Direct Detection: Theoretical Analysis

Looking at the particle nature of Dark Matter and how much information we can hope to gain from Direct Detection.

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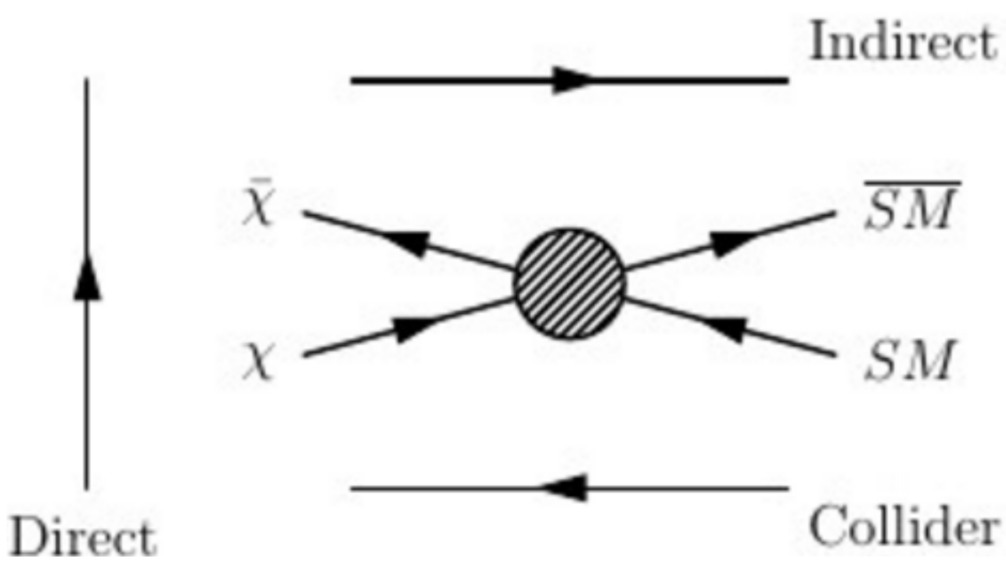
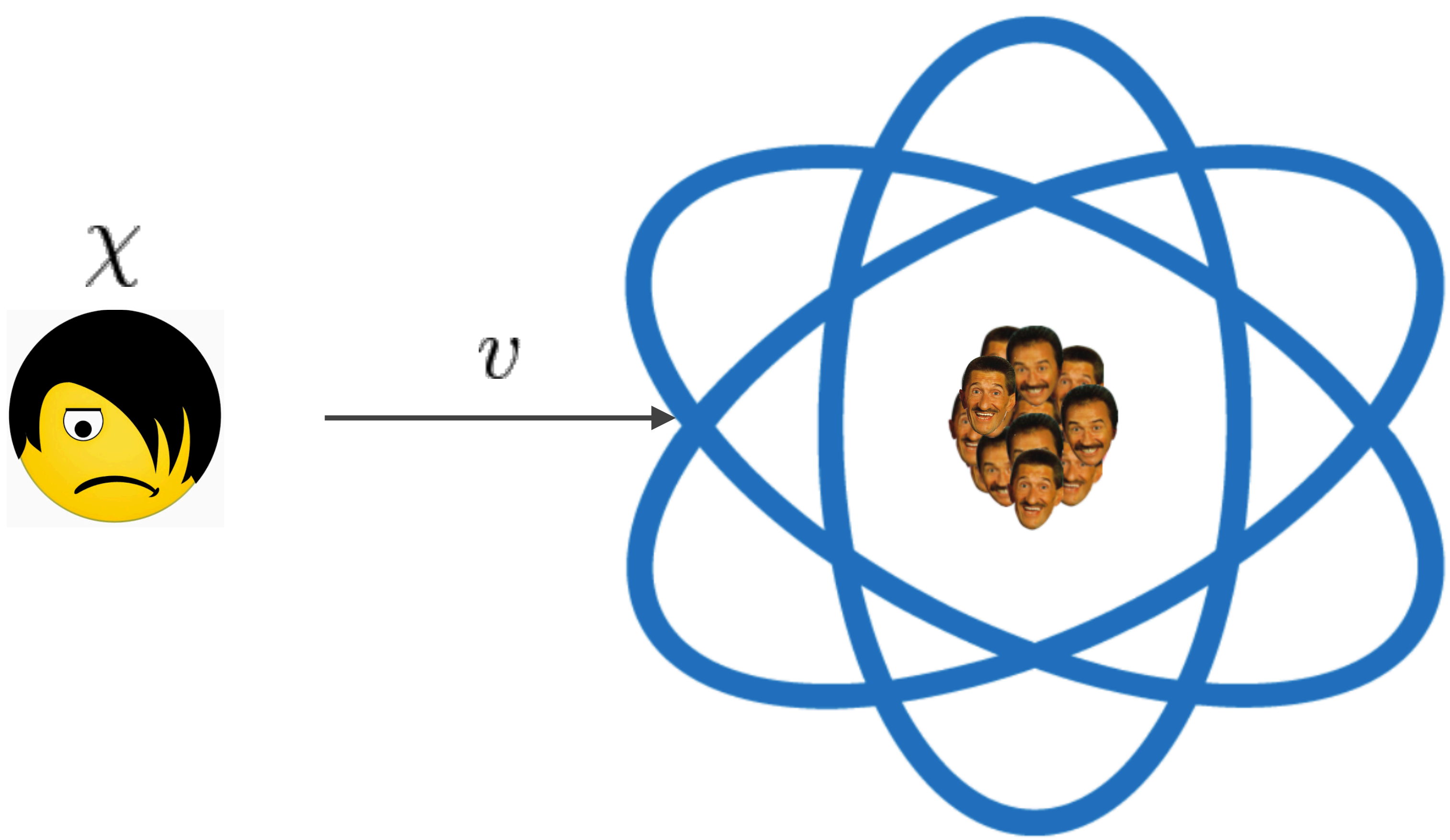
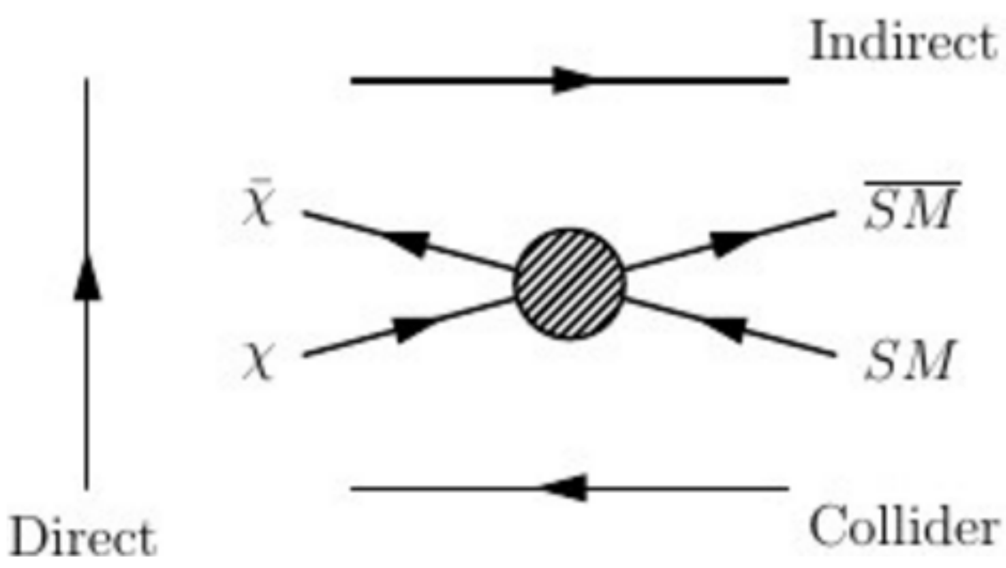
Direct Detection (DD) of Dark Matter (DM)



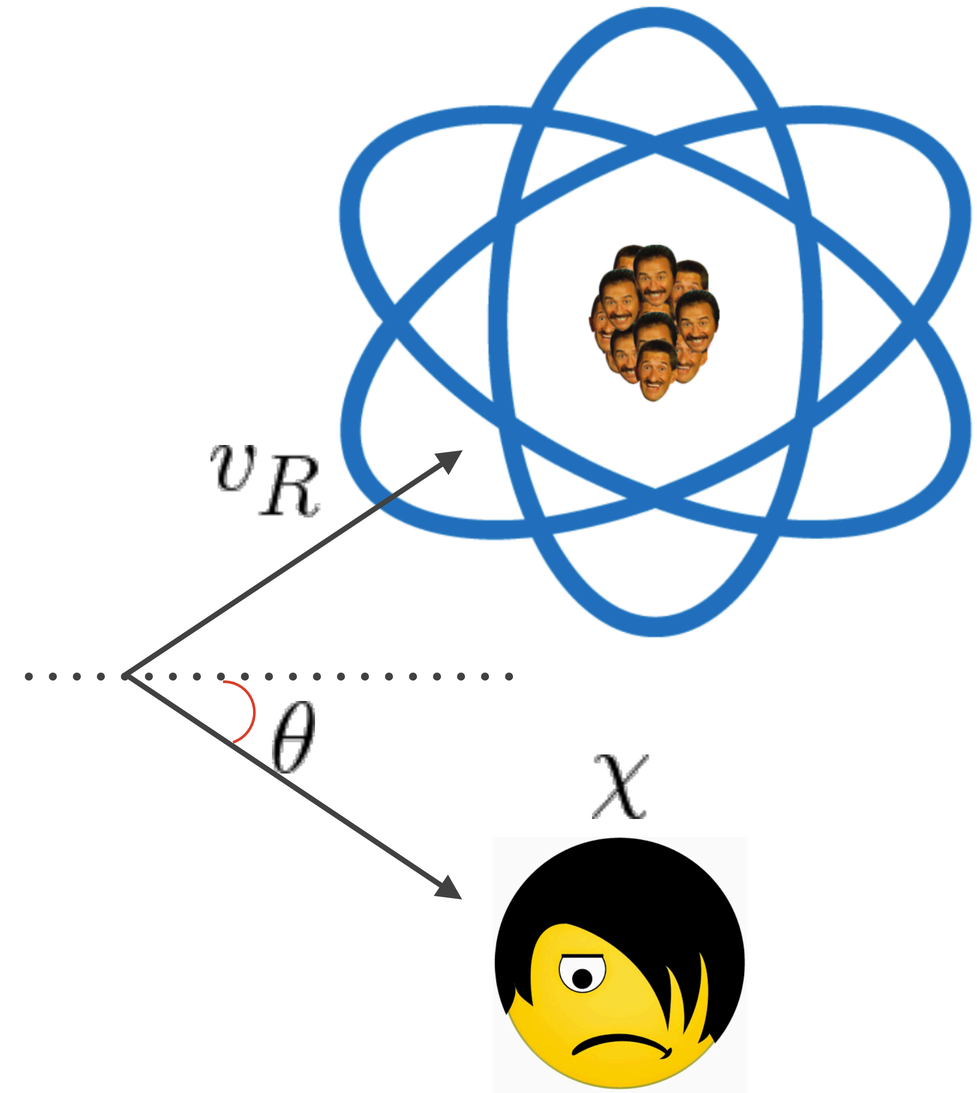
- Incoming DM particles from the galactic halo collide with SM nuclei.



Direct Detection of Dark Matter



- DD experiments count these collisions by measuring the recoiling nuclei.



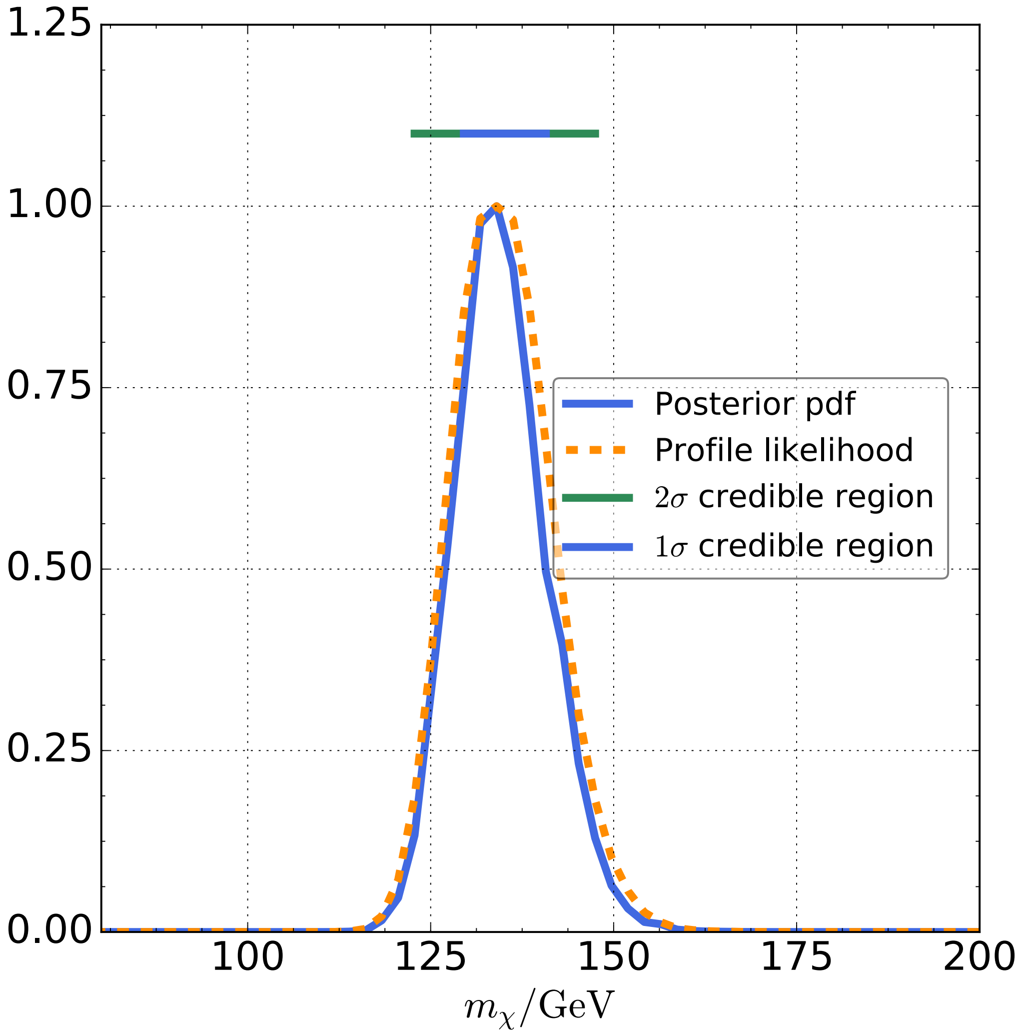
- When taking into account the kinematics and the local number density of DM, DD is most sensitive to WIMP DM.

Spin-Independent (SI) & Spin-Dependent (SD) Responses

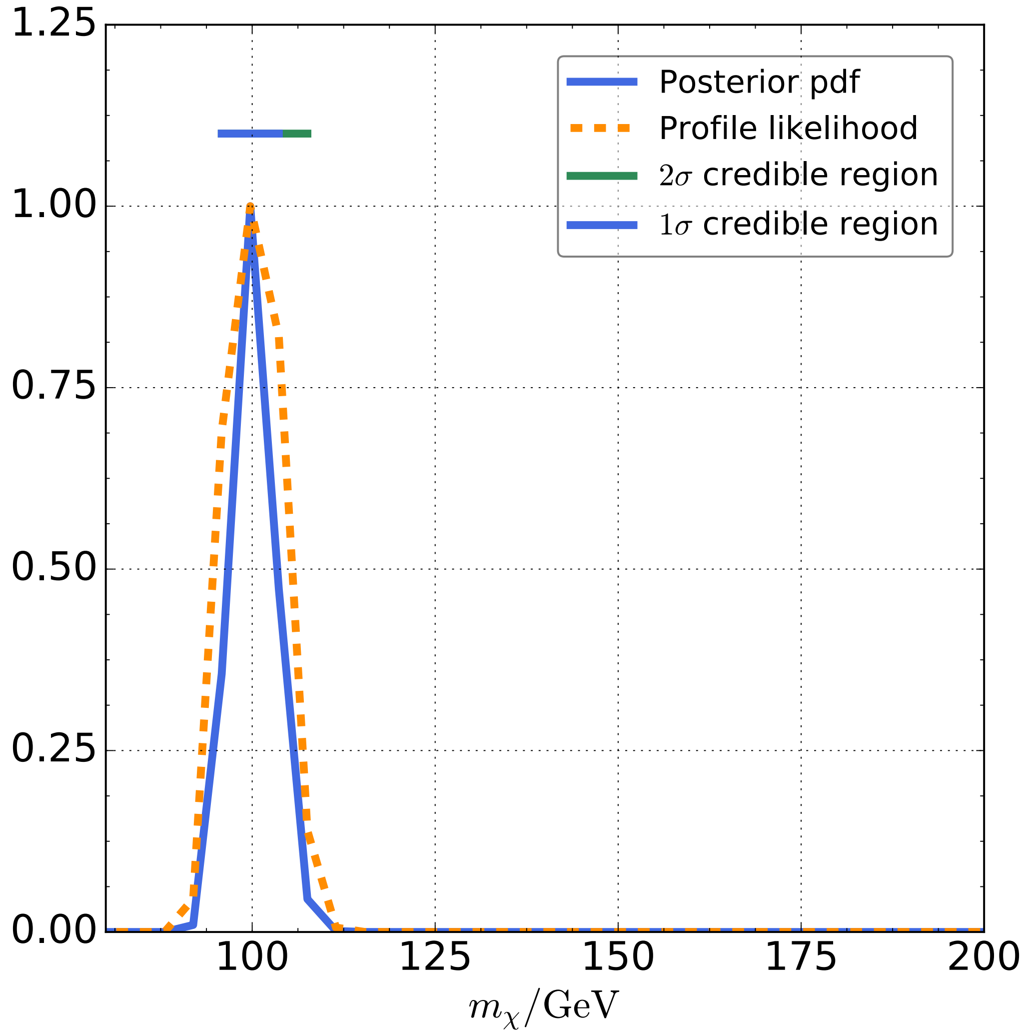
- There are many well motivated WIMP models, can DD help us distinguish between them?
- Often people assume DM matter couples to nuclei through ordinary spin-independent or spin-dependent interactions.
- More exotic situations are well motivated; interactions via a moment (magnetic dipole or anapole), pseudoscalar mediated interactions and composite DM give novel nuclear responses.
- Ignorance to the broader range of DM-nuclei interactions could reconstruct parameters incorrectly. See Gresham et al arXiv 1401.3739 for details.

Reconstructing for the wrong model

Spin Independent reconstruction



Anapole reconstruction



A Non-Relativistic Effective Theory Approach

- We can be more general by building an effective theory for DD, outlined in Fitzpatrick et al [arXiv 1203.3542]
- By taking the relevant Galilean invariant operators

$$\frac{i\vec{q}}{m_N}, \quad \vec{v}^{\perp} \equiv \vec{v} + \frac{\vec{q}}{2m_N}, \quad \vec{S}_\chi, \quad \vec{S}_N$$

- And combining them

$$\boxed{\begin{aligned} O_1 &= 1_\chi 1_N & O_7 &= \vec{S}_N \cdot \vec{v}^{\perp} \\ O_3 &= i\vec{S}_N \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^{\perp}\right) & O_8 &= \vec{S}_\chi \cdot \vec{v}^{\perp} \\ O_4 &= \vec{S}_\chi \cdot \vec{S}_N & O_9 &= i\vec{S}_\chi \cdot \left(\vec{S}_N \times \frac{\vec{q}}{m_N}\right) \\ O_5 &= i\vec{S}_\chi \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^{\perp}\right) & O_{10} &= i\vec{S}_N \cdot \frac{\vec{q}}{m_N} \\ O_6 &= \left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right) & O_{11} &= i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N} \end{aligned}}$$

A Non-Relativistic Effective Theory Approach

- These operators form a basis for all non-relativistic four-field interactions

$$\mathcal{L}_{\rm int} = \chi \mathcal{O}_\chi \chi N \mathcal{O}_N N \;=\; \sum_{N=n,p} \sum_i c_i^{(N)} \mathcal{O}_i \chi^+ \chi^- N^+ N^-$$

- The interactions to all orders of momentum can be encoded in a Form-Factor

$$c_0 \mathcal{O} + c_2 q^2 \mathcal{O} + c_4 q^4 \mathcal{O} + \ldots \;\equiv\; F_{\mathcal{O}}\left(\frac{q^2}{\Lambda^2}\right) \mathcal{O}$$

- There are only six distinct responses from Nuclei

M	Σ''	Σ'	Δ	Φ''	$\tilde{\Phi}'$
SI	**SD longitudinal**	**SD transverse**	**LD (ang.-mom.-dependent)**	**LSD (spin-orbit)**	**Tensor LSD**

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SI	**SD longitudinal**	**SD transverse**	**LD (ang.-mom.-dependent)**	**LSD (spin-orbit)**
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$\boxed{1_N}$	$\boxed{\vec{S}_N \cdot \frac{\vec{q}}{m_N}}$	$\boxed{\vec{S}_N \times \frac{\vec{q}}{m_N}}$	$\boxed{\vec{S}_\chi \cdot \vec{v}^\perp}$	$\boxed{i\vec{S}_N \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp\right)}$

SI	$\mathcal{O}_1 = 1_\chi 1_N$		$\mathcal{O}_7 = \vec{S}_N \cdot \vec{v}^\perp$
	$\mathcal{O}_3 = i\vec{S}_N \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp\right)$		$\mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^\perp$
SD	$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N$		$\mathcal{O}_9 = i\vec{S}_\chi \cdot \left(\vec{S}_N \times \frac{\vec{q}}{m_N}\right)$
	$\mathcal{O}_5 = i\vec{S}_\chi \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp\right)$		$\mathcal{O}_{10} = i\vec{S}_N \cdot \frac{\vec{q}}{m_N}$
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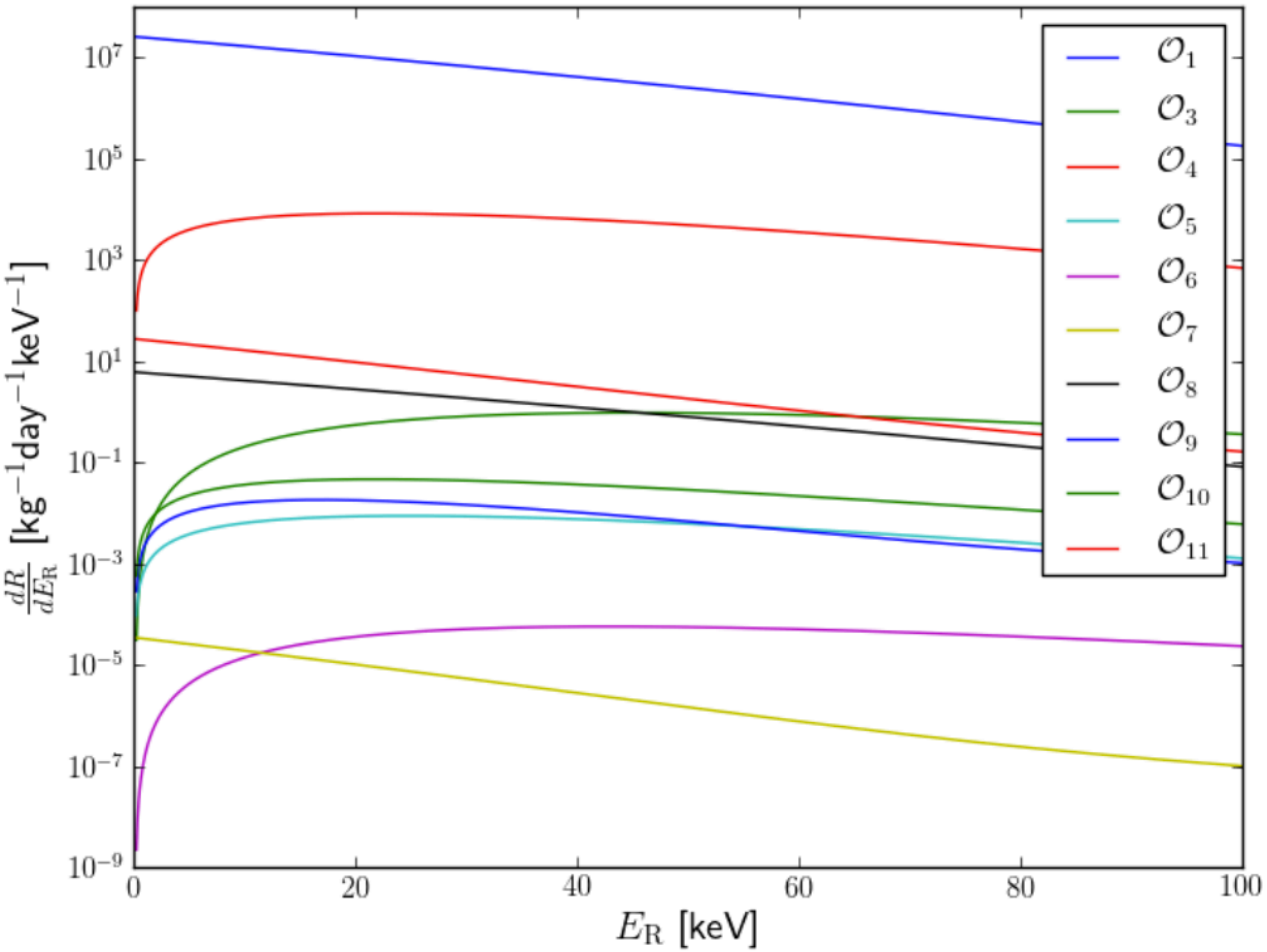
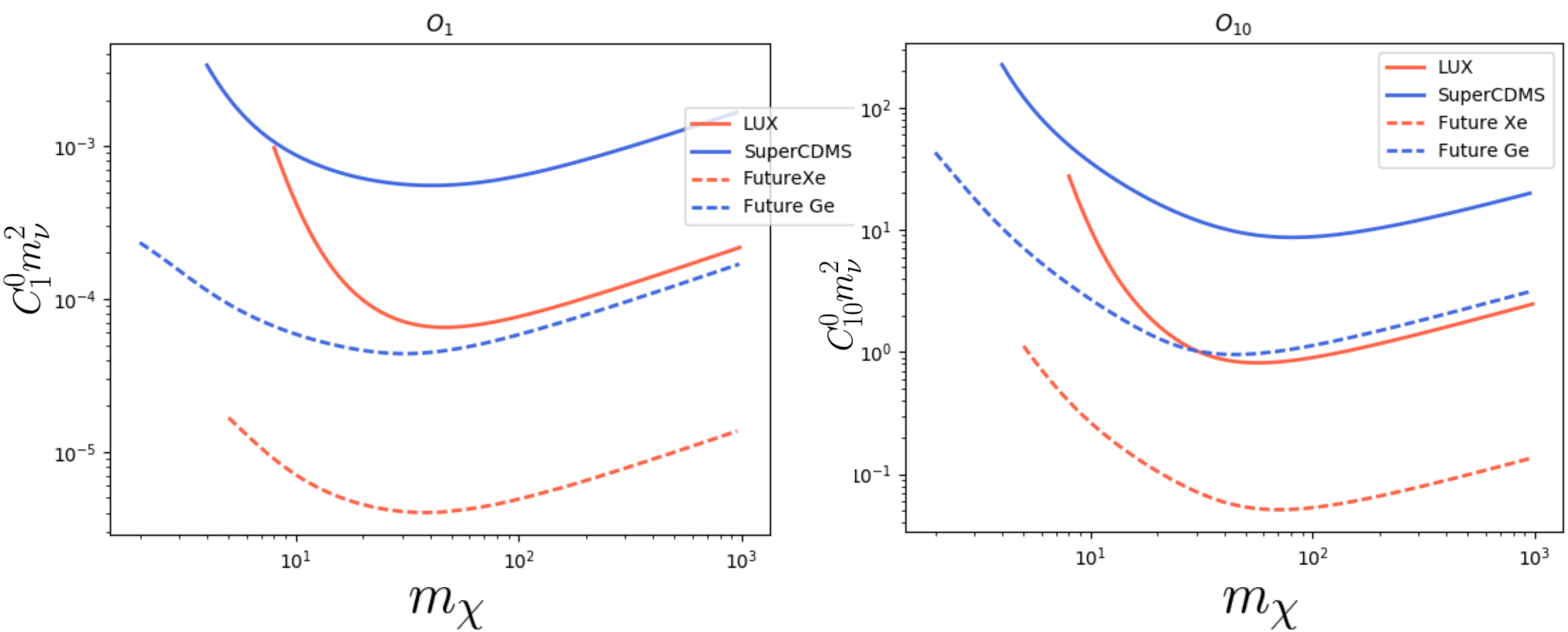
Operator Hierarchy

- Setting each operator coupling to 1 gives a strong hierarchy.
- In general these couplings can be independent



A reason to limit yourself

- Without a positive signal, exclusion limits are weakest when you consider just one operator at a time.

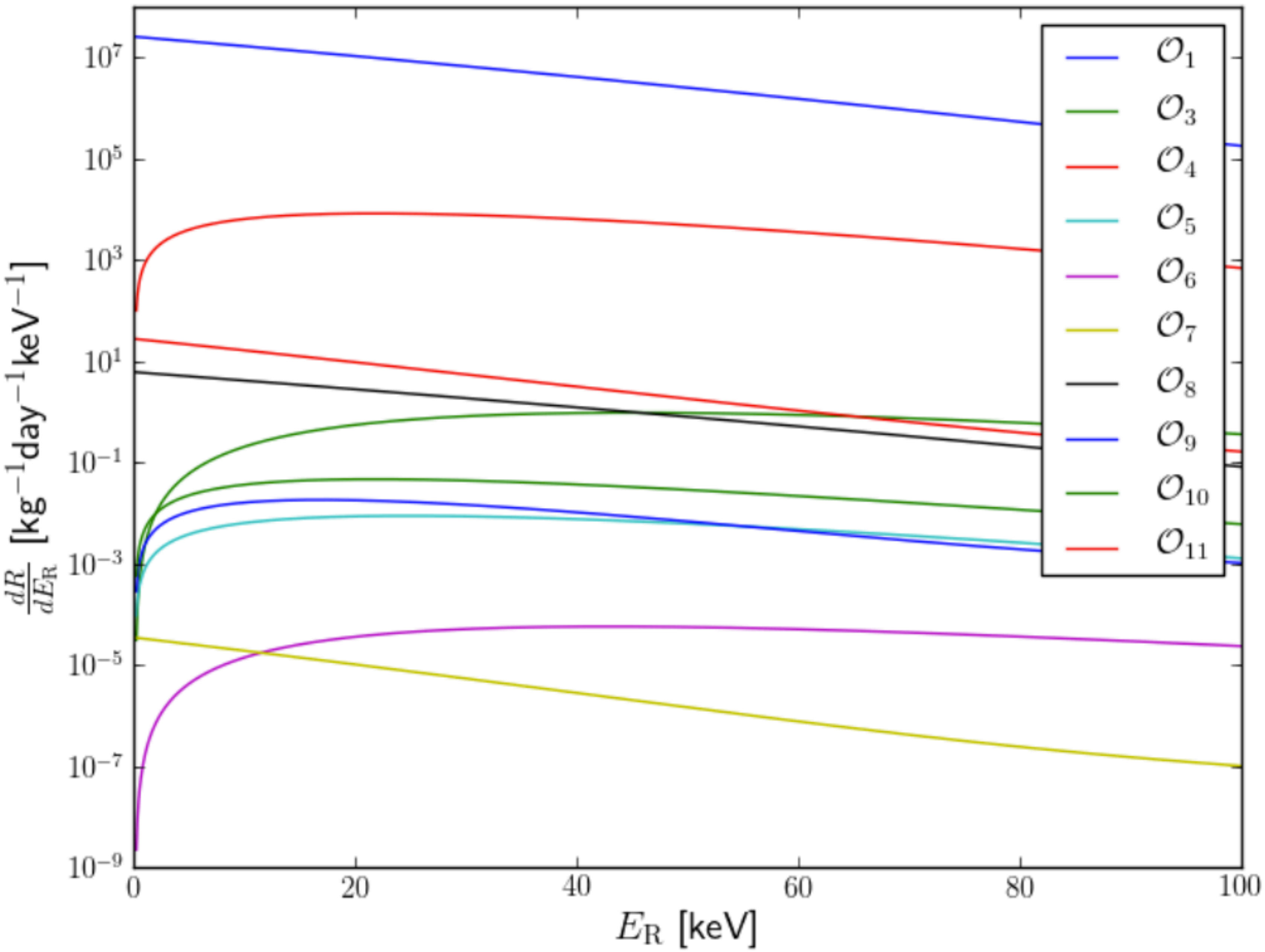


A reason to free yourself

- The coupling structure of DM and nuclei could be complicated, with interferences and cancellations. An analysis that accommodates all WIMP models is desired.
- Ensure we don't miss unconventional DM signatures in experiments.
- Determine DM parameters coupling strength and coupling structures.

With a great many parameters, comes great degeneracy

- When couplings are independent, degeneracies naturally appear.



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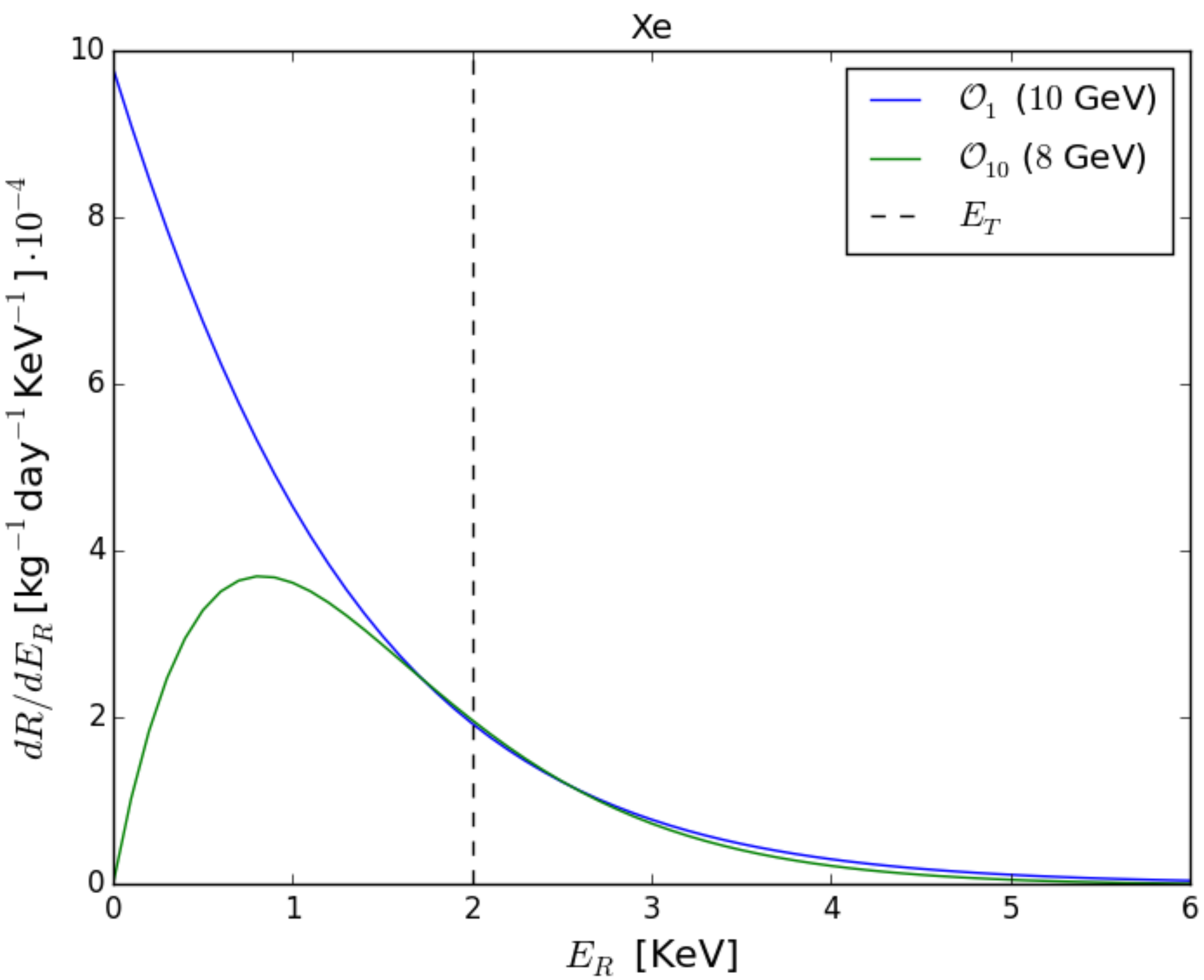
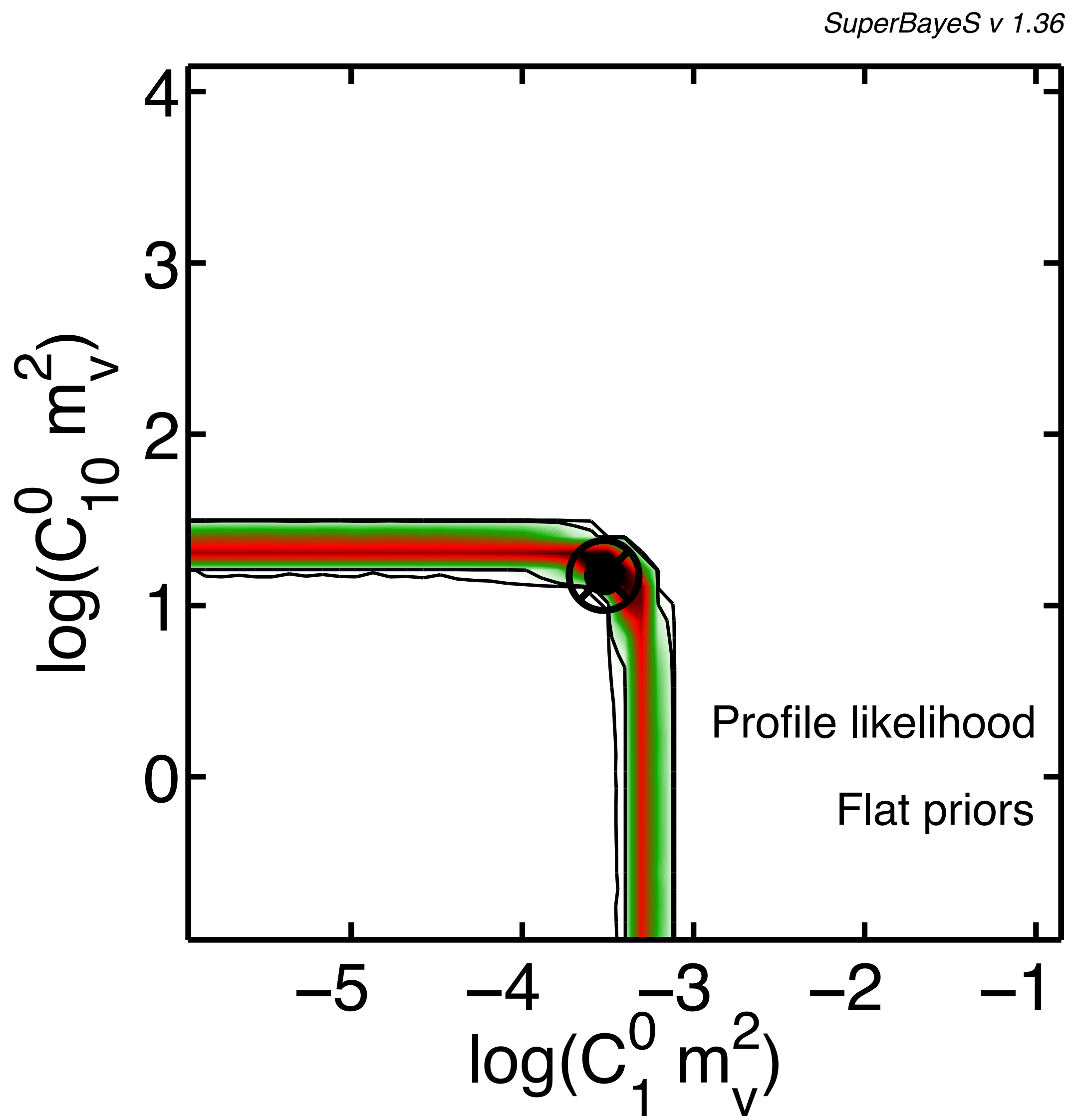
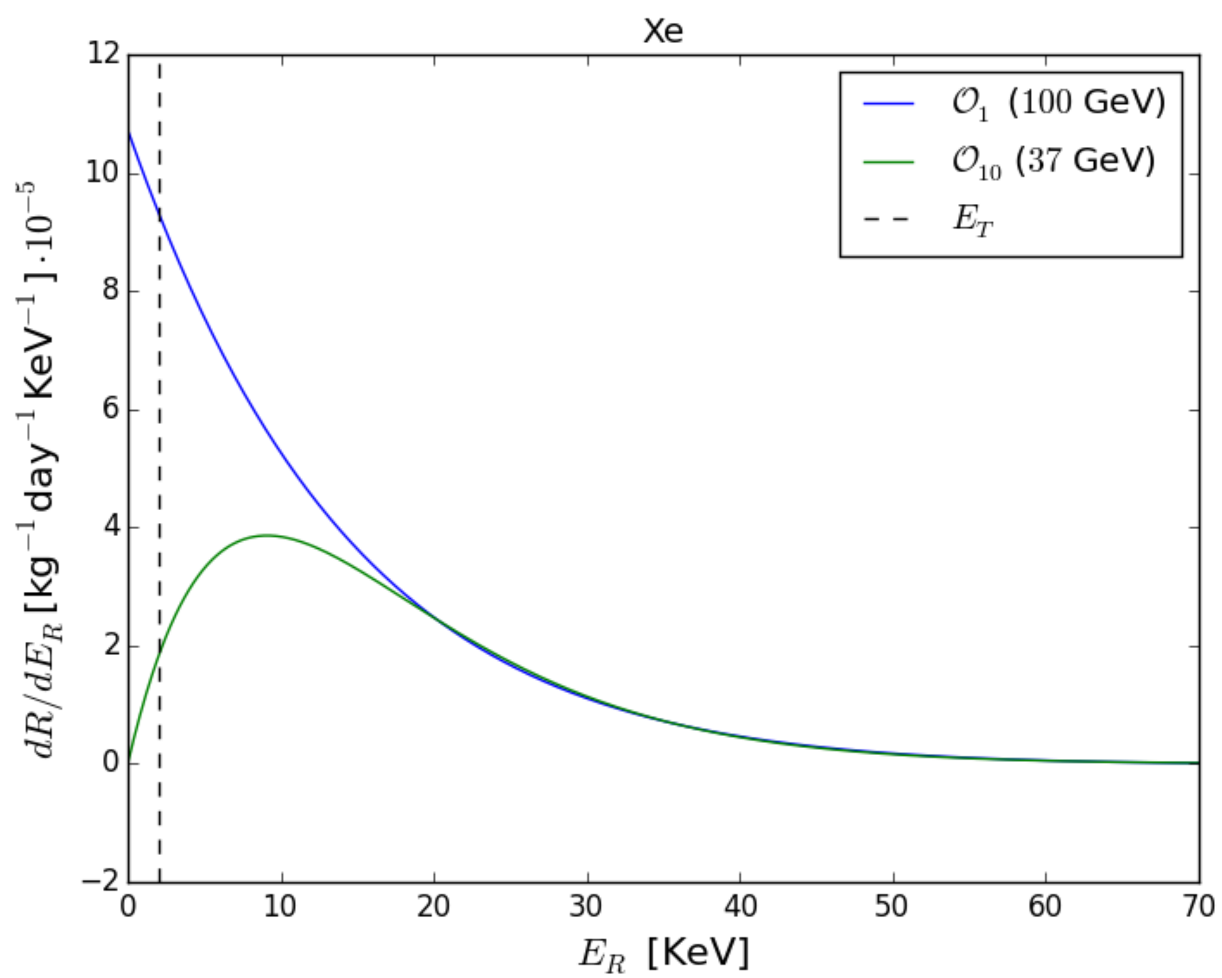




How good can we get?

- In order to constrain the operators we consider, one requires the number of constraints (experiments with different targets) to be greater than the number of free parameters (Operator couplings).
- However, analysing the spectrum can help isolate particular operators.



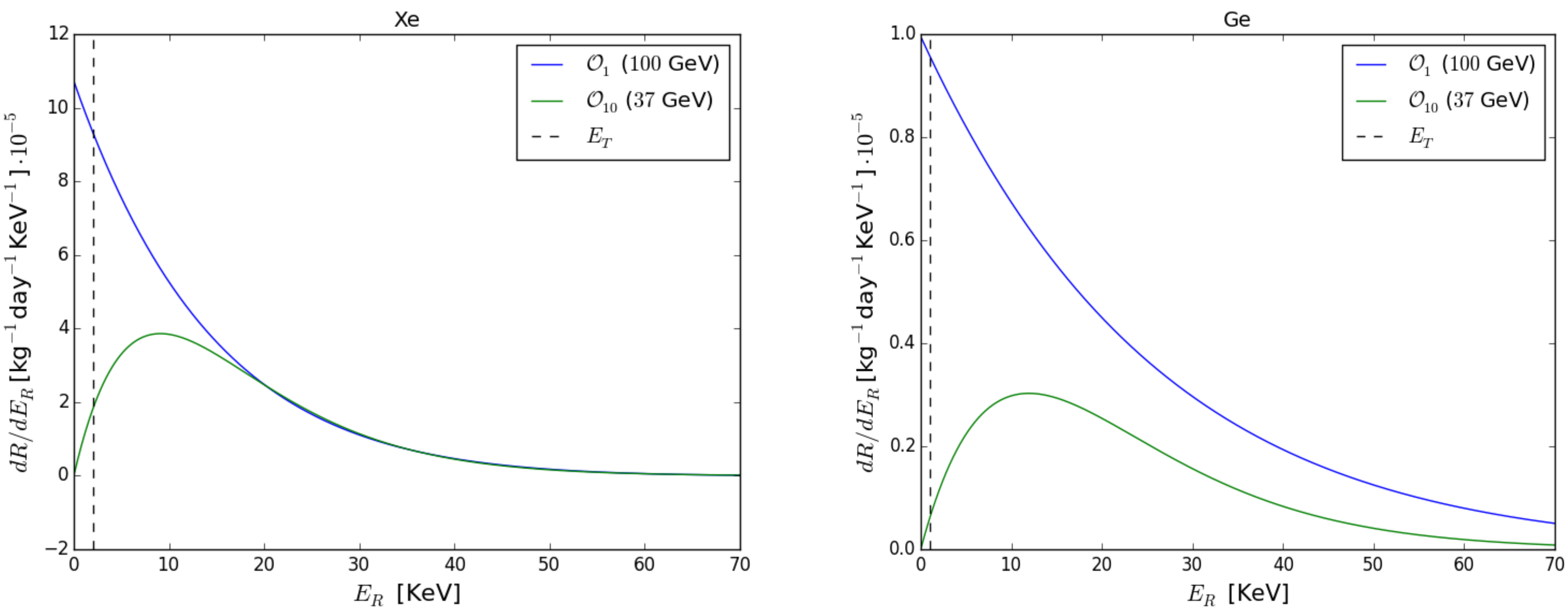
The importance of multiple target materials

- The minimum velocity for a given recoil energy is

Halo dependent

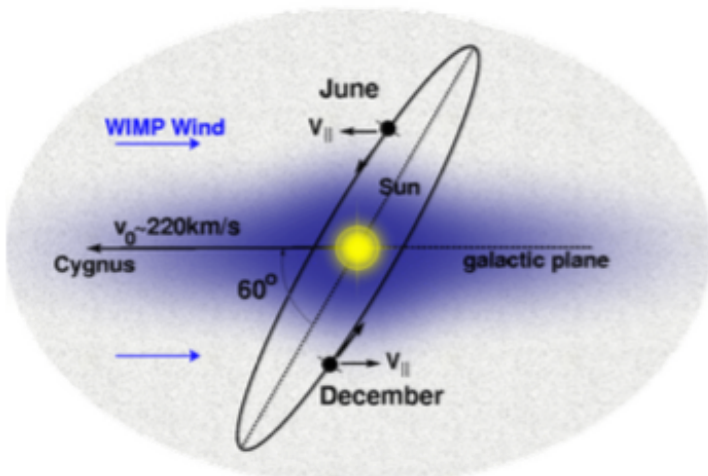
$$\frac{v_{\min}}{c} = \sqrt{\frac{1}{2m_N E_{\mathrm{R}}}}\left(\frac{m_N E_{\mathrm{R}}}{\mu_N}\right)$$

Target dependent



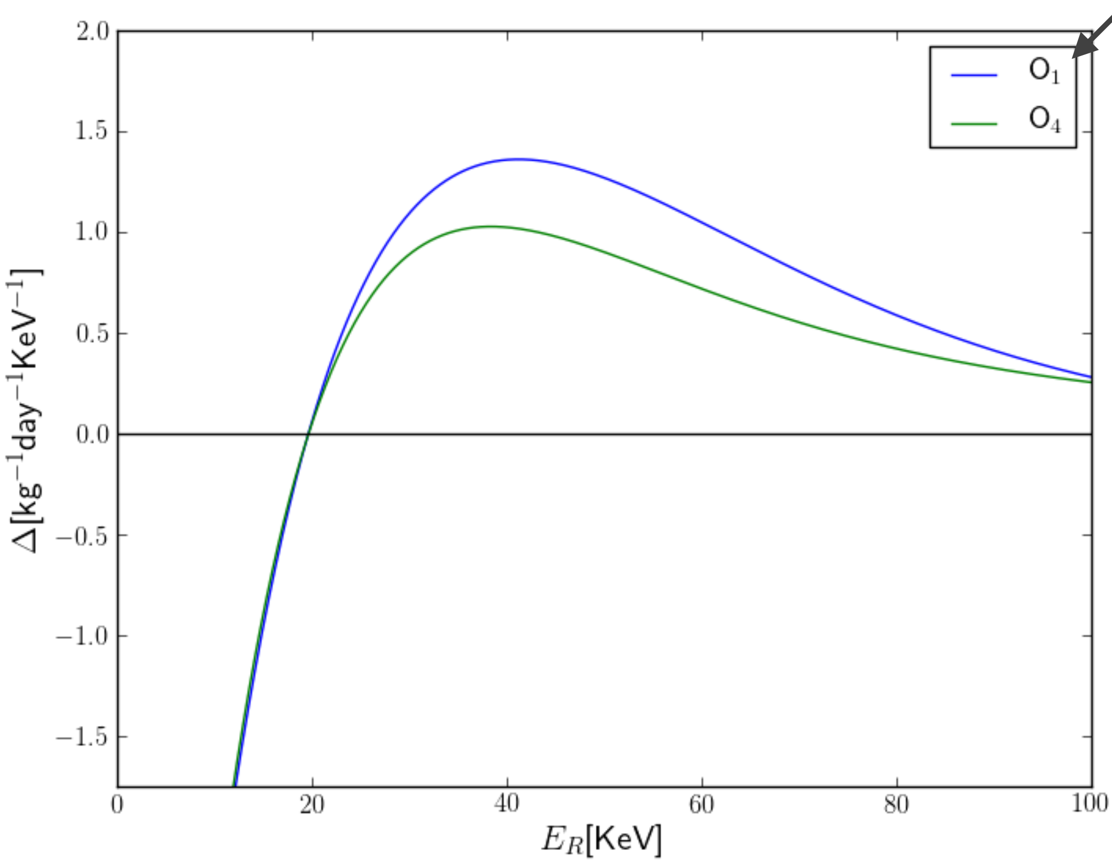
Annual Modulation

- With time, Annual Modulation will also be important to distinguish between sub-sets of operators.

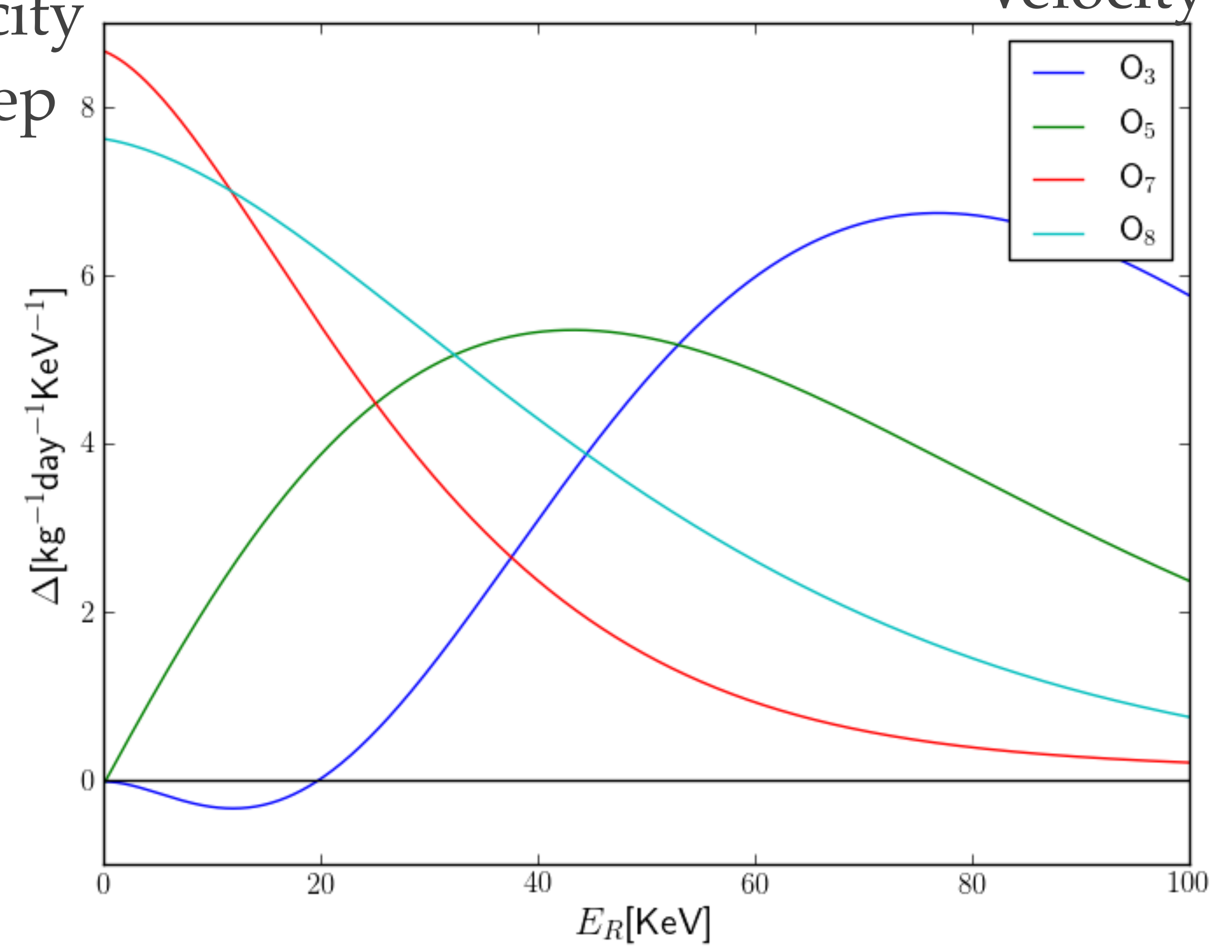


$$\Delta \approx \frac{1}{2}\left(\frac{dR}{dE_R}\Big|_{June} - \frac{dR}{dE_R}\Big|_{Dec}\right)$$

Velocity Indep



Velocity dep



Conclusion

- In order to fully appreciate what DD is telling us, a NREFT analysis should be made (have been made).
- Despite the large parameter space, signals of different operators can be distinguished, advantage complementarity between different target materials and annual modulation.
- When connecting with theoretical motivations and other detection technologies, a full appreciation for the properties of DM is possible.

Backup slides

EFT: The "New" Differential Rate

$$\frac{dR}{dE_R} = \frac{\rho_0}{m_\chi m_A} \int_{v>v_{min}} v f(\vec{v}) \frac{d\sigma}{dE_R} d^3v$$

- **The differential cross section is:**

$$\frac{d\sigma}{dE_R} = \frac{m_A}{2\pi v^2} |\mathcal{M}|^2 \text{ , where } |\mathcal{M}|^2 \equiv \frac{m_A^2}{m_N^2} \sum_{i,j} \sum_{N,N'=n,p} c_i^N c_j^{N'} F_{i,j}^{(N,N')}$$

- **c_i, c_j are the WIMP-nucleon coupling constants for operators O_i and O_j**
- **$F_{i,j}$ is a form factor that contains all the particle and nuclear physics:**

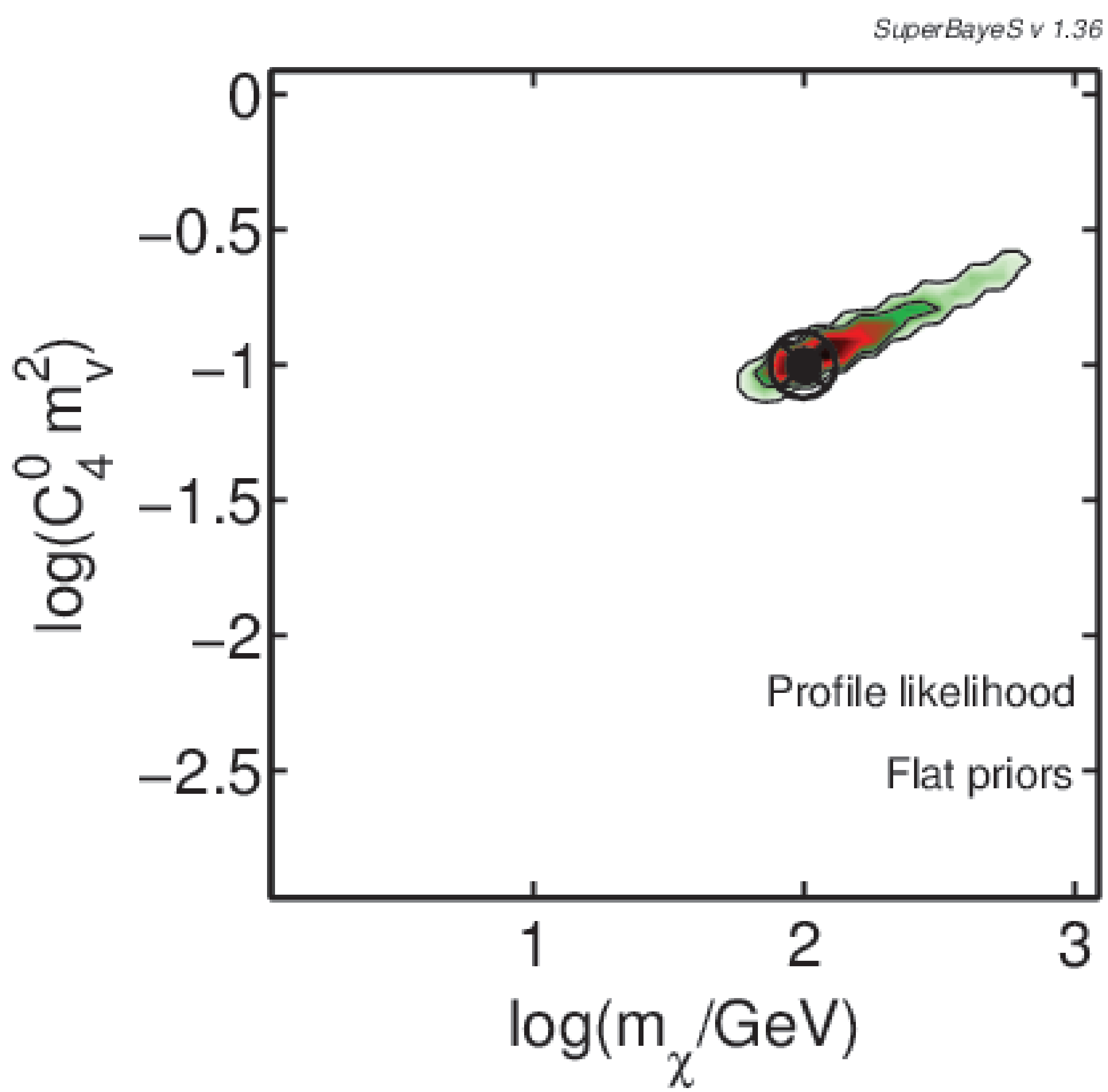
$$\frac{dR}{dE_R} = \frac{\rho_0}{32\pi m_\chi^3 m_N^2} \int_{v>v_{min}} \frac{f(\vec{v})}{v} \sum_{i,j} \sum_{N,N'=n,p} c_i^N c_j^{N'} F_{i,j}^{(N,N')}$$

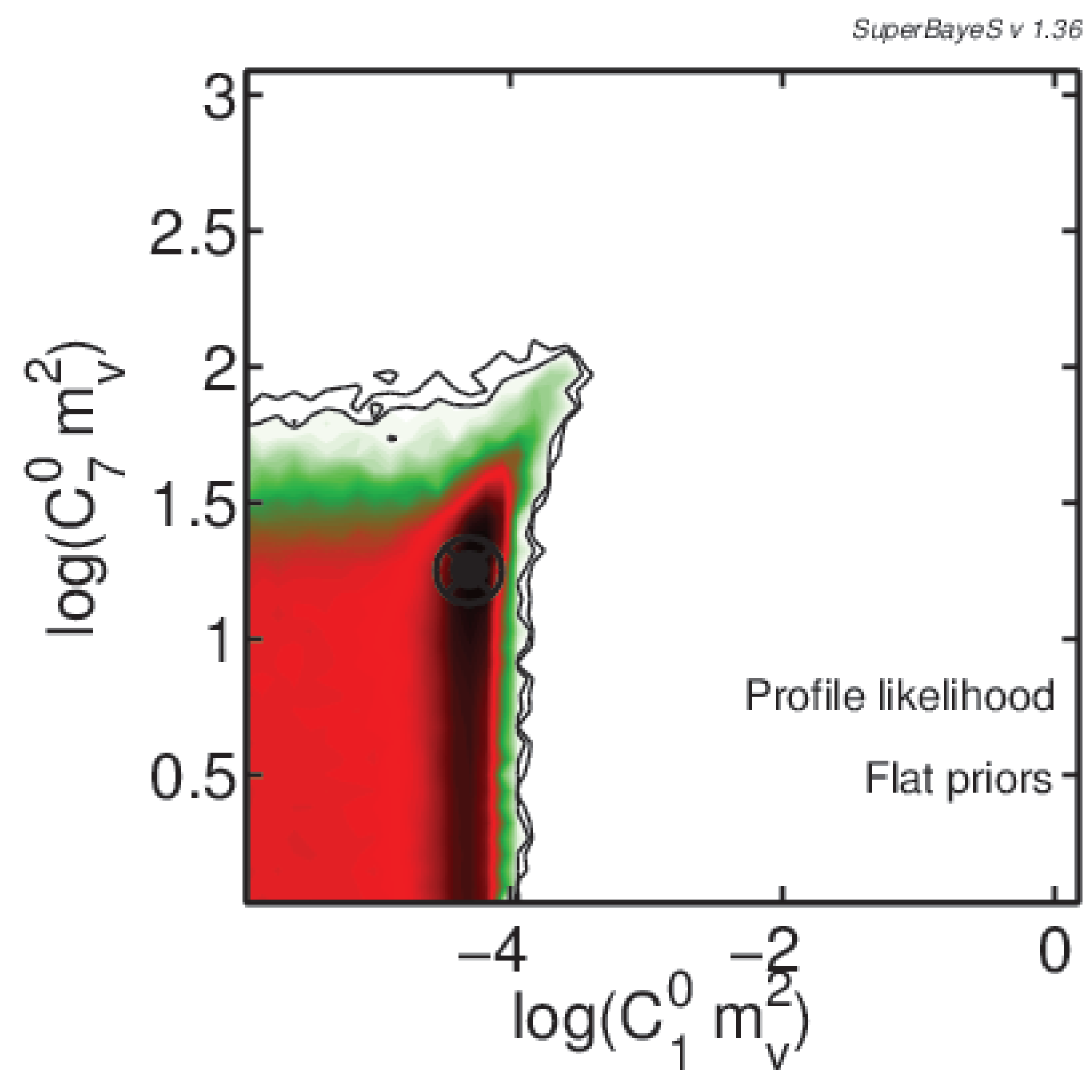
- **Each $F_{i,j}$ is a linear combination of nuclear response functions F_k:**

$$F_{i,j}^{(N,N')} = \sum_{k=M,\Sigma'',\Sigma',\Delta,\Phi'',\tilde{\Phi}'} a_{ijk} F_k^{(N,N')}$$

5D parameter reconstructions

- Performing a 5 dimensional analysis for simulated data of three idealised future detectors (Ge, F and Xe).





Including halo uncertainties

- The situation gets even worse if you wanted to be more general and include uncertainties in the halo model.
- Here we adopted a generalised halo model and allowed the four additional parameters to vary.

$$f(v) = N_k(e^{v^2/kv_0^2} - e^{-v_{esc}/kv_0^2})^k$$