Weakly Interacting Massive Particles (WIMPs)

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International School on Astroparticle Physics

Dark Matter in Astrophysics and Particle Physics
Miraflores de la Sierra (Madrid), June 21st-July 1st 2008
Theoretical/phenomenological issues covered by the following lectures on WIMPs:

- WIMP relic abundance
- WIMP detection rates
- WIMP distribution in the galactic halo
- strategies for WIMP detection
- examples of WIMPs: LKK, introductory ideas about neutralinos

Will rely on other lectures on Cosmology from S. Bonometto, F. Prada, A. Klypin

Experimental issues covered by M.L. Sarsa and A. Morselli
Details of susy dark matter covered by K. Olive

Physics Beyond the Standard Model covered by A. Masiero
References

• WIMP relic density: Kolb and Turner, “The Early Universe”, Frontier in Physics, chapter 5 (and page 310…)
• Eddington’s equation & isothermal sphere model: Binney and Tremaine, “Galactic Dynamics”, chapter 4
• + research papers cited in individual slides
The concordance model
Evidence for Dark Matter

- Spiral galaxies
  - rotation curves
- Clusters & Superclusters
  - Weak gravitational lensing
  - Strong gravitational lensing
  - Galaxy velocities
  - X rays
- Large scale structure
  - Structure formation
- CMB anisotropy: WMAP
  - $\Omega_{\text{tot}} = 1$
  - $\Omega_{\text{dark energy}} \sim 0.7$
  - $\Omega_{\text{matter}} \sim 0.27$
  - $\Omega_{\text{baryons}} \sim 0.05$
  - $\Omega_{\text{visible}} \sim 0.005$
  - $\Omega_{\text{dark matter}} \sim 0.22$
Apart from being unable to drive galaxy formation (they decouple too late from photons, not enough time for gravitational instabilities to grow) baryons are too few in the Universe in order to explain the dark matter because of nucleosynthesis.

Observations give $0.6 < h < 0.8$

Big Bang nucleosynthesis (deuterium abundance) and cosmic microwave background (WMAP) determine baryon contribution $\Omega_B h^2 \approx 0.023$, so $\Omega_B \approx 0.04$

$\Omega_{lum} \approx (4 \pm 2) \cdot 10^{-3}$ (stars, gas, dust) => baryonic dark matter has to exist (maybe as warm intergalactic gas?)

But, now we know that $\Omega_M > 0.2$, so there has to exist non-baryonic dark matter

**Figure 20.1**: The abundances of $^4\text{He}$, $^3\text{He}$, and $^7\text{Li}$ as predicted by the standard model of big-bang nucleosynthesis. Boxes indicate the observed light element abundances (smaller boxes: $2\sigma$ statistical errors; larger boxes: $\pm 2\sigma$ statistical and systematic errors added in quadrature). The narrow vertical band indicates the CMB measure of the cosmic baryon density. See full-color version on color pages at end of book.

*Fields & Sarkar, 2004*
A lot of matter in the Universe is dark and non-baryonic but also some baryonic DM needed - as explained in E. Masso’s lectures.
The properties of a good Dark Matter candidate:

- stable (protected by a conserved quantum number)
- no charge, no colour (weakly interacting)
- cold, non dissipative
- relic abundance compatible to observation*
- motivated by theory (vs. “ad hoc”)

Subdominant candidates – variety is common in Nature → may be easier to detect
The first place to look for a DM candidate...

### The Standard Model

<table>
<thead>
<tr>
<th>GAUGE</th>
<th>Gauge bosons</th>
<th>$(\text{SU}(3)_c \times \text{SU}(2)_L)_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-boson</td>
<td>$A^{(1)}<em>\mu = B</em>\mu$</td>
<td>$(1, 1)_0$</td>
</tr>
<tr>
<td>W-bosons</td>
<td>$A^{(2)}<em>\mu = W^\alpha</em>\mu$</td>
<td>$(1, 3)_0$</td>
</tr>
<tr>
<td>gluon</td>
<td>$A^{(3)}<em>\mu = G^\alpha</em>\mu$</td>
<td>$(8, 1)_0$</td>
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</table>

<table>
<thead>
<tr>
<th>MATTER</th>
<th>Fermions</th>
<th>$(\text{SU}(3)_c \times \text{SU}(2)_L)_Y$</th>
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</thead>
<tbody>
<tr>
<td>leptons $I = 1, 2, 3$</td>
<td>$L^I = \begin{pmatrix} \nu^I_L \ e^{-I}_L \end{pmatrix}$</td>
<td>$(1, 2)_{-1}$</td>
</tr>
<tr>
<td></td>
<td>$E^I = e^{-I}_R$</td>
<td>$(1, 1)_{+2}$</td>
</tr>
<tr>
<td>quarks $I = 1, 2, 3$</td>
<td>$Q^I = \begin{pmatrix} u^I_L \ d^I_L \end{pmatrix}$</td>
<td>$(8, 2)_{+1}$</td>
</tr>
<tr>
<td></td>
<td>$U^I = u^I_R$</td>
<td>$(\bar{3}, 1)_{-\frac{4}{3}}$</td>
</tr>
<tr>
<td></td>
<td>$D^I = d^I_R$</td>
<td>$(\bar{3}, 1)_{+\frac{2}{3}}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>HIGGS</th>
<th>Higgs Boson</th>
<th>$(\text{SU}(3)_c \times \text{SU}(2)_L)_Y$</th>
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<tbody>
<tr>
<td>Higgs</td>
<td>$\phi = \begin{pmatrix} \phi^+ \ \omega^0 \end{pmatrix}$</td>
<td>$(1, 2)_{+1}$</td>
</tr>
</tbody>
</table>
GRAVITY OF NEUTRINOS OF NONZERO MASS IN ASTROPHYSICS

R. COWSIK* AND J. MCCLELLAND
Department of Physics, University of California, Berkeley
Received 1972 July 24

ABSTRACT
If neutrinos have a rest mass of a few eV/c², then they would dominate the gravitational dynamics of the large clusters of galaxies and of the Universe. A simple model to understand the virial mass discrepancy in the Coma cluster on this basis is outlined.

Subject headings: cosmology — galaxies, clusters of — neutrinos
2 early bounds on neutrino mass from cosmology (relic abundance):

- **Cowsik-McClelland bound:** $m_\nu < \text{few eV}$

- **Lee-Weinberg limit:** $m_\nu > \text{few GeV}$
Neutrino

• \( \Sigma m_\nu < 0.66 \text{ eV} \) (WMAP+LSS+SN)
• LEP:
  \( N_\nu = 2.994 \pm 0.012 \)
  \[ \rightarrow m_\nu \geq 45 \text{ GeV} \]
  \[ \rightarrow \Omega_\nu h^2 \leq 10^{-3} \]
• DM searches exclude: \( 10 \) GeV \( \leq m_\nu \leq 4.7 \) TeV
  (similar constraints for sneutrinos and KK-neutrinos)
  
  does not work

\[
\Omega_\nu h^2 = \frac{\sum m_\nu}{91.5 \text{ eV}}
\]

\[
\Omega_\nu h^2 \propto <\sigma_{\text{ann}}\nu>^{-1}
\]

![Diagram showing Neutrino Masses and Constraints](image)

Lee-Weinberg

3 – 7 GeV

30 eV

Cowsik-McClelland

mix with sterile component
(both for neutrinos and sneutrinos)
Pioneering work on direct DM searches @ Homestake mine in late ’80s:

**LIMITS ON COLD DARK MATTER CANDIDATES FROM AN ULTRALOW BACKGROUND GERMANIUM SPECTROMETER**

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h Institute for Advanced Study, Princeton, NJ 08540, USA

Received 5 May 1987

An ultralow background spectrometer is used as a detector of cold dark matter candidates. A realistic model for the galactic halo, large regions of the mass–cross section space are excluded. In particular, a halo dominated by heavy standard Dirac neutrinos (taken as independent $Z^0$ exchange interactions) with masses between 20 GeV and 1 TeV is excluded. Neutrinos is $<0.4$ GeV/cm$^3$ for masses between 17.5 GeV and 2.5 TeV, at the 68% confidence level. However, today the sneutrino is not completely dead (rescaling due to relic density not applied to the signal at the time, see later).

few GeV<M<few TeV
excluded both for neutrinos ad sneutrinos

*
Neutrinos don’t’s work also because they are hot dark matter (=relativistic at decoupling, erase density perturbation through free-streaming):

(from S. Bonometto and A. Klypin’s lectures)

(see Mark Tegmark home page)
CLUSTERING IN A NEUTRINO-DOMINATED UNIVERSE

Simon D. M. White,1, 2 Carlos S. Frenk, 1 and Marc Davis1, 3

University of California, Berkeley
Received 1983 June 17; accepted 1983 July 1

ABSTRACT

We have simulated the nonlinear growth of structure in a universe dominated by massive neutrinos using initial conditions derived from detailed linear calculations of earlier evolution. Codes based on a direct N-body integrator and on a fast Fourier transform Poisson solver produce very similar results. The coherence length of the neutrino distribution at early times is directly related to the mass of the neutrino and thence to the present density of the universe. We find this length to be too large to be consistent with the observed clustering scale of galaxies if other cosmological parameters are to remain within their accepted ranges. The conventional neutrino-dominated picture appears to be ruled out.
Structure formation (i.e.: the very existence of galaxies) needs Cold Dark Matter and Cold Dark Matter implies physics beyond the Standard Model (light neutrinos don’t work)
Have to go non-baryonic and beyond the Standard Model
Two main guiding principles:
   1. simplicity
   2. theoretical motivation

not always coinciding!
A recent example of a “minimal extensions” of the SM

Cirelli et al, NPB753(2006)

\[ \mathcal{L} = \mathcal{L}_{\text{SM}} + c \begin{cases} \bar{\chi} (i \slashed{D} + M) \chi & \text{when } \chi \text{ is a spin } 1/2 \text{ fermionic multiplet}, \\ |D_\mu \chi|^2 - M^2 |\chi|^2 & \text{when } \chi \text{ is a spin } 0 \text{ bosonic multiplet}, \end{cases} \]

- add to SM extra \( n \)-tuplets of \( \text{SU}(2)_L \) with minimal spin, isospin and hypercharge and search for assignements that provide most of all of the following properties:
  - lightest particle stable, no strong interactions,
  - only 1 parameter free: \( M \)
  - QC induce mass splitting \( \Delta M \), the lightest \( \chi \) is neutral
  - DM candidate not excluded by DM searches

NB: in the SM the proton does not decay simply because decay modes consistent with renormalizability do not exist (accidental B-L symmetry)

Minimal DM can be stable for the same reason.

The trick: choose \( n \) sufficiently high
<table>
<thead>
<tr>
<th>Quantum numbers</th>
<th>DM can decay into</th>
<th>DM mass in TeV</th>
<th>$m_{DM}^+ - m_{DM}^-$ in MeV</th>
<th>Events at LHC $\int L dt = 100$ fb</th>
<th>$\sigma_{SI}$ in $10^{-45}$ cm$^2$</th>
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</thead>
<tbody>
<tr>
<td>SU(2)$_L$</td>
<td>U(1)$_Y$</td>
<td>Spin</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1/2</td>
<td>0</td>
<td>$E_L$</td>
<td>$0.54 \pm 0.01$</td>
<td>350</td>
</tr>
<tr>
<td>2</td>
<td>1/2</td>
<td>1/2</td>
<td>$E_H$</td>
<td>$1.2 \pm 0.03$</td>
<td>341</td>
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<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>$H H^*$</td>
<td>$2.0 \pm 0.05$</td>
<td>166</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1/2</td>
<td>$L H$</td>
<td>$2.5 \pm 0.06$</td>
<td>166</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0</td>
<td>$H H$, $L L$</td>
<td>$1.6 \pm 0.04$</td>
<td>540</td>
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<tr>
<td>3</td>
<td>1</td>
<td>1/2</td>
<td>$L H$</td>
<td>$1.9 \pm 0.05$</td>
<td>526</td>
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<td>4</td>
<td>1/2</td>
<td>0</td>
<td>$H H H^*$</td>
<td>$2.4 \pm 0.06$</td>
<td>353</td>
</tr>
<tr>
<td>4</td>
<td>1/2</td>
<td>1/2</td>
<td>$(L H H^*)$</td>
<td>$2.4 \pm 0.06$</td>
<td>347</td>
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<tr>
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<td>3/2</td>
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<td>$H H H$</td>
<td>$2.9 \pm 0.07$</td>
<td>729</td>
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<tr>
<td>4</td>
<td>3/2</td>
<td>1/2</td>
<td>$(L H H)$</td>
<td>$2.6 \pm 0.07$</td>
<td>712</td>
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<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>$(H H H^+ H^*)$</td>
<td>$5.0 \pm 0.1$</td>
<td>166</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>1/2</td>
<td>$\sim$</td>
<td>$4.4 \pm 0.1$</td>
<td>166</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
<td>$\sim$</td>
<td>$8.5 \pm 0.2$</td>
<td>166</td>
</tr>
</tbody>
</table>

$n \geq 5$ for fermions  \quad n \geq 7$ for scalars

M is the only free parameter fixed by relic abundance!

also direct detection is fixed

more candidates if stabilization mechanism added
Note that DM candidates with the same quantum numbers of the previous table already exist in different contexts:

- scalar triplets in little Higgs models
- inert Higgs + $Z_2$ symmetry
- fermion or scalar triplet in see-saw models
- KK excitations of lepton doublets or of Higgses in extradimensions
- Higgsinos, sneutrinos, Winos in Supersymmetry

the above candidates are stable because of some symmetry
many free parameters with variable interaction rates motivations from particle physics building
...acronym “WIMP” eventually coined in mid ‘80

WIMP=Weakly Interacting Massive Particle
in which case one could conclude that $H_0 t_0 > 0.66$, in contradiction with the inflationary prediction.

Recalling that primordial nucleosynthesis restricts the baryonic contribution to be $\Omega_B \lesssim 0.15$, we see that inflation requires non-baryonic matter to be the dominant form of matter in the Universe. The simplest and most plausible form of such is relic WIMPs.\(^{40}\) Prime candidates for the

\(^{39}\)Since density perturbations correspond to fluctuations in the curvature, the density perturbations on the present Hubble scale imply that a very accurate measurement of $\Omega_0$ would actually yield a value, $\Omega_0 = 1.0 \pm (\delta \rho/\rho)_{\text{HOB}} = 1.0 \pm \mathcal{O}(10^{-5})$.

\(^{40}\)WIMP\(^{\circledR}\) is a copyrighted trademark of the Chicago group, standing for Weakly Interacting Massive Particle.
Wimp.com

Updated Daily

Mar 26 - Obese man - video
Mar 26 - Sexy pole dancing - video
Mar 26 - Truth about firearms - video
Mar 26 - Amazing jumps - video
Mar 25 - Grandpa punched - video
Mar 25 - John Locke painting - video
Mar 25 - Cosby's joke stolen - video
Mar 25 - Men never change - video
Jun 2 - Painful jump - video
Jun 2 - Deadly car chase - video
Jun 2 - Crazy weather guy - video
Jun 2 - Hydrofoil board surfing - video
Jun 2 - Funny abortion interviews - video
Jun 2 - Playful cats compilation - video
Jun 2 - Three armed Chinese baby - video
Jun 2 - Mexican molester beat up - video
Jun 2 - Funny pong shots - video
Jun 2 - Orleans censorship - video
Why are WIMPs so popular?
The standard lore:

- the Cosmic Microwave Background Radiation (CMBR) is the remnant of the hot plasma that dominated the energy density in the early Universe.
- from the measured temperature of the CMBR ($T_0 = 2.7$ K) we know the CMBR density today ($\rho_{\gamma,0} \sim 422$ photons cm$^{-3}$).
- Weak interactions kept WIMPs in thermal equilibrium with those photons in the early Universe (THERMAL RELICS).
- working out the decoupling between WIMPs and the plasma we can calculate the WIMP density normalizing it to $\rho_{\gamma,0} \sim 422$ photons cm$^{-3}$.
- since WIMPs are COLD, they were non relativistic at decoupling, so their equilibrium density was exponentially suppressed compared to photons.
- however, after decoupling (freeze-out) WIMPs density in a comoving volume stayed almost the same, while photons were deluted and redshifted away so that now they contribute $\Omega_\gamma \sim 10^{-3}$.
Thermal equilibrium simplifies things!

$T >> M_{WIMP}$  

Decoupling: $T << M_{WIMP}$  

now

$n_x \sim n_y$

$n_{x,f} << n_{y,f}$  
(exponential suppression, $n_{x,f} \sim e^{-M/T}$)

normalize WIMP density to CMB

Decoupling is the key!
WIMP Boltzmann equations

\[ \hat{L}[f] = C[f] \]

Liouville operator \[ \rightarrow \]

Collisional operator

\[ f(x^\mu, p^\nu, t) = f(E, t) \]

Phase-space density depends on E only
(in FWR model spatially homogeneous and isotropic)

\[ n(t) = \frac{g}{(2\pi)^3} \int d^3 \vec{p} f(E, t) \]

WIMP number density
Liouville operator

\[ \hat{L} = \frac{d}{d\tau} = \frac{d}{dx} \frac{dx^\mu}{d\tau} + \frac{d}{dp} \frac{dp^\mu}{d\tau} = p^\mu \frac{d}{dx^\mu} - \Gamma^\mu_{\sigma\rho} p^\sigma p^\rho \frac{d}{dp^\mu} \]

\[ \rightarrow \quad E \frac{d}{dt} - \Gamma^0_{jk} p^j p^k \frac{d}{dE} = E \frac{d}{dt} - H |\vec{p}|^2 \frac{d}{dE} \]

\[ d\tau \equiv \sqrt{-g^{\mu\nu} dx_\mu dx_\nu} \]

proper time

\[ A = - \int d\tau = \int L dt \rightarrow L = -\dot{\tau} \]

action for free particle

\[ p^\mu = \frac{dL}{dx^\mu} = - \frac{d\dot{\tau}}{dt} = \frac{dx^\mu}{d\tau} \]

\[ \frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\sigma\rho} \frac{dx^\sigma}{d\tau} \frac{dx^\rho}{d\tau} = 0 \rightarrow \frac{dp^\mu}{d\tau} = -\Gamma^\mu_{\sigma\rho} p^\sigma p^\rho \]

(=background-force term from geodesic equation)

affine connection
Dependence on gravitational background through affine connection

\[ \Gamma^\sigma_{\lambda \mu} = \frac{1}{2} \left\{ \frac{\partial g_{\mu \nu}}{\partial x^\lambda} + \frac{\partial g_{\lambda \nu}}{\partial x^\mu} - \frac{\partial g_{\mu \lambda}}{\partial x^\nu} \right\} \]

\[ ds^2 = dt^2 - a^2(t) \left\{ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right\} \]  \hspace{1cm} \text{RW metric}

non-zero terms:

\[ \Gamma^i_{jk} = \frac{1}{2} h^{il} \left( \frac{\partial g_{lj}}{\partial x^k} + \frac{\partial g_{lk}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^l} \right) \]

\[ \Gamma^0_{ij} = \frac{\dot{a}}{a} h_{ij} = H h_{ij} \]

\[ \Gamma^i_{0j} = \frac{\dot{a}}{a} \delta^i_j \]

\[ h_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]
Integration over phase-space of Liouville operator:

\[
\frac{g}{(2\pi)^3} \int \frac{d^3 \vec{p}}{E} \hat{L}[f] = \frac{g}{(2\pi)^3} \int \frac{d^3 \vec{p}}{E} \left( \frac{df}{dt} + H \frac{|\vec{p}|^2}{E} \frac{df}{dE} \right) = \frac{dn}{dt} + 3Hn = s \frac{dY}{dt}
\]

\[Y \equiv \frac{n}{s}\]

\[s = \frac{\rho + p}{T} = g_s \frac{2\pi^2}{45} T^3\]

\[a^3 s = \text{constant}\]

\[\frac{dY}{dt} = \frac{d}{dt} \left( \frac{Y}{s} \right) = \frac{d}{dt} \left( \frac{a^3 Y}{a^3 s} \right) = \frac{1}{a^3 s} \frac{d}{dt} (a^3 n) = \frac{1}{a^3 s} \left( a^3 \frac{dn}{dt} + 3a^2 \dot{a} n \right) = \frac{1}{s} \left( \frac{dn}{dt} + 3Hn \right)\]
Collisional operator

\[
\frac{g}{(2\pi)^3} \int C[f] \frac{d^3\vec{p}}{E} = 
\]

\[
- \int d\Pi_1 \ d\Pi_2 \ d\Pi_a \ d\Pi_b \ (2\pi)^4 \delta^4(p_1 + p_2 - p_a - p_b) [|M_{12\rightarrow ab}|^2 f_1 f_2 - |M_{ab\rightarrow 12}|^2 f_a f_b] = 
\]

\[
- \int d\Pi_1 \ d\Pi_2 \ d\Pi_a \ d\Pi_b \ (2\pi)^4 \delta^4(p_1 + p_2 - p_a - p_b) |M|^2 [f_1 f_2 - f_1^{eq} f_2^{eq}] = - <\sigma v > \ (n^2 - n_{eq}^2)
\]

\[
f_{a,b} = f_{a,b}^{eq} = e^{-\frac{E_{a,b}}{T}}
\]

SM particles assumed in thermal equilibrium

\[
f_{a}^{eq} f_{b}^{eq} = e^{\frac{E_{a} + E_{b}}{T}} = e^{\frac{E_{1} + E_{2}}{T}} = f_{1}^{eq} f_{2}^{eq}
\]
detailed balance

thermally averaged annihilation cross section:

\[
<\sigma v > \equiv \frac{1}{n_{eq}} \int d\Pi_1 \ d\Pi_2 \ d\Pi_a \ d\Pi_b \ (2\pi)^4 \delta^4(p_1 + p_2 - p_a - p_b) |M|^2 f_{1}^{eq} f_{2}^{eq}
\]
A small technicality: annihilation cross-section and Moeller velocity

Definition of unpolarized annihilation cross section for the process $12 \rightarrow 34$:

$$\sigma = \frac{1}{4Fg_1g_2g_3g_4} \sum_{\text{final spins}} \int |\mathcal{M}_{12 \rightarrow 34}|^2 d\Pi_3 d\Pi_4$$

$$F = \left[(p_1 \cdot p_2)^2 - m_1^2 m_2^2\right]^{1/2}$$

so that, writing:

$$v = \frac{F}{E_1 E_2} = \left[|\vec{v}_1 - \vec{v}_2|^2 - |v_1 \times v_2|^2\right]^{1/2} = v_{\text{Møller}}$$

one recovers the previous expression for the thermal average, which, in particular, reads:

$$\langle \sigma v_{\text{Møller}} \rangle = \frac{\int \sigma v_{\text{Møller}} \, dn_1^{eq} \, dn_2^{eq}}{dn_1^{eq} \, dn_2^{eq}}$$

$$\left(\frac{dn_i}{(2\pi)^3} d^3 \vec{p}_i f(E_i)\right)$$

N.B.: covariant expression, different from relative velocity in c.m. or lab frame
Boltzmann’s equation

\[
\frac{dn}{dt} + 3Hn = - < \sigma v > \left( n^2 - n_{eq}^2 \right)
\]

\[
\frac{dY}{dt} = - s < \sigma v > \left( Y^2 - Y_{eq}^2 \right)
\]

\[
\frac{dY}{dx} = - \frac{x s}{H(T = m)} < \sigma v > \left( Y^2 - Y_{eq}^2 \right)
\]

\[
x = \frac{m}{T}
\]

(t→x tranformation through iso-entropic expansion:

\[
\frac{d}{dt} (a^3 s) = 0 \rightarrow \frac{d}{dt} (aT) = 0 \rightarrow \frac{d}{dt} \left( \frac{\dot{a}}{x} \right) = \frac{\ddot{a}}{x} - \frac{a}{x^2} \ddot{x} = 0 \rightarrow \frac{dx}{dt} = Hx
\]
\[
\frac{dY}{dx} = -\frac{xs}{H(T = m)} < \sigma v > (Y^2 - Y_{eq}^2)
\]

[Riccati equation, no closed-form solutions – can feed to computer]

\[x > x_f:\]

\[
\frac{dY}{dx} \approx -\frac{xs}{H(T = m)} < \sigma v > Y^2
\]

separable: analytic solution:

\[
-\frac{dY}{Y^2} = \frac{xs}{H(T = m)} < \sigma v > \, dx
\]

\[
\frac{1}{Y_f} - \frac{1}{Y_0} \approx \int_{x_0}^{x_f} g_*^{1/2} < \sigma v > \left( \frac{1}{x} \right) \, dx
\]

\[
\approx \left( \frac{\pi}{45G} \right)^{1/2} g_*^{1/2} (T_f) \frac{m}{x_f} \left( a + \frac{b}{2x_f} \right) = \left( \frac{\pi}{45G} \right)^{1/2} g_*^{1/2} (T_f) \frac{m}{x_f} < \tilde{\sigma} v >
\]

\[
< \sigma v > \approx a + \frac{b}{x}
\]

\[
< \tilde{\sigma} v > = \frac{m}{T_f} \int_0^{T_f/m} < \sigma v > d\left( \frac{T}{m} \right) \approx a + \frac{b}{2x_f}
\]
So, finally, the thermal cosmological density of a WIMP $X$ is given by

$$\Omega_X h^2 \sim 1/\langle \sigma_{\text{ann}} v \rangle_{\text{int}}$$

$$\langle \sigma_{\text{ann}} v \rangle_{\text{int}} = \int_{x_f}^{x_0} \langle \sigma_{\text{ann}} v \rangle dx$$

$x_0 = M/T_0$

$T_0 =$ present (CMB) temperature

$x_f = M/T_f$

$T_f =$ freeze-out temperature

$X_f >> 1$, $X$ non relativistic at decoupling, low temp expansion for

$$\langle \sigma_{\text{ann}} v \rangle: \langle \sigma_{\text{ann}} v \rangle \sim a + b/x$$

if $\sigma_{\text{ann}}$ is given by weak-type interactions $\rightarrow \Omega_X \sim 0.1-1$

...+ coannihilations with other particle(s)

close in mass + resonant annihilations
Joining the pieces together: the WIMP relic abundance

\[ \Omega_{WIMP} \equiv \frac{\rho_{WIMP}}{\rho_c} = \frac{m \, n_{WIMP}}{\rho_c} = \frac{m S_0 Y_0}{\rho_c} \]

\[ S_0 = 2970 \, \text{cm}^{-3} \quad \text{today's entropy} \]

\[ \rho_c = 1.054 h^2 \times 10^{-5} \, \text{GeV} \, \text{cm}^{-3} \quad \text{critical density} \]

\[ x_f \approx 20 \quad \text{freeze-out temperature} \]

\[ g_*^{1/2} \approx 10 \quad \# \text{ of degrees of freedom} \]

\[ m = \text{WIMP mass} \]

\[ Y_0 : \text{from Boltzmann equation} \]

\[ \Omega_{WIMP} h^2 = \frac{x_f}{g_*^{1/2}} \frac{3.45 \times 10^{-38}}{\langle \tilde{\sigma} v \rangle} \approx \frac{0.1 \, \text{pbarn}}{\langle \tilde{\sigma} v \rangle} \]

\[ h \equiv H_0/100 \, \text{km sec}^{-1}\text{Mpc}^{-1} \]
N.B. Very different scales conjure up to lead to the weak scale!

\[ T_0 \simeq K \simeq 10^{-13} \text{ GeV} \]

\[ H_{100} = 100 \text{ km sec}^{-1} \text{ Mpc} \simeq 10^{-42} \text{ GeV} \]

\[ m_{Planck} = \frac{1}{G^{1/2}} = 10^{19} \text{ GeV} \]

\[
\frac{x_f}{g_*^{1/2}} \left( \frac{45G}{\pi} \right)^{1/2} \frac{S_0}{\rho_c} G^{1/2} \simeq 10^2 \frac{T_0^3}{H_{100}^2 m_{Planck}^3} \]

\[
\simeq 10^2 \frac{10^{-39}}{10^{-84} \times 10^{57}} \text{ GeV}^{-2} \simeq 10^{-9} \text{ GeV}^{-2} \simeq 1 \text{ pbarn} \]

the WIMP “miracle”???

CMB temp.
Hubble par.
Planck scale
on dimensional grounds:

\[ \sigma v \simeq \frac{\alpha^2}{M^2} \]
\[ \alpha \simeq 0.1 \]

\[ \sigma v \simeq 1 \text{ pbarn} \rightarrow M \simeq \text{TeV} \]

WIMP are non relativistic, so typically (m=WIMP mass):

\[ \sigma v \simeq \alpha^2 \frac{m^2}{M_X^4} = G_X^2 m^2 \]

(cfr.: \[ \sigma v \simeq G_X^2 T^2 \]
  for a relativistic particle)

\[ \Omega \sim \frac{1}{\sigma v} \sim \frac{1}{m^2} \]
if \( m \rightarrow 0 \) \( \Omega \rightarrow \infty \)

\Rightarrow \text{cosmological lower bound on } m \]

(Lee-Weinberg limit and alike)

N.B.: \( \Omega \leq 1 \) condition leads also to an upper unitarity bound on the WIMP mass, \( m \leq 340 \text{ TeV} \] [Griest, Kamionkowski, PRL64(1990) 615]
Low-temperature expansion $<\sigma_{\text{ann}}v> \sim a + b/x$ is not valid in some cases:

- resonant annihilations
- thresholds
- coannihilations with other particle(s) close in mass

* i.e., whenever $\sigma(s)$ is a strongly varying function of the center-of-mass energy $s$
Example: WIMP Resonant annihilation by exchange of some particle $A$

\[ \sigma_{\text{ann}} \text{ gets a boost by resonant annihilation when } m_\chi \approx M_A/2 \]
Resonant annihilation through $A$ exchange

The annihilation cross section to the final state $f$ can be derived from the relation (brackets=thermal average):

$$\langle \Gamma(\chi \chi \to f) \rangle = \langle \Gamma(\chi \chi \to A) B(A \to f) \rangle$$

$$\frac{n_{\chi}^2}{2} \langle \sigma_{\text{ann}} v \rangle_{\text{res,} f} \quad \quad \quad n_{H_1} \Gamma_{\chi} \frac{K_1(x_{H_1})}{K_2(x_{H_1})} B_f$$

$$(x_i \equiv m_i / T,$$

$B_f=$branching ratio to $f,$

$K_i=$Bessel functions)

$$< \sigma_{\text{ann}} v >_{\text{res}} = \frac{\pi^2 M_{H_1}^2}{m_\chi^5} \frac{x_\chi K_1(x_{H_1})}{K_2(x_\chi)} \Gamma(H_1) B_\chi (1 - B_\chi) \Theta \left( \frac{x_{H_1}}{x_\chi} - 2 \right)$$
Temperature average of a decay rate (Boltzmann approx)

\[ \langle \Gamma \rangle = \left\langle \frac{1}{\tau} \right\rangle \simeq \Gamma_0 \left\langle \frac{M}{E} \right\rangle \simeq \Gamma_0 \frac{\frac{d^3p}{(2\pi)^3} \frac{M}{E} e^{-E/T}}{\int \frac{d^3p}{(2\pi)^3} e^{-E/T}} = \Gamma_0 \frac{K_1(z)}{K_2(z)} \]

zero-temp amplitude \hspace{1cm} E/M=\gamma=boost (time-dilation) factor

asymptotic behavior of Bessel functions:

\[ K_1(z) = z \int_1^{\infty} (t^2 - 1)^{1/2} e^{-zt} \, dt, \quad z \gg 1 \]
\[ K_2(z) = \frac{z^2}{3} \int_1^{\infty} (t^2 - 1)^{3/2} e^{-zt} \, dt, \quad z \gg 1 \]

non-dimensional variables:
\[ y = \frac{E}{M} \]
\[ z = \frac{M}{T} \gg 1 \]

\[ \frac{d^3p}{(2\pi)^3} e^{-E/T} = \frac{4\pi}{(2\pi)^3} \int p^2 \, dp \, e^{-E/T} = \]
\[ \frac{4\pi}{(2\pi)^3} \int p \, dp \, dE \, e^{-E/T} = \]
\[ \frac{1}{2\pi^2} \int \sqrt{E^2 - M^2} \, dE \, e^{-E/T} = \]
\[ \frac{M^3}{2\pi^2} \int \sqrt{y^2 - 1} \, y e^{-zy} \, dy \]
\[ \frac{M^3 z}{6\pi^2} \int (y^2 - 1)^{3/2} e^{-zy} \, dy = \frac{M^3}{2\pi^2} \frac{K_2(z)}{z} \]

\[ \frac{d^3p}{(2\pi)^3} \frac{M}{E} e^{-E/T} = \frac{4\pi}{(2\pi)^3} \int p^2 \frac{M}{E} \, dp \, e^{-E/T} = \]
\[ \frac{4\pi M}{(2\pi)^3} \int p \, dp \, dE \, e^{-E/T} = \]
\[ \frac{M}{2\pi^2} \int \sqrt{E^2 - M^2} \, dE \, e^{-E/T} = \]
\[ \frac{M^3}{2\pi^2} \int \sqrt{y^2 - 1} \, e^{-zy} \, dy \]
\[ \frac{M^3 z}{2\pi^2} \int (y^2 - 1)^{1/2} e^{-zy} \, dy = \frac{M^3}{2\pi^2} \frac{K_1(z)}{z} \]
Using the approximation, valid for \( z \gg 1 \):

\[
K_1(z) \sim K_2(z) \sim \left( \frac{\pi}{2z} \right)^{1/2} \exp^{-z}
\]

the integral over temperature can be done analytically, yielding:

\[
\langle \sigma_{\text{ann}} v \rangle_{\text{res}} \simeq 4\pi^2 x_f \Gamma(H_1) \frac{B_\chi (1-B_\chi)}{\beta_\chi} \sqrt{\frac{\delta(\delta+1)}{2}} \left[ 1 - \text{erf} \left( \sqrt{2(\delta - 1)x_f} \right) \right]
\]

\[
\delta \equiv \frac{M_A}{2m_\chi}
\]

very sensitive to the neutralino mass
The relic abundance interval from observation

exponential dependence on \( m_\chi \)

asymmetric shape: thermal motion allows resonant annihilation for \( m_\chi < M_H/2 \), while this is not possible for \( m_\chi > M_H/2 \)
General expression of thermal average of WIMP annihilation cross section (Gondolo, Gelmini, NPB360(1991)145)

\[
\langle \sigma v_{\text{M\øller}} \rangle = \frac{\int \sigma v_{\text{M\øller}} \frac{dn^e_1}{dn^e_1} \frac{dn^e_2}{dn^e_2}}{\int \frac{dn^e_1}{dn^e_1} \frac{dn^e_2}{dn^e_2}} = \frac{\int \sigma v_{\text{M\øller}} e^{E_1/T} e^{E_2/T} d^3p_1 d^3p_2}{\int e^{E_1/T} e^{E_2/T} d^3p_1 d^3p_2}
\]

\[
\Rightarrow \quad \langle \sigma v_{\text{M\øller}} \rangle = \frac{1}{M^4TK^2_2(x)} \int_{4m^2}^{\infty} \sigma(s) \cdot (s - 4m^2) \sqrt{s}K_1(\sqrt{s}/T) \, ds
\]
thermal expansion (x=M/T>>1):

\[
\langle \sigma v_{\text{Møller}} \rangle = a^{(0)} + \frac{3}{2} a^{(1)} x^{-1} + \left[ \frac{9}{2} a^{(1)} + \frac{15}{8} a^{(2)} \right] x^{-2} \\
+ \left[ \frac{15}{16} a^{(1)} + \frac{195}{16} a^{(2)} + \frac{35}{16} a^{(3)} \right] x^{-3} + \left[ \frac{675}{32} a^{(2)} + \frac{735}{32} a^{(3)} + \frac{315}{128} a^{(4)} \right] x^{-4} + \mathcal{O}(x^{-5})
\]

\[
a^{(i)} \equiv \left. \frac{\partial (\sigma v_{\text{lab}})}{\partial \epsilon_n} \right|_{\epsilon=0} (\epsilon \equiv \frac{s - 4M^2}{4M^2}, \text{“available kinetic energy”})
\]

for a two-particle final state (most common case):

\[
\sigma v_{\text{lab}} = \frac{1}{64\pi^2 (s - 2M^2)} \beta_f \int d\Omega |\vec{M}|^2
\]

\[
\beta_f = \left[ 1 - \frac{m_3 + m_4}{s} \right]^{1/2} \left[ 1 - \frac{m_3 - m_4}{s} \right]^{1/2}
\]
Coannihilation [Griest, Seckel, PRD43(1991)3191]

• should be the norm – evolution of all coupled species should be taken into account to determine WIMP density evolution and decoupling
• all particles carrying the quantum number preventing the DM WIMP decay (R-parity, KK-parity, T-parity etc) should eventually contribute to the density of the lightest one
• however, usually at the time of the DM WIMP decoupling the density of all other heavier exotic species is exponentially suppressed and they play no role
• exception when some mass degeneracy occurs between the DM candidate and some heavier exotic particle
In this case Lee-Weinberg’s formula can be easily generalized to account for coannihilation (X,Y=SM particles)

\[
\frac{d n_i}{dt} + 3H n_i = - \sum_{j=1}^{N} \left[ \langle \sigma_{ij} v_{ij} \rangle (n_i n_j - n_i^{eq} n_j^{eq}) - \sum_{j \neq i} \left[ \langle \sigma'_{Xij} v_{ij} \rangle (n_i n_X - n_i^{eq} n_X^{eq}) - \langle \sigma'_{Xji} v_{ij} \rangle (n_j n_X - n_j^{eq} n_X^{eq}) \right] - \sum_{j \neq i} \left[ \Gamma_{ij} (n_i - n_i^{eq}) - \Gamma_{ji} (n_j - n_j^{eq}) \right] \right].
\]

- the first term describes $\chi_i \chi_j$ (I \(\neq\) j) annihilation, whose total annihilation cross section is:

\[
\sigma_{ij} = \sum_X \sigma(\chi_i \chi_j \rightarrow X)
\]

- the second term describe $\chi_i$ to $\chi_j$ conversions by scattering off the cosmic thermal background, with inclusive scattering cross section:

\[
\sigma'_{Xij} = \sum_Y \sigma(\chi_i X \rightarrow \chi_j Y)
\]
In this case Lee-Weinberg’s formula can be easily generalized to account for coannihilation \((X,Y=\text{SM particles})\)

\[
\frac{dn_i}{dt} + 3Hn_i = -\sum_{j=1}^{N} \langle \sigma_{ij} v_{ij} \rangle \left( n_i n_j - n_i^{eq} n_j^{eq} \right)
- \sum_{j \neq i} \left[ \langle \sigma'_{Xij} v_{ij} \rangle \left( n_i n_X - n_i^{eq} n_X^{eq} \right) - \langle \sigma'_{Xji} v_{ij} \rangle \left( n_j n_X - n_j^{eq} n_X^{eq} \right) \right]
- \sum_{j \neq i} \left[ \Gamma_{ij} \left( n_i - n_i^{eq} \right) - \Gamma_{ji} \left( n_j - n_j^{eq} \right) \right].
\]

- the last term describes \(\chi_i\) decays, with inclusive rate:

\[
\Gamma_{ij} = \sum_{X} \Gamma(\chi_i \rightarrow \chi_j X)
\]

- \(v_{ij}\) is the relative velocity of particles i and j, described as

\[
v_{ij} = \frac{\sqrt{(p_i \cdot p_j)^2 - m_i^2 m_j^2}}{E_i E_j}
\]
In this case Lee-Weinberg’s formula can be easily generalized to account for coannihilation (X,Y=SM particles)

\[
\frac{dn_i}{dt} + 3Hn_i = - \sum_{j=1}^{N} \langle \sigma_{ij} v_{ij} \rangle \left( n_i n_j - n_i^{eq} n_j^{eq} \right) \\
- \sum_{j \neq i} \left[ \langle \sigma'_{X,ij} v_{ij} \rangle \left( n_i n_X - n_i^{eq} n_X^{eq} \right) - \langle \sigma'_{X,j} v_{ij} \rangle \left( n_j n_X - n_j^{eq} n_X^{eq} \right) \right] \\
- \sum_{j \neq i} \left[ \Gamma_{ij} \left( n_i - n_i^{eq} \right) - \Gamma_{ji} \left( n_j - n_j^{eq} \right) \right].
\]

- \( n_i^{eq} \) is as usual the equilibrium density of particle \( i \), given by:

\[
n_i^{eq} = \frac{g_i}{(2\pi)^3} \int d^3 p \, f_i
\]

- the decay rate of particles \( j \neq i \) are usually much faster than the age of the Universe. This implies that they all eventually decay to the lightest one, so that the abundance of the latter is given by:

\[
n = \sum_{i=1}^{N} n_i
\]
In this case Lee-Weinberg’s formula can be easily generalized to account for coannihilation (X,Y=SM particles)

\[
\frac{dn}{dt} = -3Hn - \sum_{i,j=1}^{N} \langle \sigma_{ij} v_{ij} \rangle \left( n_i n_j - n_{i\text{eq}} n_{j\text{eq}} \right)
\]

• summing-up over \( i \) the expression simplifies! (last two terms cancel because scattering and decays don’t change the overall conserved quantum number that prevents DM decay)

• note that the scattering rate of exotic particles off particles in the thermal background is much faster than their annihilation rate (cross sections are of the same order, but background particle densities are much larger than each of the exotic particle densities when the former are relativistic and the latter are non-relativistic and so suppressed by a Boltzmann factor. In this case, the \( \chi_i \) distributions remain in thermal equilibrium and in particular:

\[
\frac{n_i}{n} \approx \frac{n_{i\text{eq}}}{n_{\text{eq}}}
\]
In this case Lee-Weinberg’s formula can be easily generalized to account for coannihilation \((X,Y=\text{SM particles})\)

\[
\frac{dn}{dt} = -3Hn - \langle \sigma_{\text{eff}} v \rangle \left( n^2 - n_{\text{eq}}^2 \right)
\]

- using the last condition the equation simplifies further! It maintains the “usual” form without coannihilation provided that:

\[
\langle \sigma_{\text{eff}} v \rangle = \sum_{ij} \langle \sigma_{ij} v_{ij} \rangle \frac{n_{i,\text{eq}}}{n_{\text{eq}}} \frac{n_{j,\text{eq}}}{n_{\text{eq}}}
\]

\(m_j/T >> 1\) (all \(n_j\) exponentially suppressed)

N.B.: when \(\Delta_{j_1}/T = (m_j - m_1)/T >> 1\) (with \(j>1\)) then \(n_j/n_1 \sim \exp(-\Delta_{j_1}/T) << 1\) and only the first term in the sum gives a non-negligible contribution → usual non-coannihilating case

when \(\Delta_{j_1}/T = (m_j - m_1)/T << 1\) (with \(j>1\)) then \(n_j/n_1 \sim 1\) and the \(j^{\text{th}}\) term in the sum gives a non-negligible contribution
when $n_j/n_1 \sim 1$ COANNIHILATION is possible, and the effective annihilation cross section can be much different than the one containing only the WIMP processes

two possibilities:

- $\sigma_{1j} > \sigma_{11}$ the total relic density is *suppressed*
- $\sigma_{1j} < \sigma_{11}$ “bottle-neck” in $j$ annihilation – when $j$ particles eventually decay they provide an additional contribution to the relic density $\rightarrow$ the relic density is *enhanced*
Eventually, it turns out that the general expression of $\langle \sigma_{\text{eff}} v \rangle$ can be written in a compact form which is reminiscent of the non-coannihilating case [Edsjo, Gondolo, PRD56(1997)1879]

$$\langle \sigma_{\text{eff}} v \rangle = \int_0^\infty dp_{\text{eff}} p_{\text{eff}}^2 W_{\text{eff}} K_1 \left( \frac{\sqrt{s}}{T} \right) \frac{m_1^4 T \left[ \sum_i g_i \frac{m_i^2}{g_1^2} K_2 \left( \frac{m_i}{T} \right) \right]^2}{m_1^4 T \left[ \sum_i g_i \frac{m_i^2}{g_1^2} K_2 \left( \frac{m_i}{T} \right) \right]^2}$$

\[ W_{\text{eff}} = \sum_{ij} \frac{p_{ij}}{p_{11}} \frac{g_i g_j}{g_1^2} W_{ij} = \sum_{ij} \sqrt{\frac{[s - (m_i - m_j)^2][s - (m_i + m_j)^2]}{s(s - 4m_1^2)}} \frac{g_i g_j}{g_1^2} W_{ij} \]

\[ W_{ij} = 4p_{ij} \sqrt{s} \sigma_{ij} = 4\sigma_{ij} \sqrt{(p_i \cdot p_j)^2 - m_i^2 m_j^2} = 4E_i E_j \sigma_{ij} v_{ij} \]

\[ p_{\text{eff}} = p_{11} = \frac{1}{2} \sqrt{s - 4m_1^2} \]

\[ p_{ij} = \frac{[s - (m_i + m_j)^2]^{1/2}}{2\sqrt{s}} \left[ \frac{s - (m_i - m_j)^2}{2\sqrt{s}} \right]^{1/2} \]

Remember, input from theory are the $\sigma_{ij} = \sum_X \sigma(\chi_i \chi_j \rightarrow X)$!
So Dark Matter seems to be naturally related to New Physics at the TeV scale
Hierarchy problem:

Higgs mass expected to be below a few TeV (on general grounds, perturbativity of the theory) radiative corrections to the Higgs boson of the Standard Model $\rightarrow$ loop of the type:

$$\int d^4P \left[ \frac{1}{(P - m_f)(P + K - m_f)} \right]$$

for a Higgs of momentum $K$. Quadratically divergent for large $P$ independently of $K$: $\delta m_H^2 \sim \lambda^2$, where $\lambda$ is the scale beyond which the low-energy theory no longer applies ($\lambda$=cut-off of the SM)

N.B.: technically, not SM’s business - the quadratic divergence is independent on the momentum of the Higgs and may be subtracted off

However the problem arises when embedding the SM in a more general theory: in this case $\delta m_H^2 \sim a \lambda^2$ is cancelled by new contribution $\delta m'_H^2 \sim b \lambda^2$ in such a way that $(a-b) \lambda^2 \sim$ TeV scale – a huge cancellation unless $\lambda \sim$ TeV itself
The bottom line

new physics at the TeV scale is cool because it kills two birds with one stone:*  
1. solves the hierarchy problem  
2. explains the Dark Matter

in *split susy* (Arkani-Hamed, Dimopoulos, 2004) not necessarily in this order of priority!
LEP’S COSMOLOGICAL LEGACY

- Simple solution: impose a discrete parity, so all interactions require pairs of new particles. This also makes the lightest new particle stable.
  
  Cheng, Low (2003); Wudka (2003)

- LEP’s Cosmological Legacy:
  LEP constraints ↔ Discrete symmetry ↔ Stability

- Dark matter is easier to explain than no dark matter
- The WIMP paradigm is more natural

WIMP signal: missing energy+new particles produced in pairs
What WIMP?
Never run short of candidates…
(Incomplete) List of DM candidates

- Neutrinos
- Axions
- Lightest Supersymmetric particle (LSP) – neutralino, sneutrino, axino
- Lightest Kaluza-Klein Particle (LKP)
- Heavy photon in Little Higgs Models
- Solitons (Q-balls, B-balls)
- Black Hole remnants
- Hidden-sector tecnipions
- ...
most popular thermal WIMP candidates from particle physics (solve hierarchy problem: $M_W/M_{Pl} \sim 10^{-16}$)

- conserved symmetry
- DM candidate

- susy *
  - R-parity
  - $\chi$ (neutralino)

- extra dimensions
  - K-parity
  - $B^{(1)}$ (KK photon)

- little Higgs
  - T-parity
  - $B_H$ (heavy photon)

all thermal candidates, massive, with weak-type interactions (WIMPs)

the most popular – see K. Olive’s lectures
$<\sigma_{\text{ann}} v> \sim a + b/x$:

- $a \neq 0$ : “s-wave” annihilator
- $a = 0$ : “p-wave” annihilator

Ranges of $\sigma_{\text{ann}}$ that can provide the correct thermal relic density

Neutralino = Majorana $\rightarrow$ s-wave suppression $(m_f/M_w)^2$ for $\chi\chi \rightarrow ff$

... + coannihilations with other particle(s) close in mass + resonant annihilations
Caveat: non-standard cosmological scenarios may change the usual picture!

- low reheating temperature
  [Fornengo, Riotto, Scopel, PRD67,023514; Gelmini, Gondolo, PRD74,023510]

- inflaton $\phi$ reheats the Universe with $T_{RH} < T_f$
  - $n = \eta \left(\frac{m_\phi}{100 \text{ TeV}}\right)$ DM particles per $\phi$ decay are produced
  - as long as $\Omega_X^{\text{standard}} > 10^{-5}$ ($100$ GeV/$m_X$) appropriate choice of $T_{RH}$ and $\eta$ provides the correct relic density

- different expansion history (kination)
  [Kamionkowski, Turner, PRD42,3310; Salati, PLB571,121]