Why anthropic reasoning does not predict $\Lambda$

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Answering the big questions:

Is our Universe “special”?  

Why is our Universe hospitable for life?  

Is there an anthropic explanation to $\Lambda$?
Anthropic coincidences?

Are physical constants tuned for life?

- *Primordial fluctuations amplitude* $Q$
- $\alpha_{EM}/G$ and $\alpha_s$
- *Cosmological constant* $\Lambda$, ...

Possible viewpoints:

- *Deeper symmetry / laws of Nature*
  
  *(but what determined THAT particular symmetry in the first place?)*

- *Design* or *necessity*
  
  *(outside the scope of scientific investigation)*

- *Any parameters will do* (no explanatory power)

- *Multiverse: we must live in one “realization” favourable for life*

Life in a multiverse
The cosmological constant problem:

why is $\Lambda/M_{Pl} \approx 10^{-123}$ ?

The anthropic “solution”:

if $\Lambda \gg 1$ galaxies cannot form
hence no observers

(Weinberg, 1987)

Shortcuts & difficulties:

- What counts as observers?
- Which parameters are allowed to vary?
- Is the multiverse a scientific (ie testable) theory?
Which parameters should we vary?

(Tegmark at al 2005)

if $\Lambda$, $Q$ and $\xi$ varied:

$$\Lambda = 10^{17} \Lambda_0$$
perfectly “viable”!

(Aguirre 2001)

(Tegmark at al 2005)

“Prediction” only successfull conditional on $\xi$, $Q$ = fixed
(AND that $T_{CMB} = 2.73$ K)
Probability theory and $\Lambda$

\[ f_{\text{obs}}(\Lambda) = f(\Lambda) f_{\text{sel}}(\Lambda) \]

- prob of observing = sampling distribution * selection function

- "random sample"  "typical observer"

The sampling distribution $f(\Lambda)$

- As a frequency of outcomes? (untestable in cosmology)
- Flat distribution (the "Weinberg conjecture")? (assumed)
- Ergodic arguments? (unclear in an infinite Universe)
- No operational def' on of "random" sample: probabilities are NOT physical properties!
Which probability theory for cosmology?

**Probability as frequency**
- Repeatable sampling
- Parent distribution
- Asymptotically \( N \to \infty \)

**Probability as state of knowledge**
- Only 1 sample
- “Multiverse” approach ill-defined
- \( N \) finite & limited
On the physical reality of probability

Coin tossing: is the coin fair?

Test the null hypothesis \( H_0: p = 0.5 \)

“The numbers \( p_r \) [the frequency with which a certain face comes up in die tossing] should, in fact, be regarded as physical constants of the particular die that we are using.”

(Cramer, 1946)

Are physical probabilities meaningful?
What does it mean “to throw at random”?

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With careful adjustment, the coin started heads up always lands heads up – 100% of the time. We conclude that coin-tossing is “physics” not “random”.

(Diaconis et al 2004; Jaynes 1996)

Symmetric Lagrangian: $\Gamma_T = \Gamma_H$

$p \neq 0.5$: $\Gamma_T / \Gamma_H$ is NOT independent on location!
The selection function

\[ f_{\text{obs}}(\Lambda) = f(\Lambda) \ f_{\text{sel}}(\Lambda) \]

What counts as “observers”? (it’s the total number that counts!)
What if the Universe is infinite? (number density/Hubble volume?)
Do observers outside your causal horizon count?
Certainly important to integrate over time: we might not be “typical” in that we are early arrivals...

An explicit counter-example: MANO weighting
Maximum Number of Allowed Observations
**MANO weighting of Universes**

- Integrate over lifetime of the Universe to obtain the total number of observations that can POTENTIALLY be carried out.
- Universes that allow for more observations should weight more.
- Gauge invariant, time independent quantity.
- Maximum number of thermodynamic processes in a $\Lambda > 0$ Universe:

$$N_{\text{max}} < \frac{E_{\text{coll}}}{k_B} T ds$$

- This assumes “rare observers”, otherwise density of observers sets the limit.
- Still suffers from dependence of micro-physics + details of how civilizations arise & evolve.
**Probability of observing $\Lambda$**

- **2 parameters model:**

  \[ R = \frac{\Omega_\Lambda}{\Omega_\Lambda^0} \quad \tau = \frac{t_{\text{obs}}}{t_0} \]

  \[ \log(R) \approx -379 \] (landscape scenario)

  \[ \propto R^{-2} \]

  \[ \propto R^{-1} g(\tau) \]

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<tr>
<th>$\tau$</th>
<th>$f_{\text{obs}}(R&gt;1)$</th>
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<td>10</td>
<td>$2 \times 10^{-16}$</td>
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Final remarks

PROBABILITY THEORY AND COSMOLOGY

- Probabilities are not physical properties but states of knowledge
- Uniqueness of the Universe calls for a fully Bayesian approach

ANTHROPIC REASONING AND SELECTION EFFECTS

- Outcome depends on selection function
- Probability theory as logic at odds with multiverse approach
- Within “traditional” anthropic arguments: you should at least integrate over time
- MANO counterexample: $P(\Lambda > 0.7) \sim 10^{-5}$
- Anthropic “predictions” completely dependent on (many) assumptions