bw}{A + Bw_0}\right|^{B/(A-B)}$$

$$\times \left|\frac{1 + w}{1 + w_0}\right|^{\alpha D/(C-D)+\beta M/(M-N)-B/(A-B)} = \left(\frac{a}{a_0}\right)^{3(\alpha+\beta-1)} .$$

Model building – nontrivial EOS with the transition

$$A\rho^{2n+1} + Bp^{2n+1} = (C\rho^{2n+1} + Dp^{2n+1})^{\alpha}$$

$$\frac{w^{2n+1} + (\alpha E - F)/(\alpha - 1)}{(F + w^{2n+1})(E + w^{2n+1})} \frac{w^{2n}}{1+w} dw = 3\frac{da}{a}$$

$$\frac{w^{2n+1} + 1}{w + 1} = \xi(w) = \sum_{l=0}^{2n} (-w)^l$$

$$\frac{\xi(w)}{(F + w^{2n+1})(E + w^{2n+1})} w^{2n} dw = 3\frac{da}{a}$$